DATA-DRIVEN PARAMETRIC MODELS FOR GENERATIVE DESIGN AND RESPONSE INFERENCE

Presented by:

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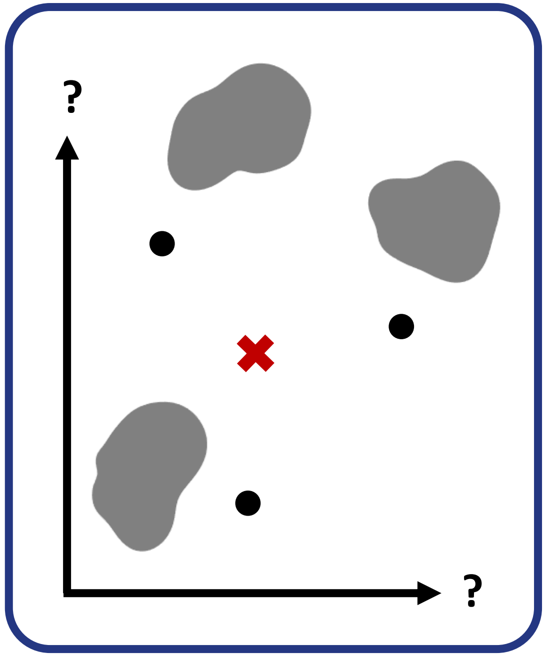
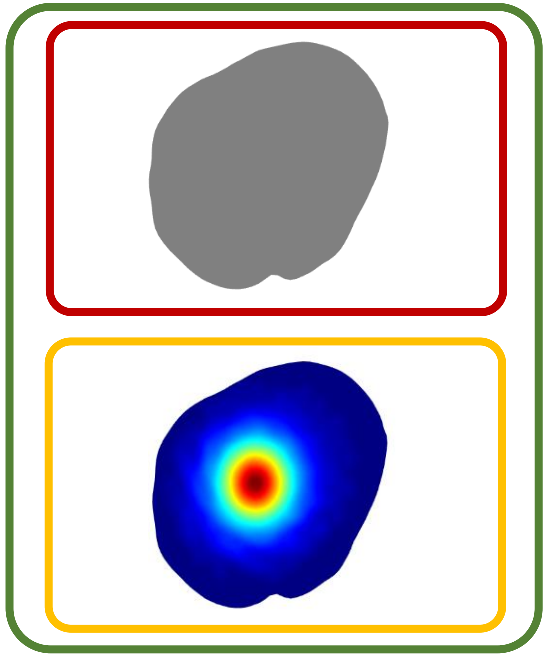
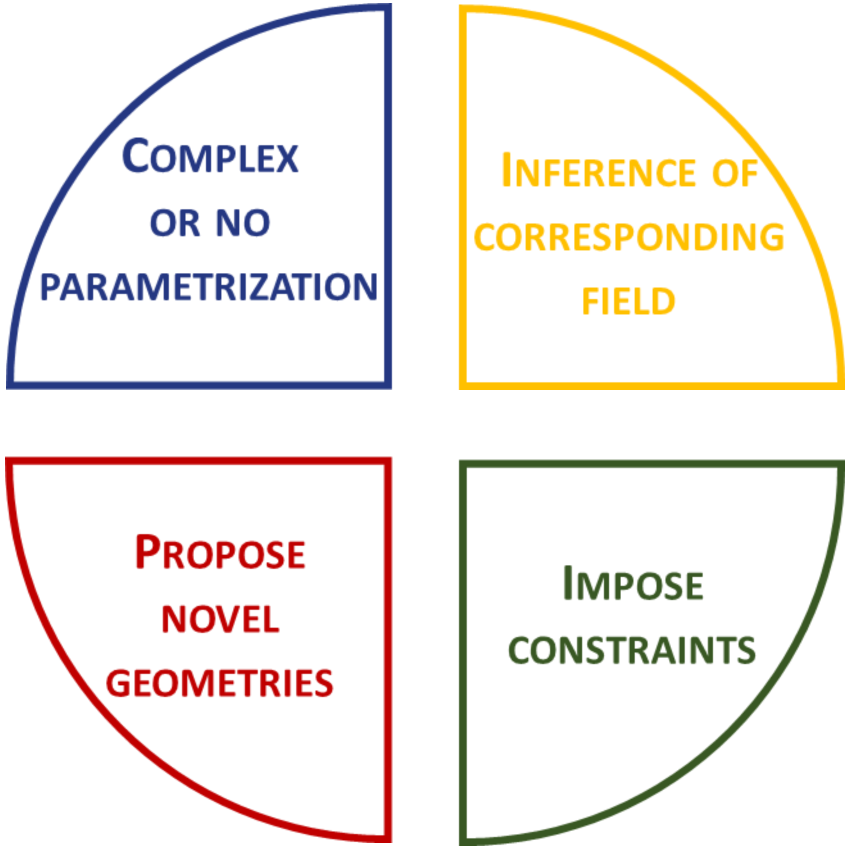
ARTS ET MÉTIERS ParisTech

10/06/2026

Generative Design: real-time parametric solutions for engineering problems across new geometrical domains to explore and refine engineering designs in order they meet predefined constraints.







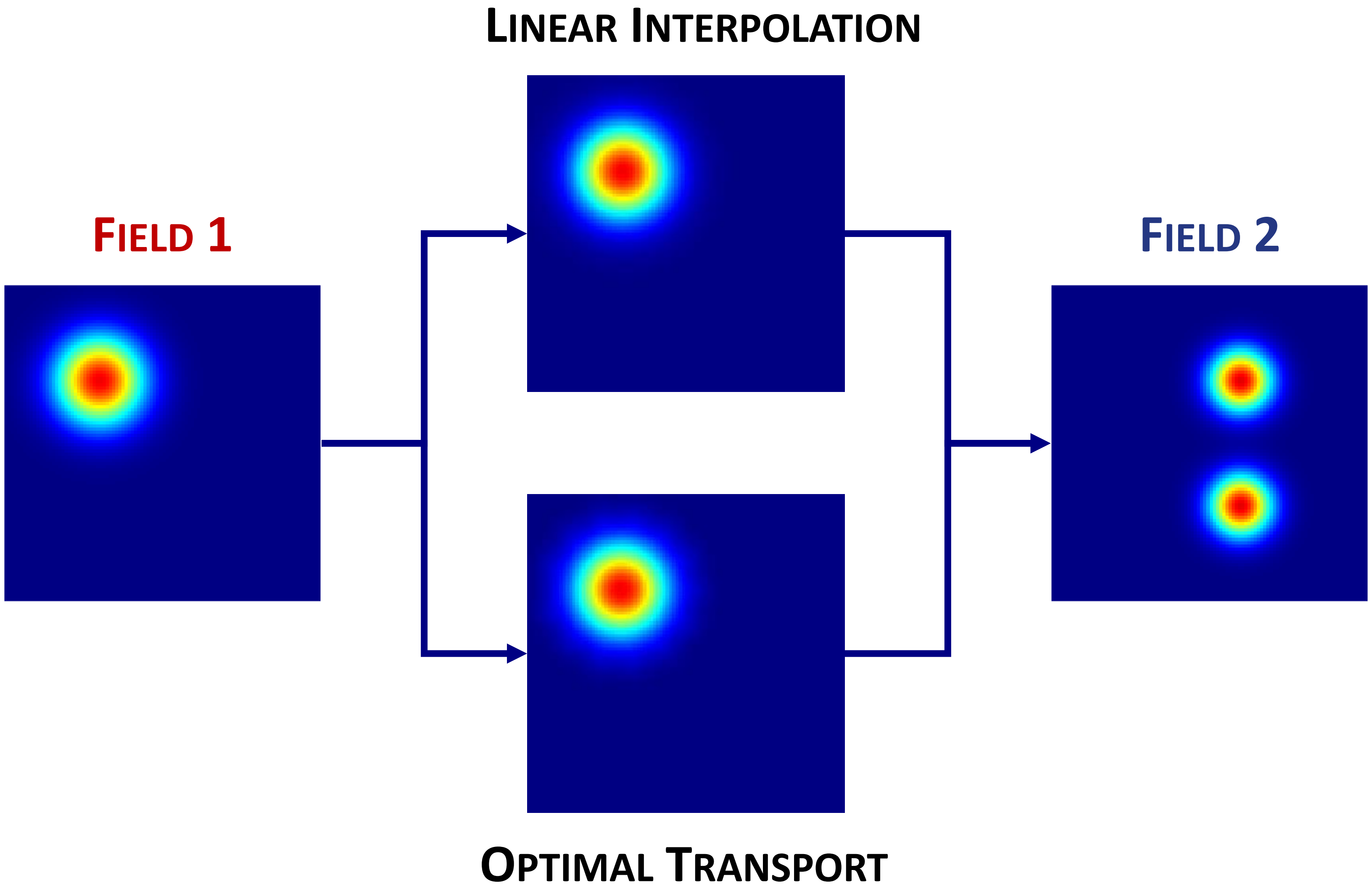
DATA ACCESS

- **Optimal Transport**
- **Rank Reduction Autoencoders**

1 REVISITING OPTIMAL TRANSPORT

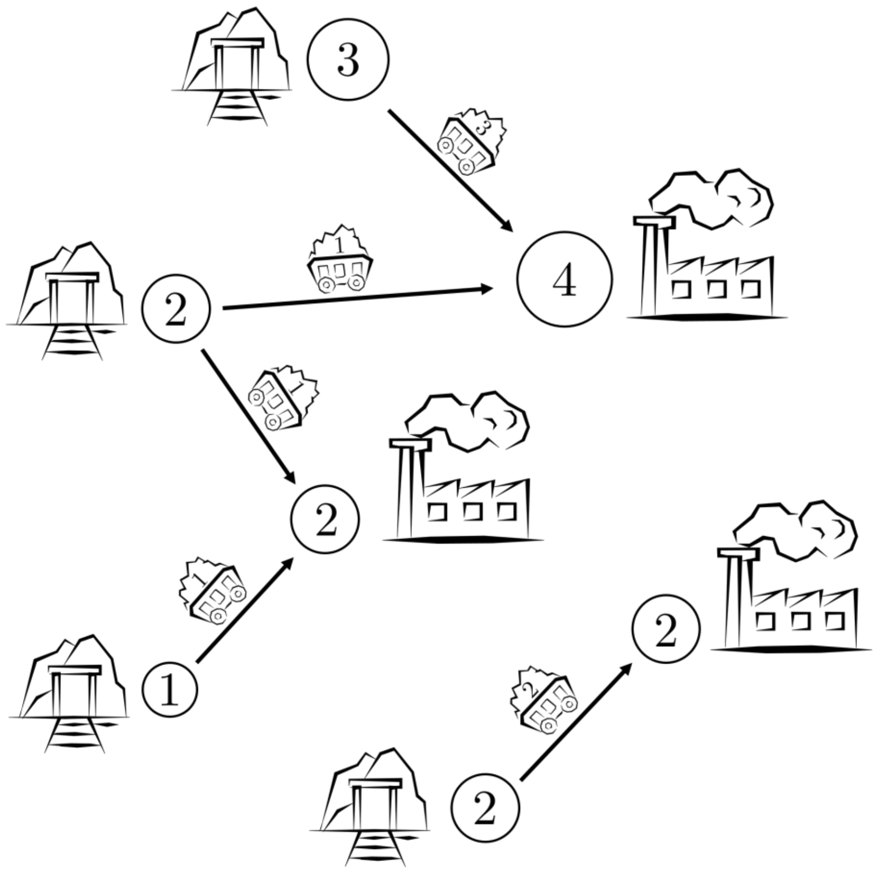
S. Torregrosa, V. Champaney, A. Ammar, D. Munoz, H. Navarro, F. Chinesta, C. Farhat

2 INDUSTRIAL USE-CASES


LINEAR INTERPOLATION
FIELD 1
FIELD 2
OPTIMAL TRANSPORT

- **Kantorovich Discrete Problem:** Each mine n ∈ $[\![N]\!]$, located at x_n, produces a quantity $\boldsymbol{a}_n$ of resource and each factory m ∈ $[\![M]\!]$, located at y_m, consumes $\boldsymbol{b}_m$. We look for the optimal transport plan $\boldsymbol{P} \in \mathbb{R}_+^{\boldsymbol{n}\times\boldsymbol{m}}$ from the distribution of mines to the distribution of factories minimizing the cost:

$$\boldsymbol{C}_{x_n,y_m} = \|x_n - y_m\|_2^2$$



- Transport plan $\boldsymbol{P} \in \mathbb{R}_+^{\boldsymbol{n}\times\boldsymbol{m}}$ where $P_{i,j}$ describes the amount of resource flowing from x_i toward y_j:

$$\min_{\boldsymbol{P}\in \boldsymbol{U}(\boldsymbol{a},\boldsymbol{b})} \sum_{i,j} \mathbf{C}_{i,j} \boldsymbol{P}_{i,j}$$

- Mass conservation:

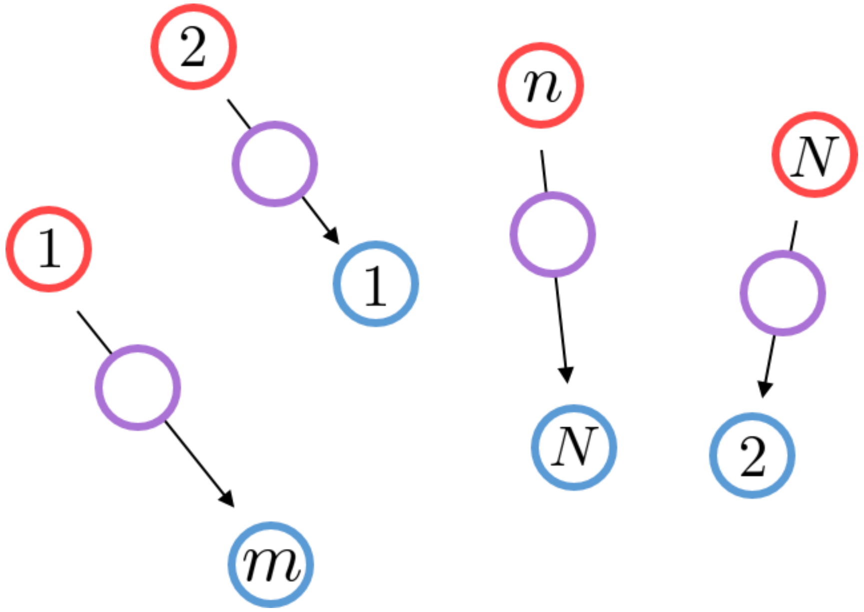
$$\boldsymbol{U}(\boldsymbol{a},\boldsymbol{b}) = \left\{ \boldsymbol{P} \in \mathbb{R}_+^{n\times m} : \sum_j \boldsymbol{P}_{i,j} = \boldsymbol{a} \text{ and } \sum_i \boldsymbol{P}_{i,j} = \boldsymbol{b} \right\}$$

- The problem can be simplified into an optimal matching problem between two particle clouds with the same number of points where each point is represented by its coordinates :

$$\min_T \sum_{n=1}^{N} \|x_n - T(x_n)\|_2^2$$

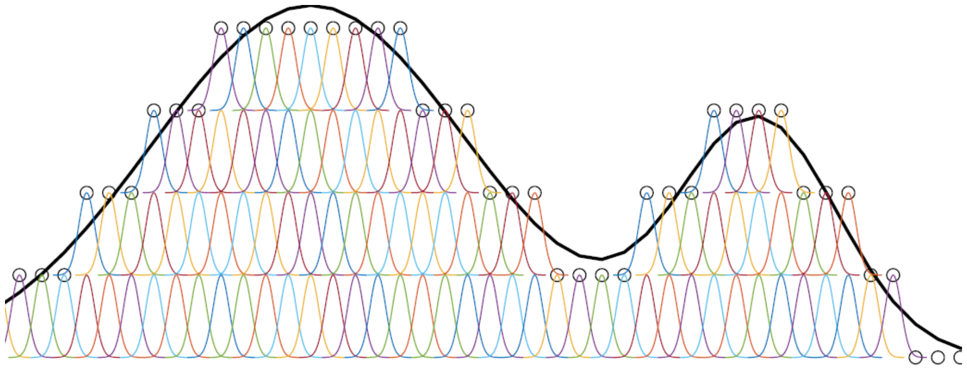
where the transport map T is a bijection:

$$T : \{x_1, \ldots, x_N\} \rightarrow \{y_1, \ldots, y_M\}$$



- Once the matching has been found, it is possible to **interpolate between the two clouds** by partially displacing all the particles over the corresponding segments.

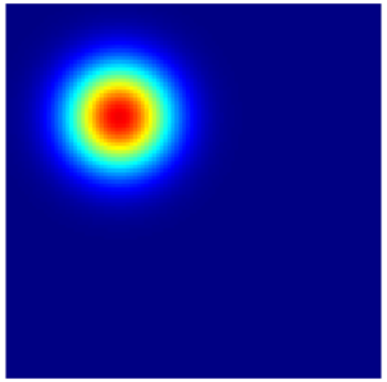
- How to work with continuous data ?



- Decomposition in sum of N Gaussian functions (also called **particles**) of standard deviation σ (***N*** **and** $\boldsymbol{\sigma}$ **are hyperparameters fixed** for all the simulations and measurements):

$$\bar{\rho}(\mathbf{x}) = \sum_{n=1}^{N} G_{\boldsymbol{\mu}_n,\sigma}(\mathbf{x}) \quad \text{where} \quad G_{\boldsymbol{\mu}_n,\sigma}(\mathbf{x}) = \frac{1}{N\sigma^2 2\pi} \exp^{\frac{-(\mathbf{x}-\boldsymbol{\mu}_n)^2}{2\sigma^2}}$$

- Thus, every surface is now described as a $\boldsymbol{N} \times \mathbf{2}$ **vector** of the x and y coordinates of the N particles:



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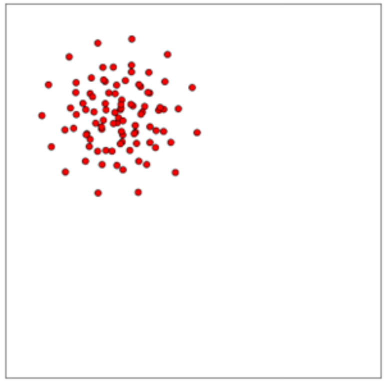
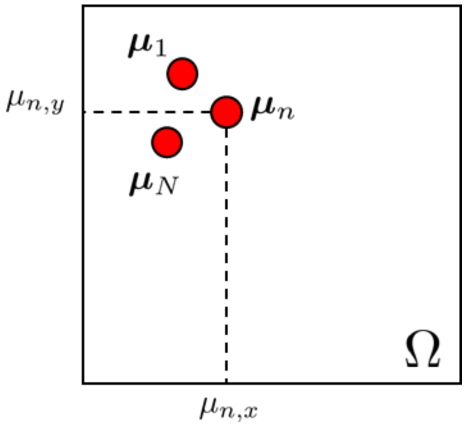


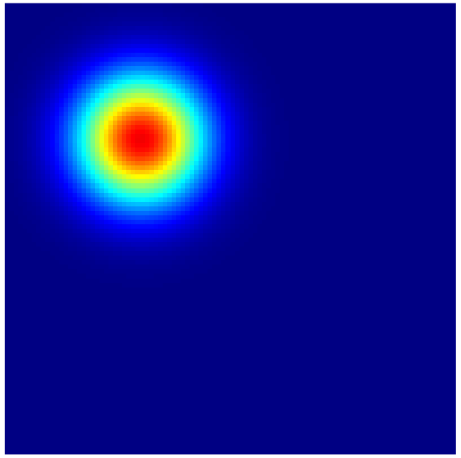
$\boldsymbol{\mu}$

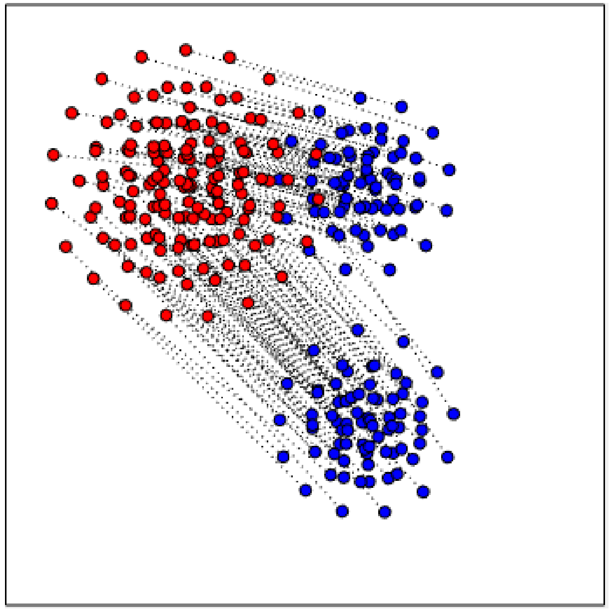


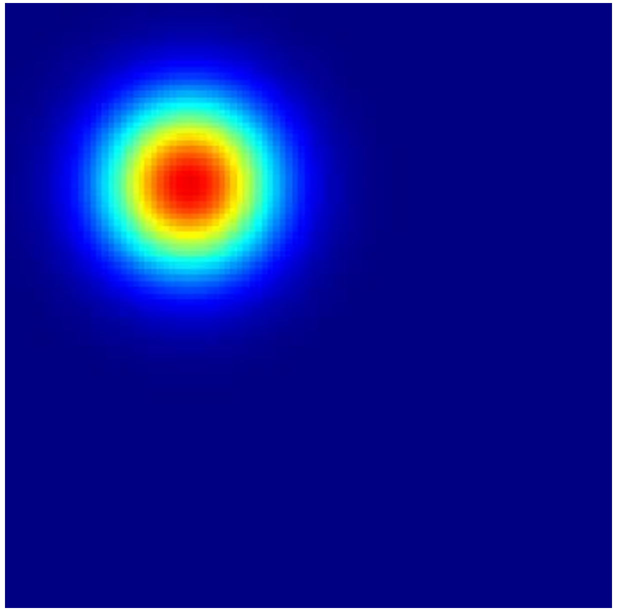
$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_1 \\ \vdots \\ \boldsymbol{\mu}_n \\ \vdots \\ \boldsymbol{\mu}_N \end{bmatrix} = \begin{bmatrix} [\mu^p_{1,x}, \mu^p_{1,y}] \\ \vdots \\ [\mu^p_{n,x}, \mu^p_{n,y}] \\ \vdots \\ [\mu^p_{N,x}, \mu^p_{N,y}] \end{bmatrix} \in \mathbb{R}^{N\times 2}$$

Optimal Transport based Interpolation

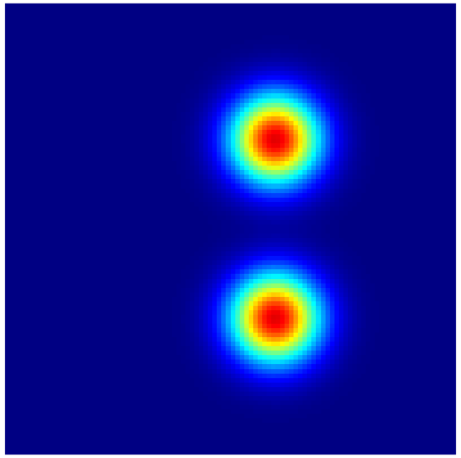
FIELD 1





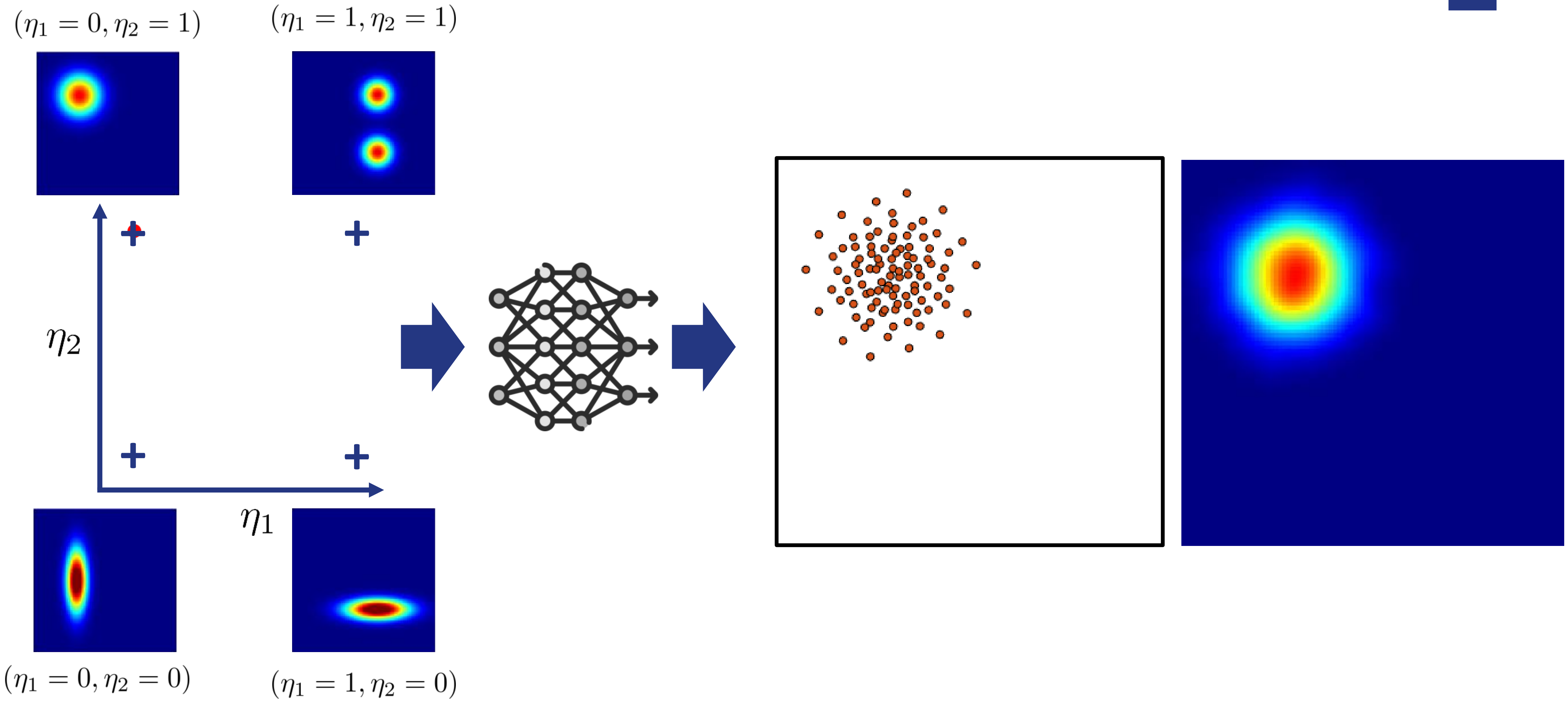


FIELD 2



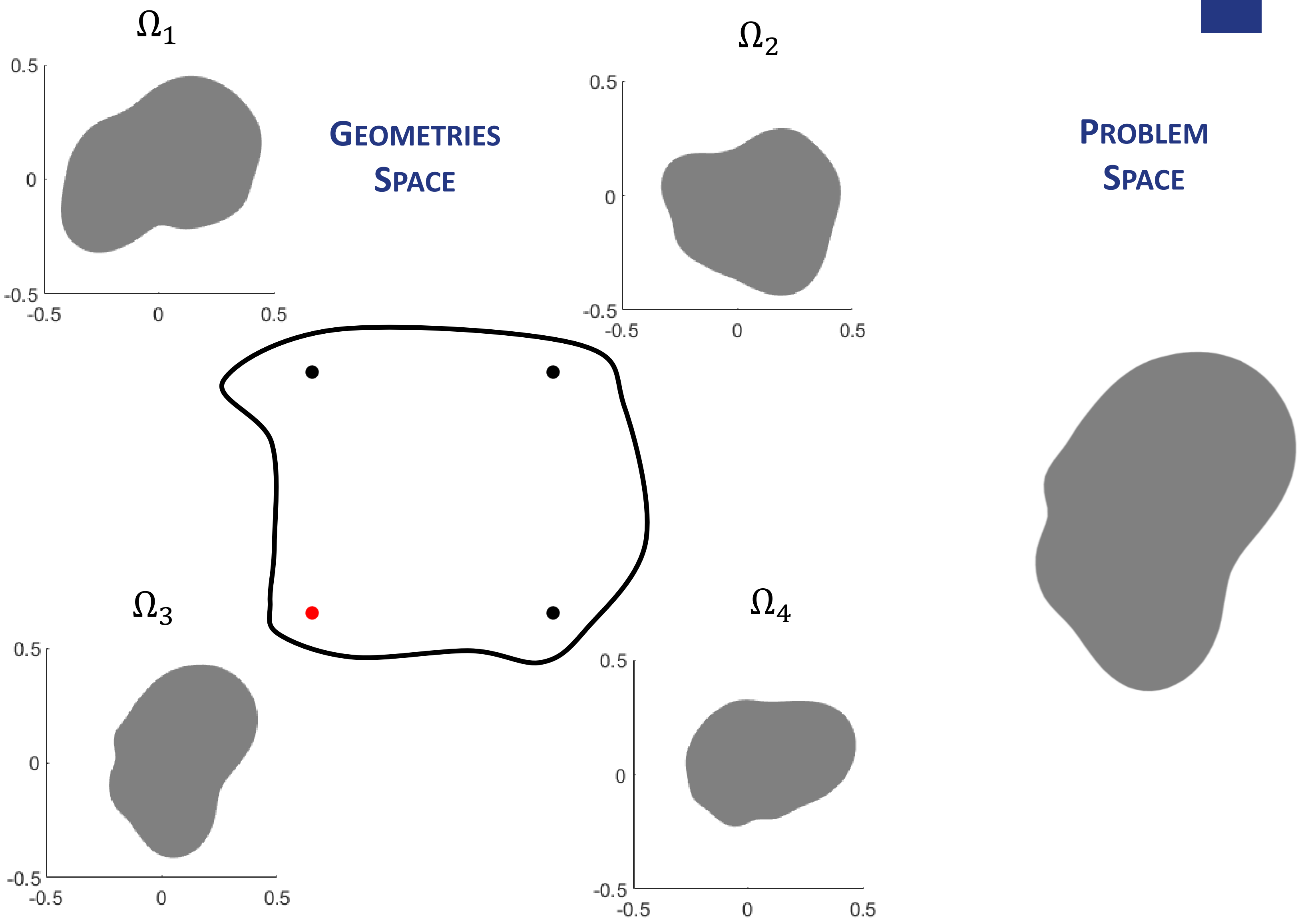
$$\min_{M \in \mathbb{R}^{N \times N}} \sum_{i,j} C_{i,j} M_{i,j}$$

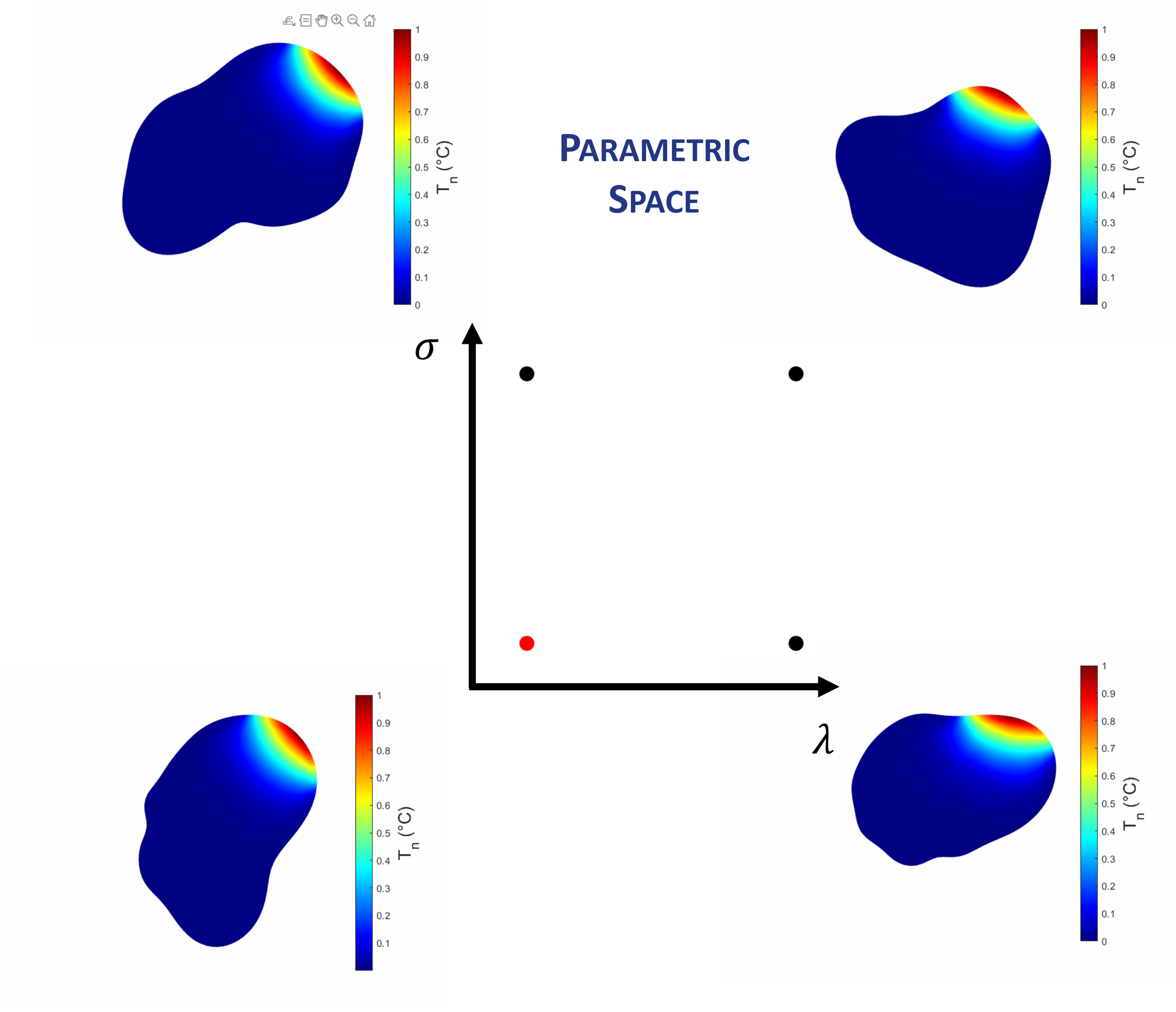
s.t. $\sum_{j=1}^{N} M_{i,j} = 1, \forall i \in [\![N]\!]$ s.t. $\sum_{i=1}^{N} M_{i,j} = 1, \forall j \in [\![N]\!]$


$(\eta_1 = 0, \eta_2 = 1)$
$(\eta_1 = 1, \eta_2 = 1)$
η_2
η_1
$(\eta_1 = 0, \eta_2 = 0)$
$(\eta_1 = 1, \eta_2 = 0)$

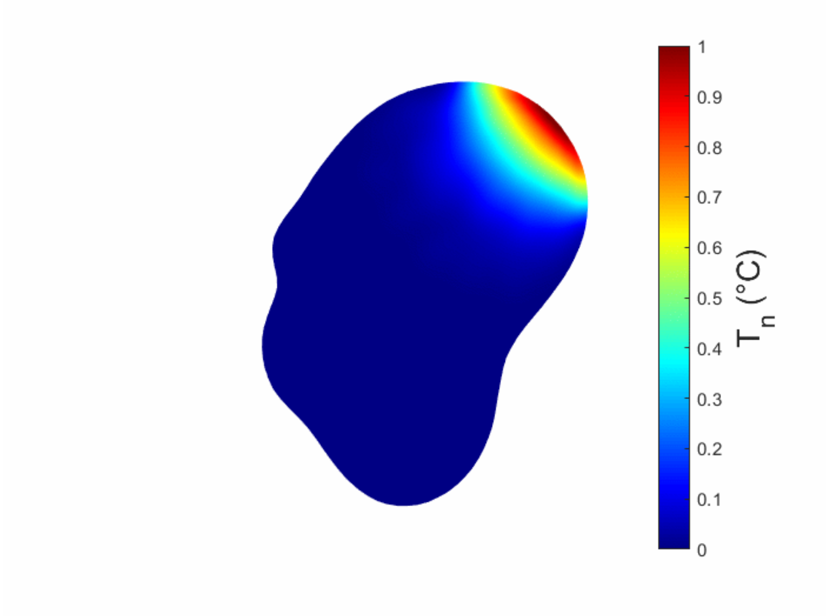
- We follow the same Gaussian functions decomposition approach for the geometries.

- Since the geometries are not parametrized, we interpolate between them using the Wassertein barycenters approach.

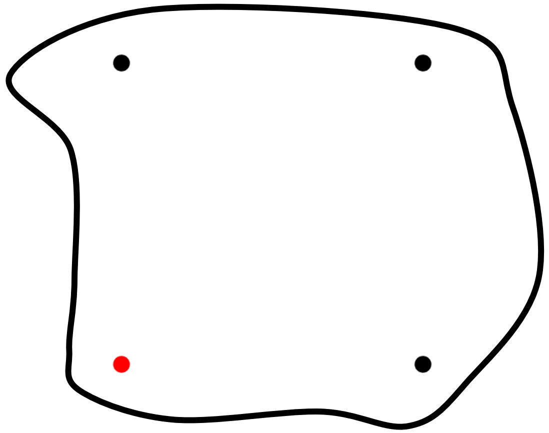




PROBLEM SPACE

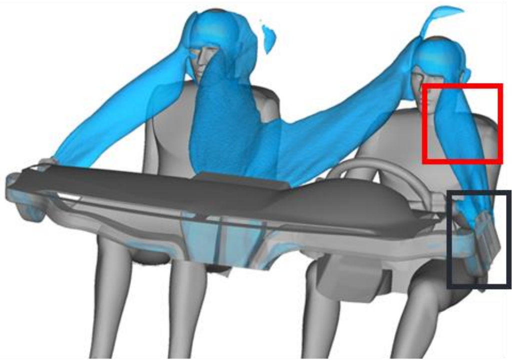


GEOMETRIES SPACE

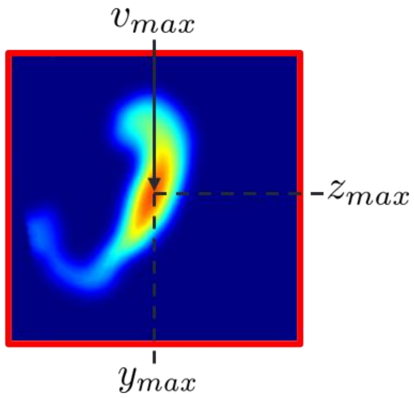


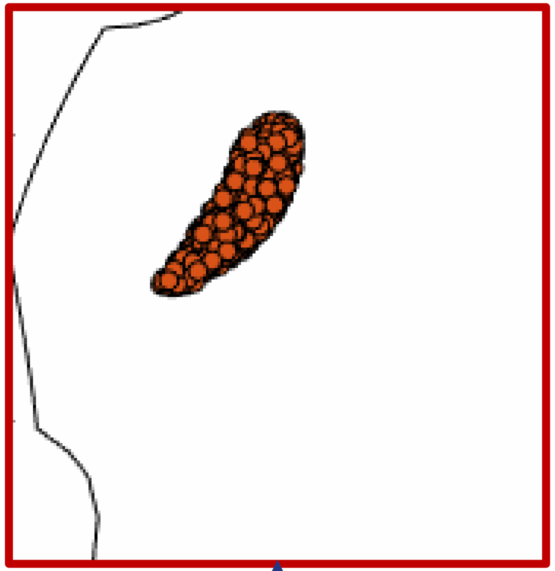
STELLANTIS

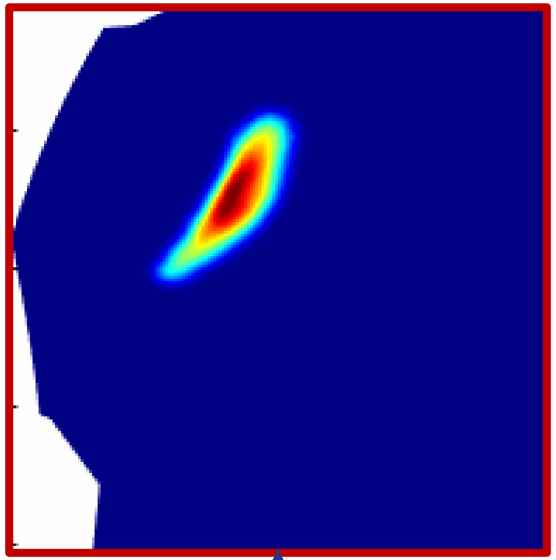
3D air-flow:



2D velocity field:

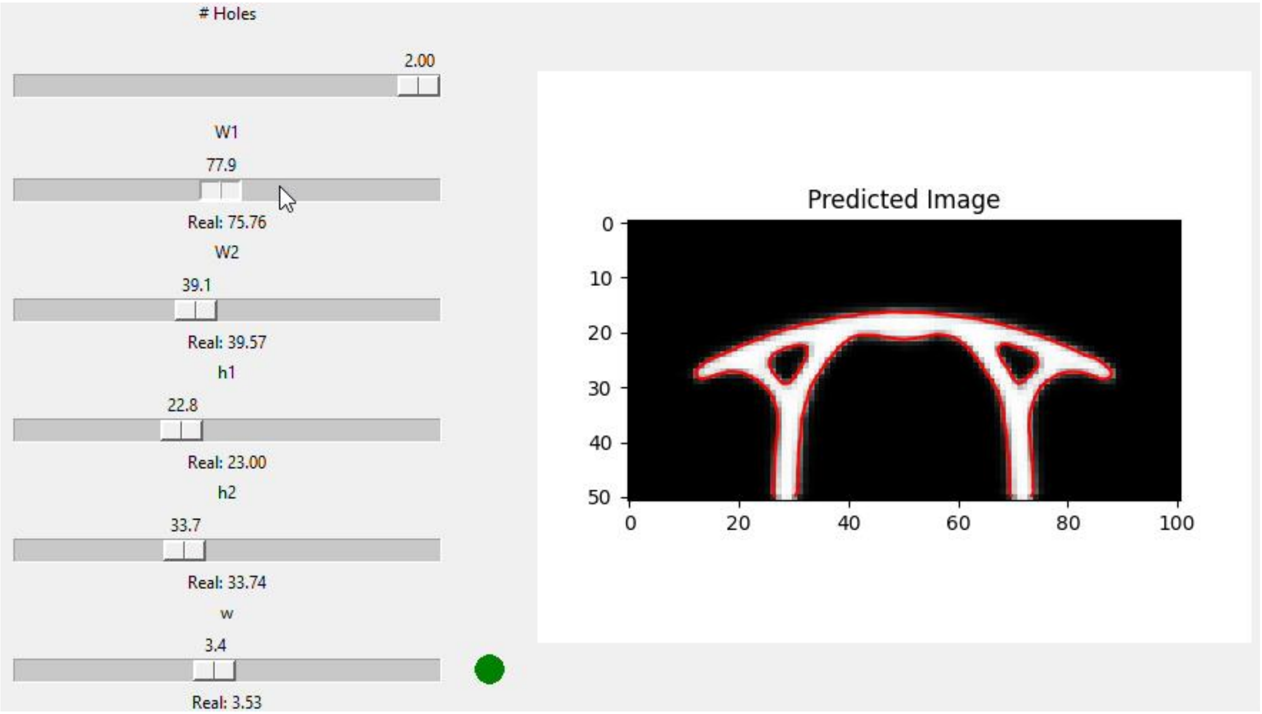


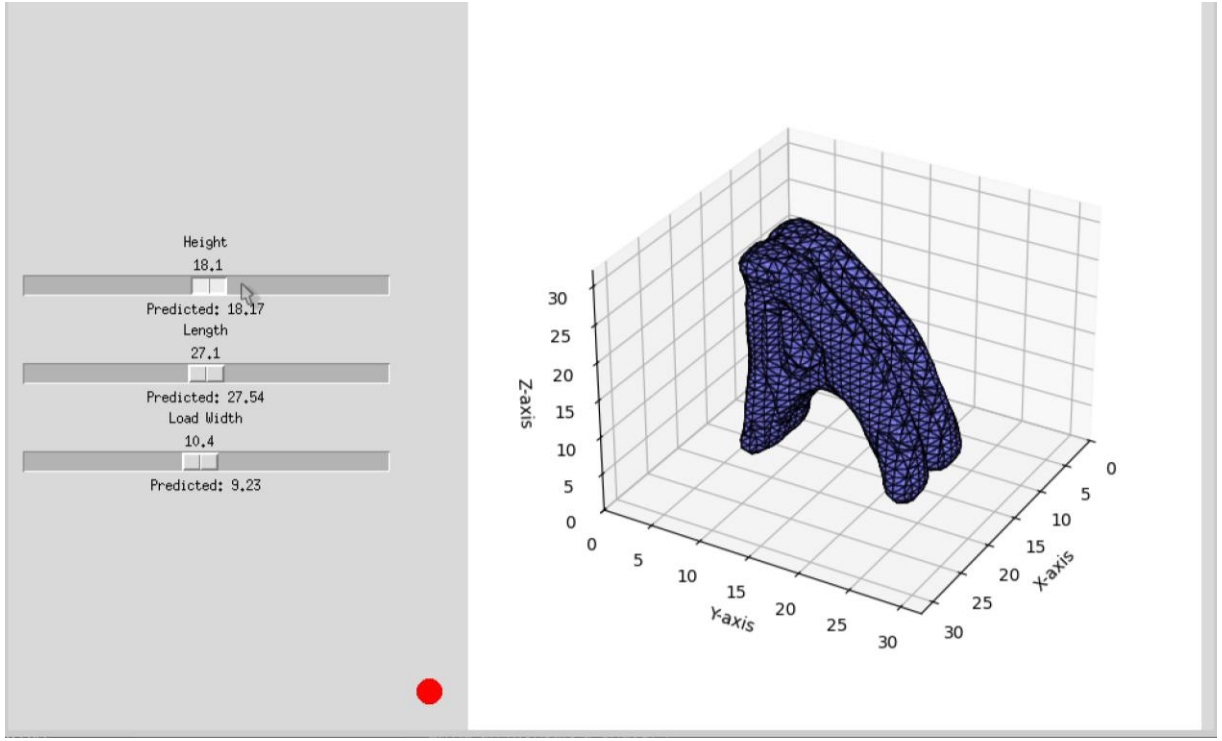




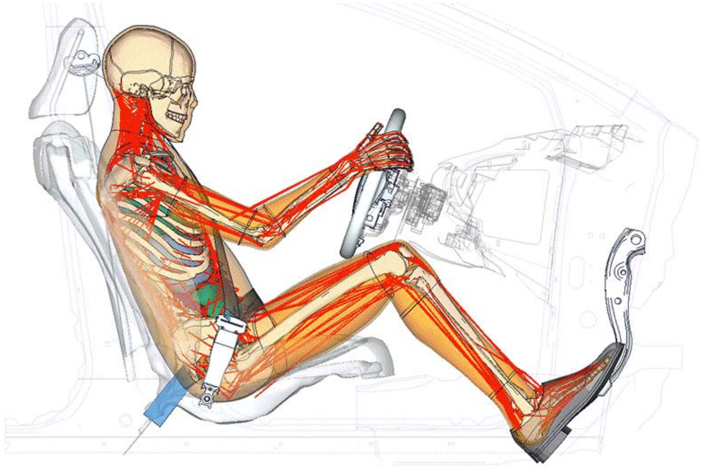
Problem Parameters → **OT-based Real-time Surrogate** → **2D point cloud** → **Sum of Gaussians** → **Reconstructed Field**

- **Car bumper:**


Holes
2.00
W1
77.9
Real: 75.76
W2
39.1
Real: 39.57
h1
22.8
Real: 23.00
h2
33.7
Real: 33.74
w
3.4
Real: 3.53
Predicted Image

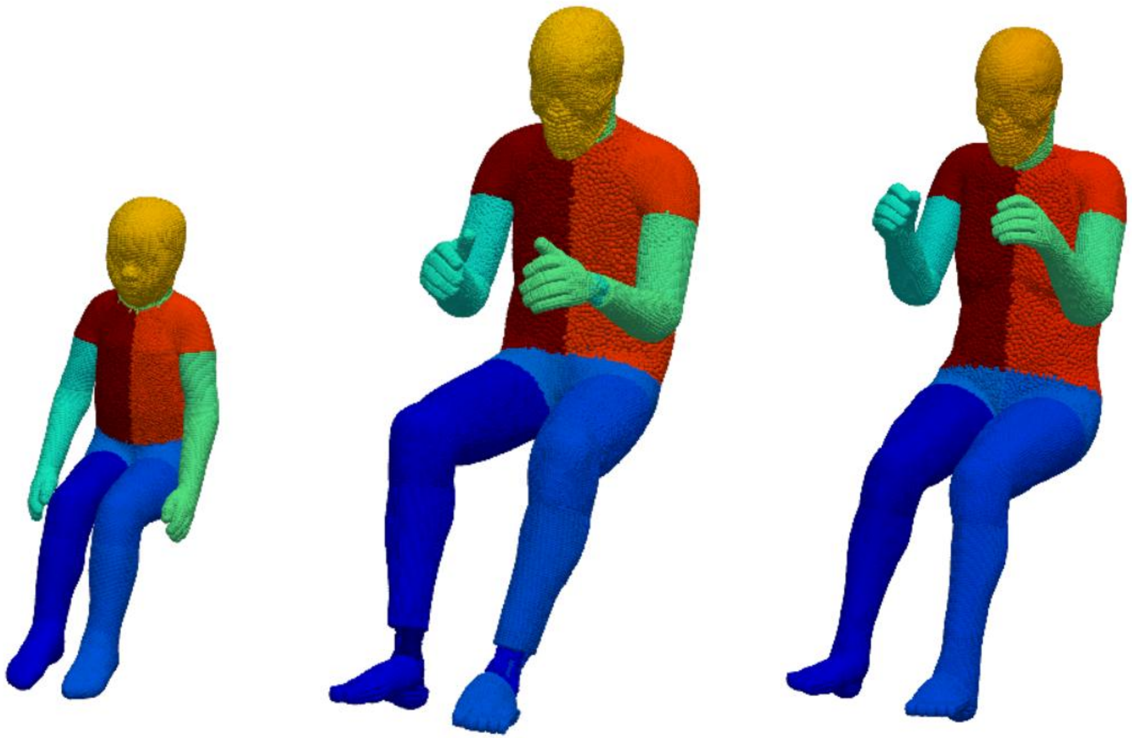

Height
18.1
Predicted: 18.17
Length
27.1
Predicted: 27.54
Load Width
10.4
Predicted: 9.23
Z-axis
Y-axis
X-axis

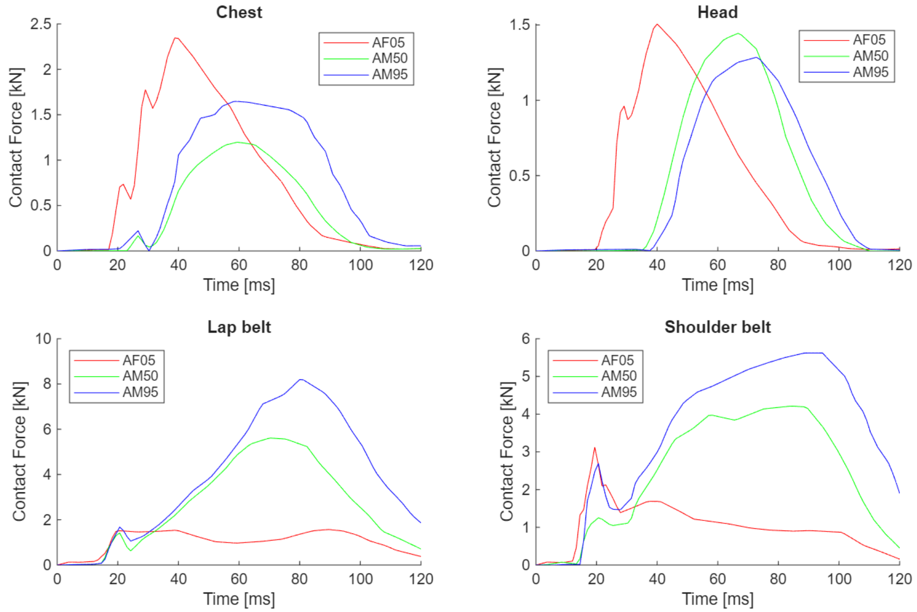
Crash test simulation:



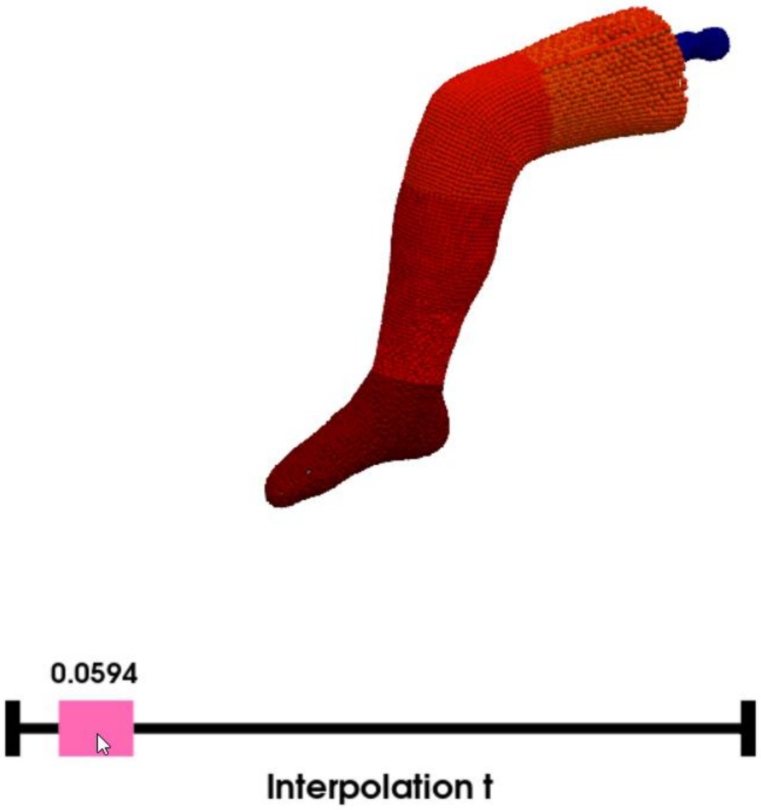
- Very reduced number of dummies.
- Very detailed and complex models

→ Create more diversified and parametrized human models.

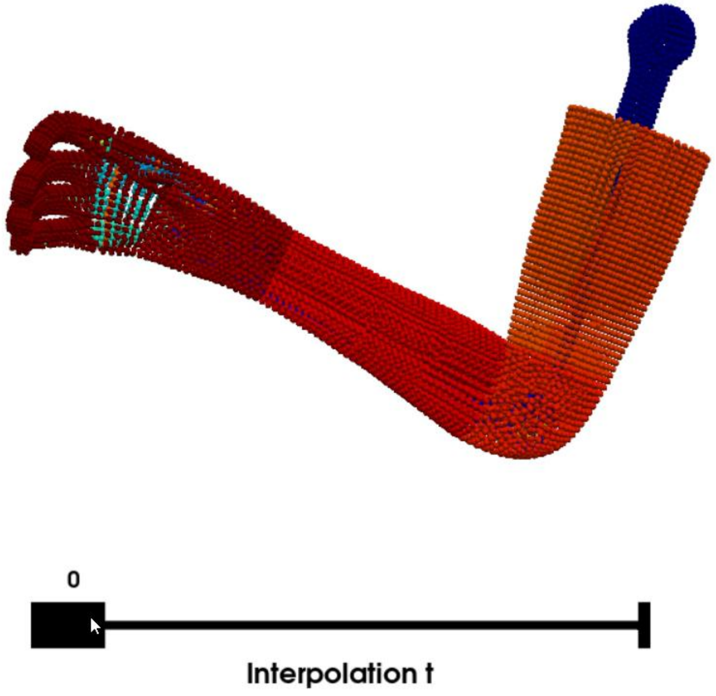




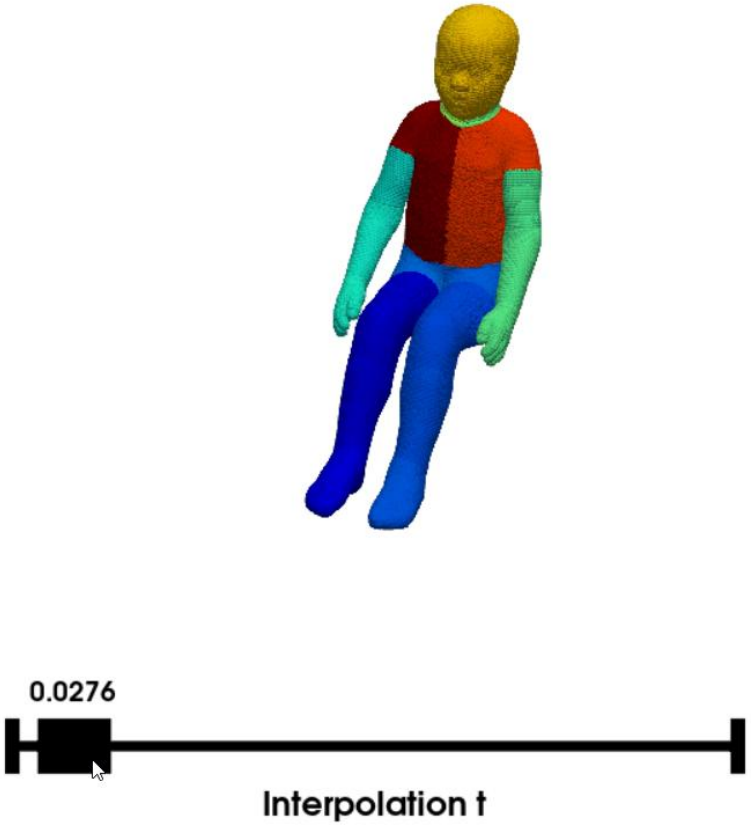
Right Leg: 6YO - AM50

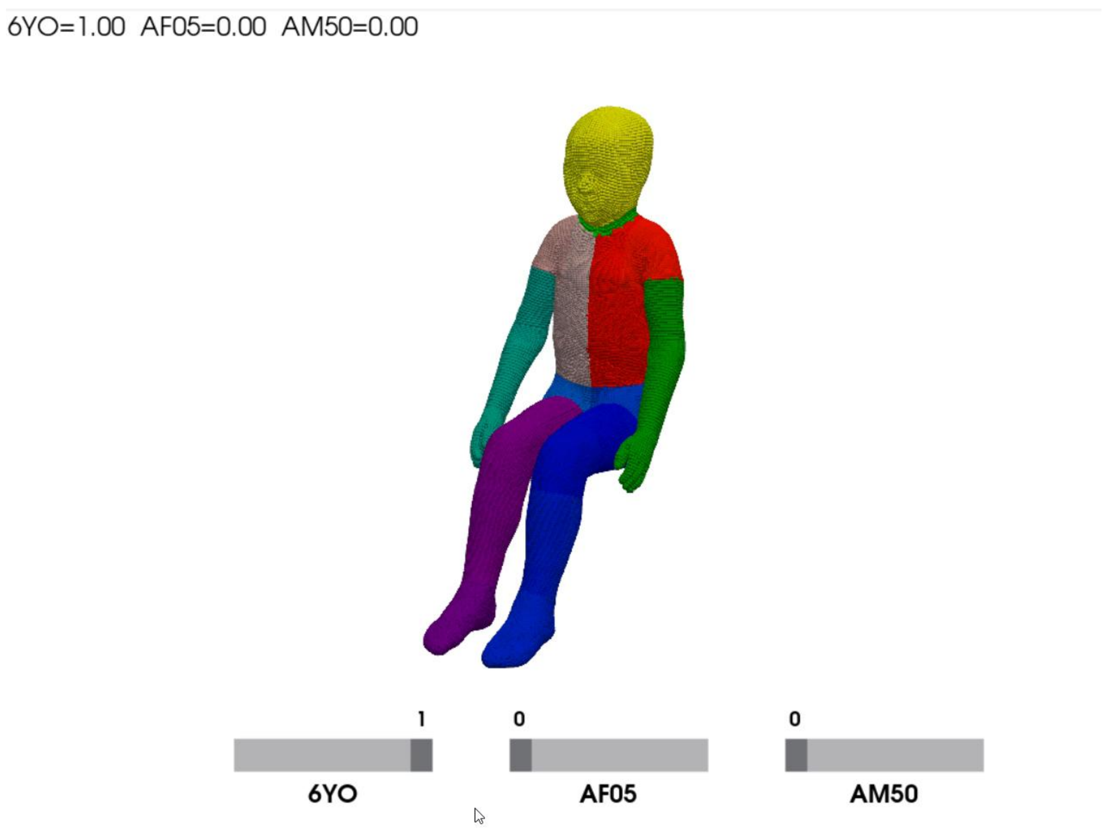


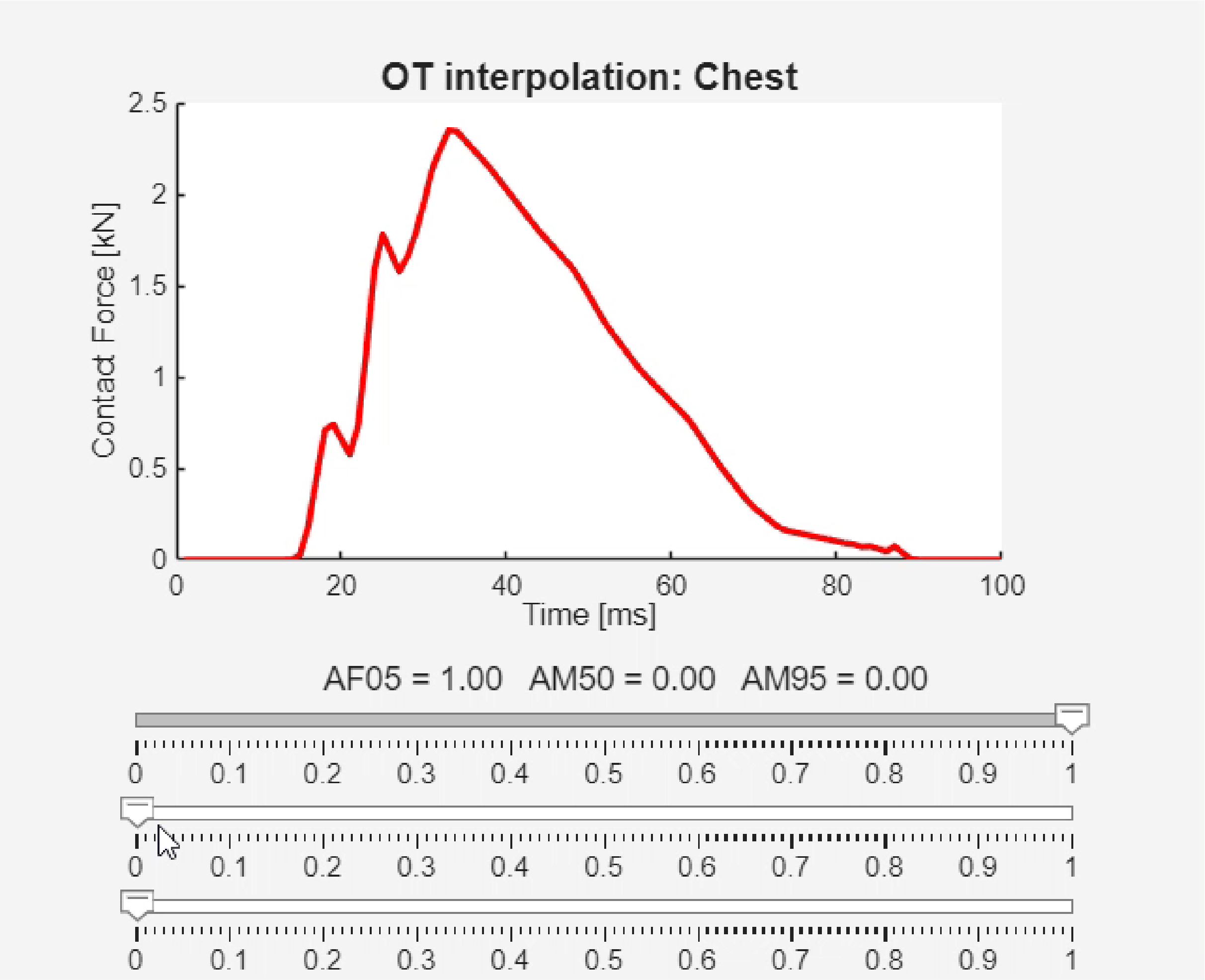
Right Arm: AF05 – AM50



Dummy: 6YO - AM50




6YO=1.00 AF05=0.00 AM50=0.00
1
0
0
6YO
AF05
AM50


OT interpolation: Chest
Contact Force [kN]
Time [ms]
AF05 = 1.00 AM50 = 0.00 AM95 = 0.00

1 RANK REDUCTION AUTOENCODERS

J. Mounayer, S. Rodriguez, M. El Fallaki Idrissi,
C. Gnatios, F. Chinesta

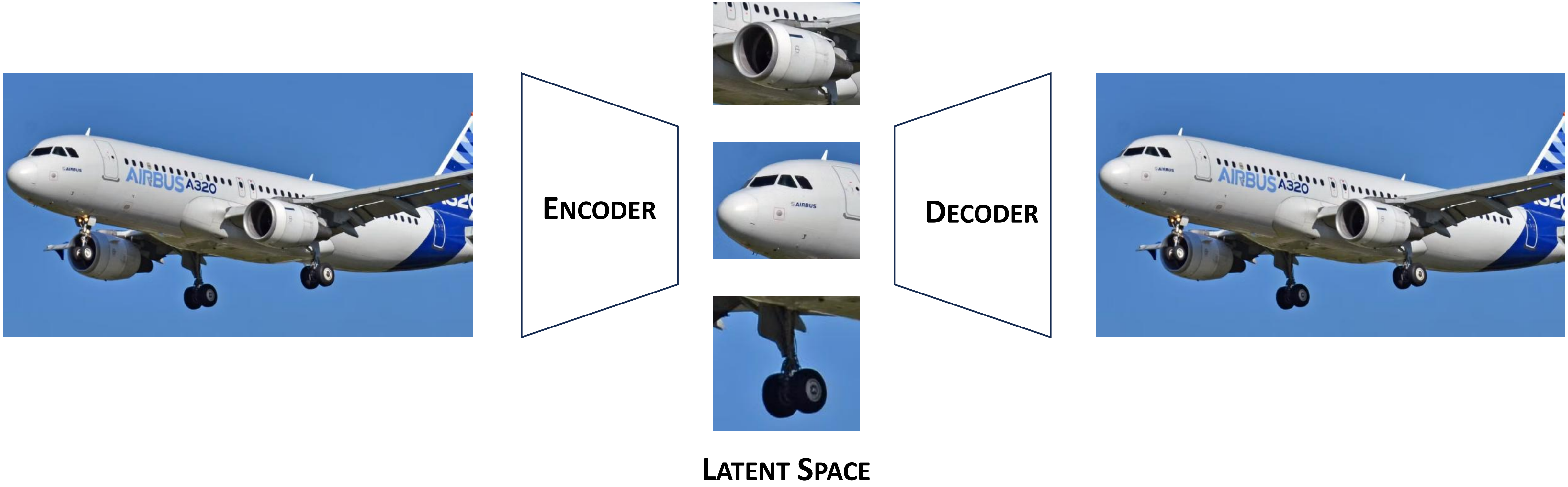
2 INDUSTRIAL USE-CASES

“ Computers are like humans, they do everything except think. ”

John von Neumann

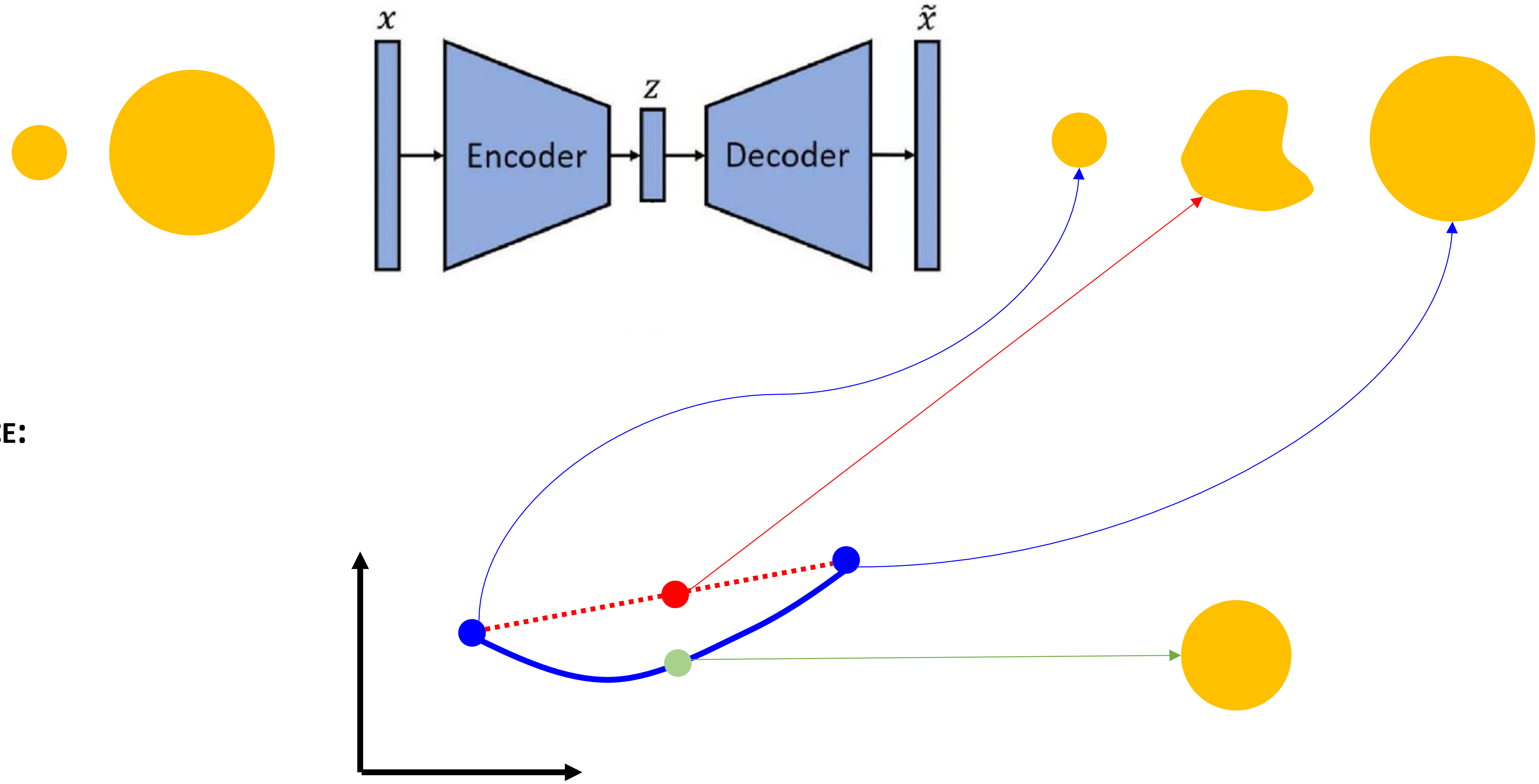
AUTOENCODER (AE):

- What is the map that take us from a high dimensional representation to a compressed one ?
- Find the smallest possible compressed representation able to reconstruct the high dimensional one.



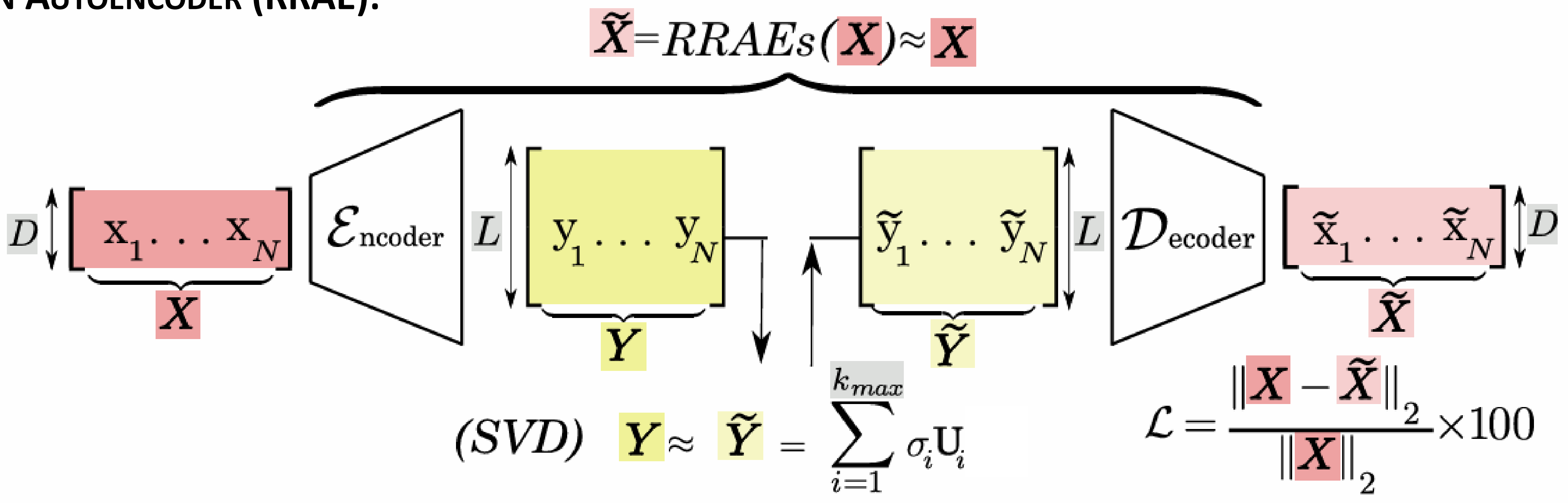
LOSS: INPUT = OUTPUT

TRAINING:

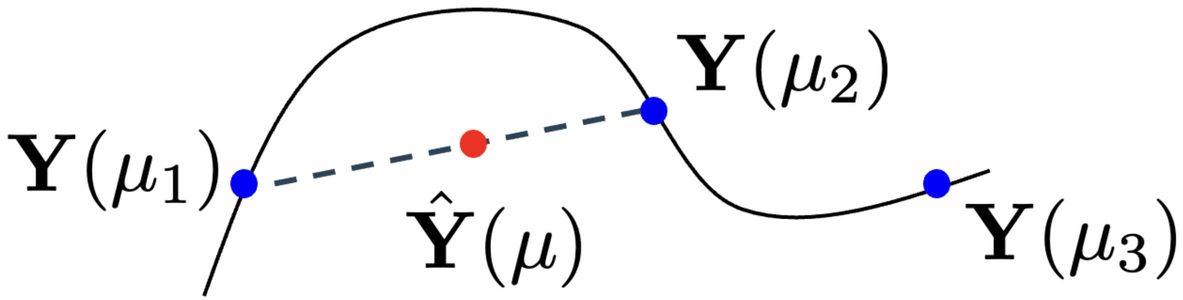


INFERENCE:

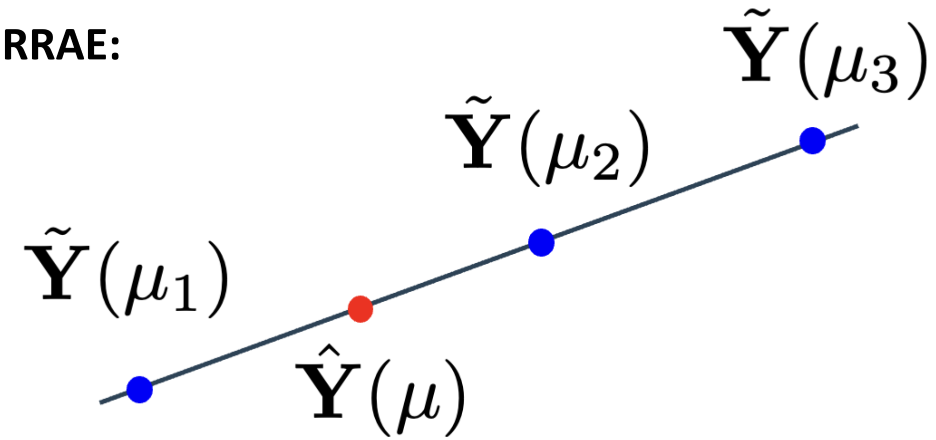
RANK REDUCTION AUTOENCODER (RRAE):

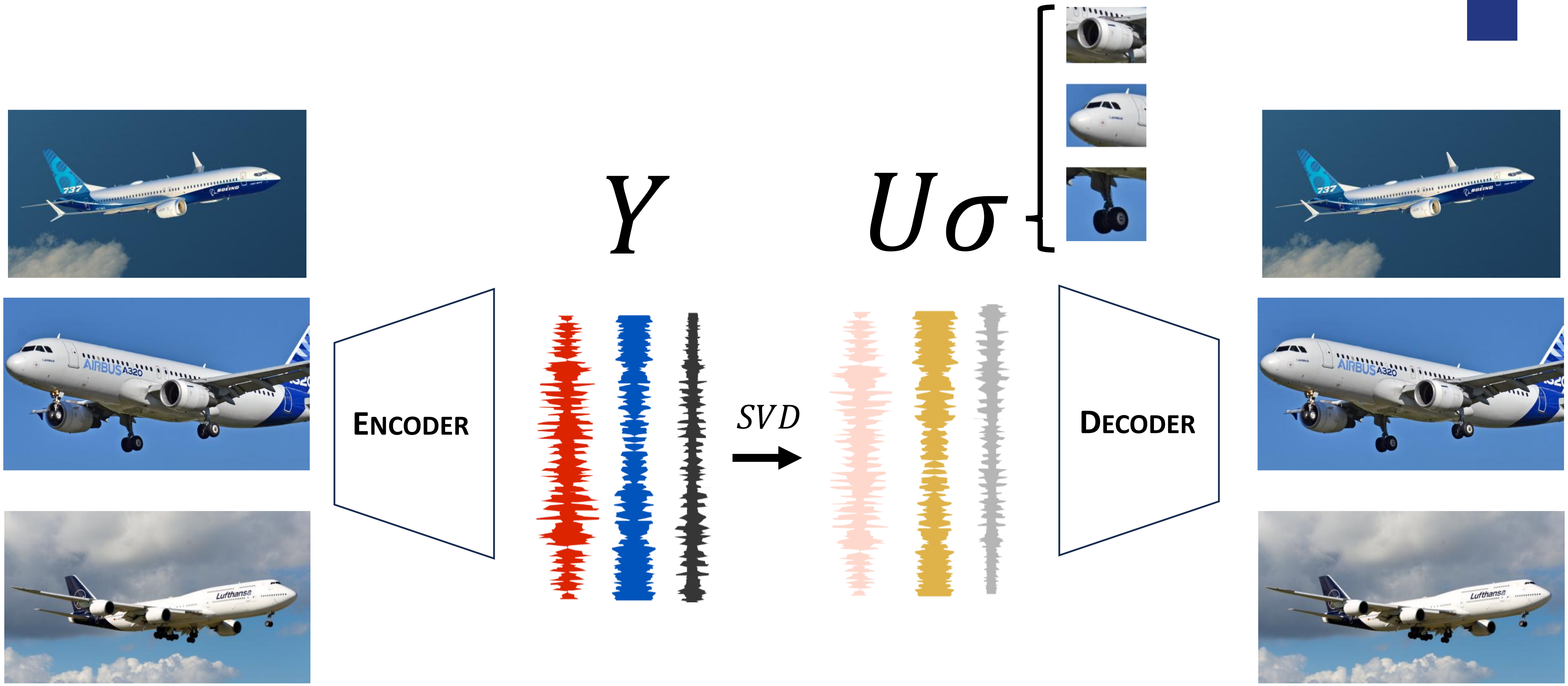


AE:

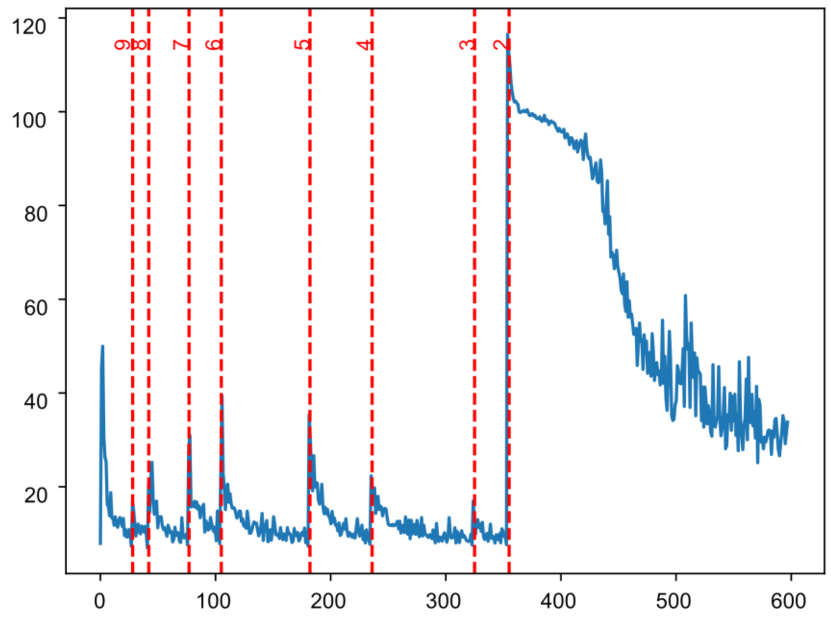


RRAE:



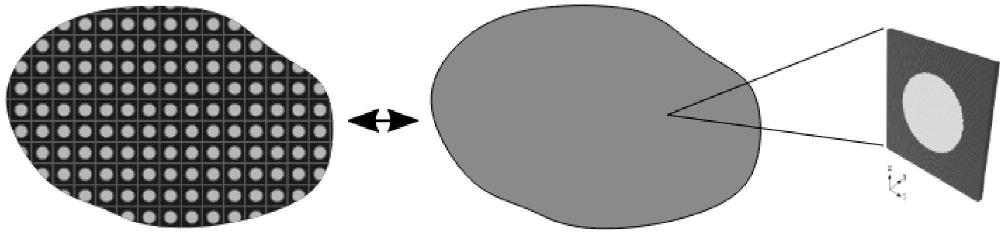


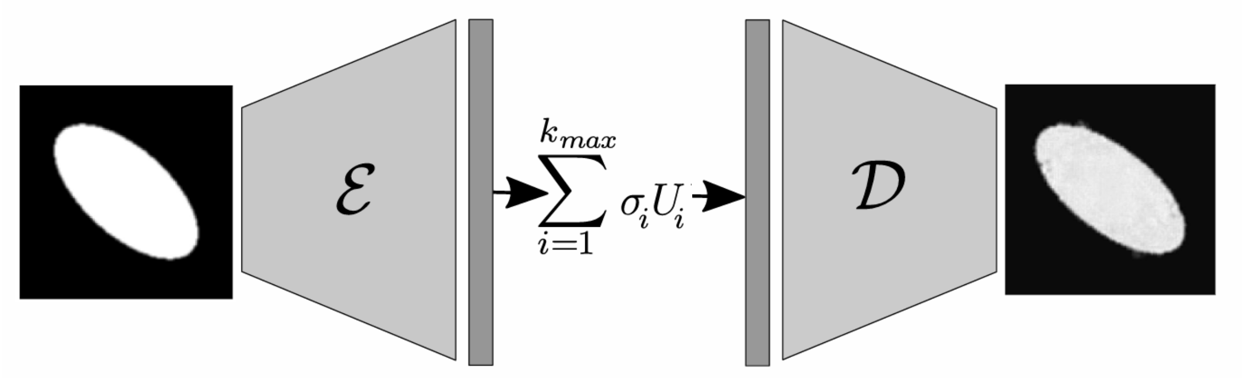
LOSS: INPUT = OUTPUT



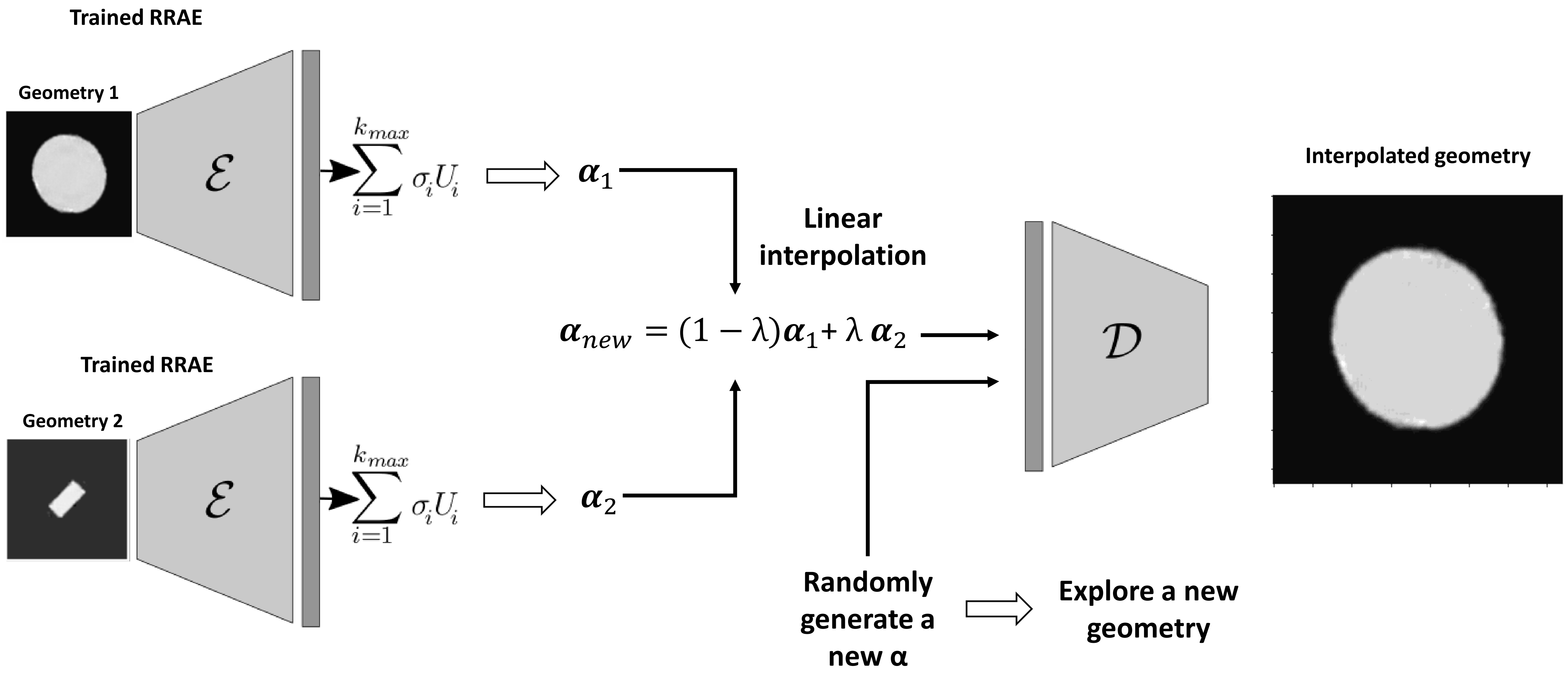
- Bottleneck exists
- Latent space regularized by the SVD
- No dependence on latent space dimension
- Only one loss: Input = Output
- Sorted Bottleneck.

- Composite Materials:

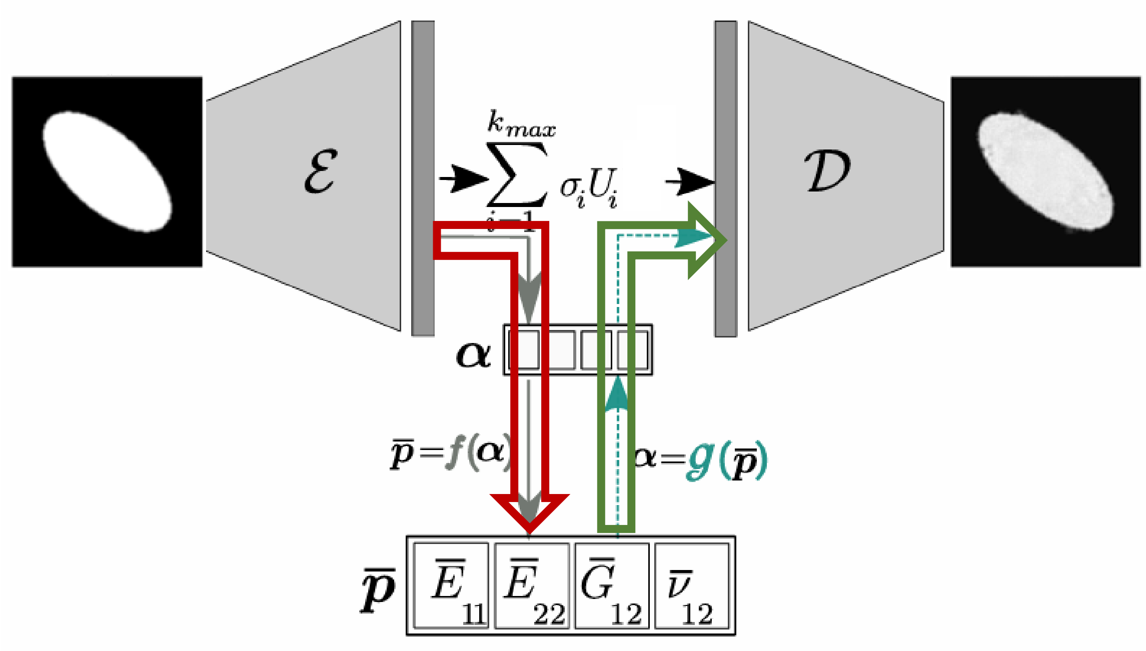




INTERPOLATION AND EXPLORATION



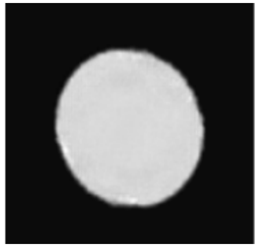
- **DIRECT PROBLEM:** From Geometry to Solution
- **INVERSE PROBLEM:** From Solution to Geometry



Effective properties

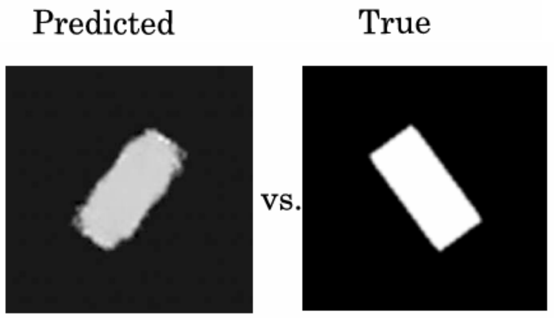
INPUT

OUTPUT

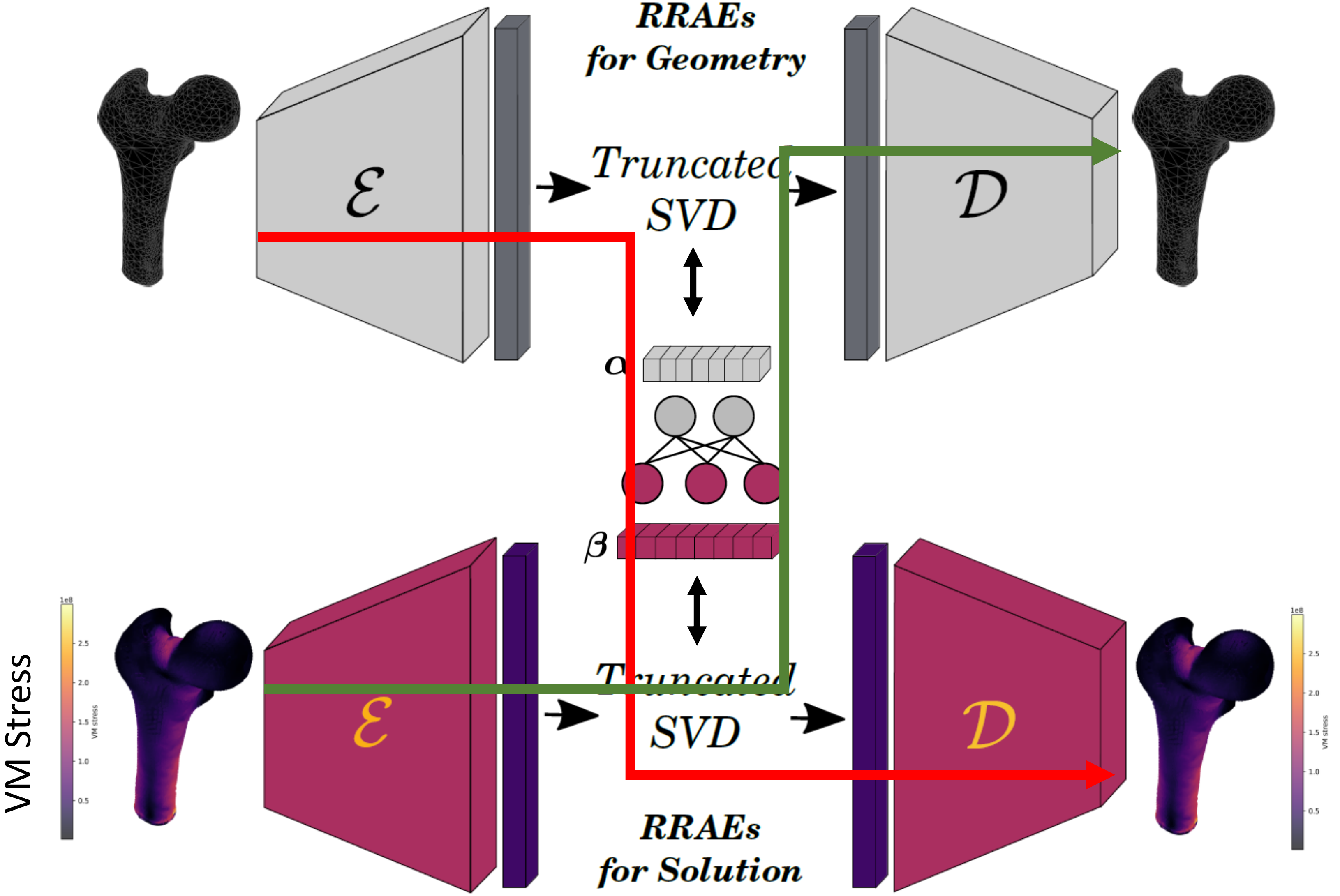


$\overline{\boldsymbol{p}}$	True	Predicted	Error
$\overline{E}_{11}$ [MPa]	3602	3539	1.75%
$\overline{E}_{22}$ [MPa]	3709	3663	1.24%
$\overline{G}_{12}$ [MPa]	1137	1131	0.52%
$\overline{\nu}_{12}$ [-]	0.342	0.345	0.87%

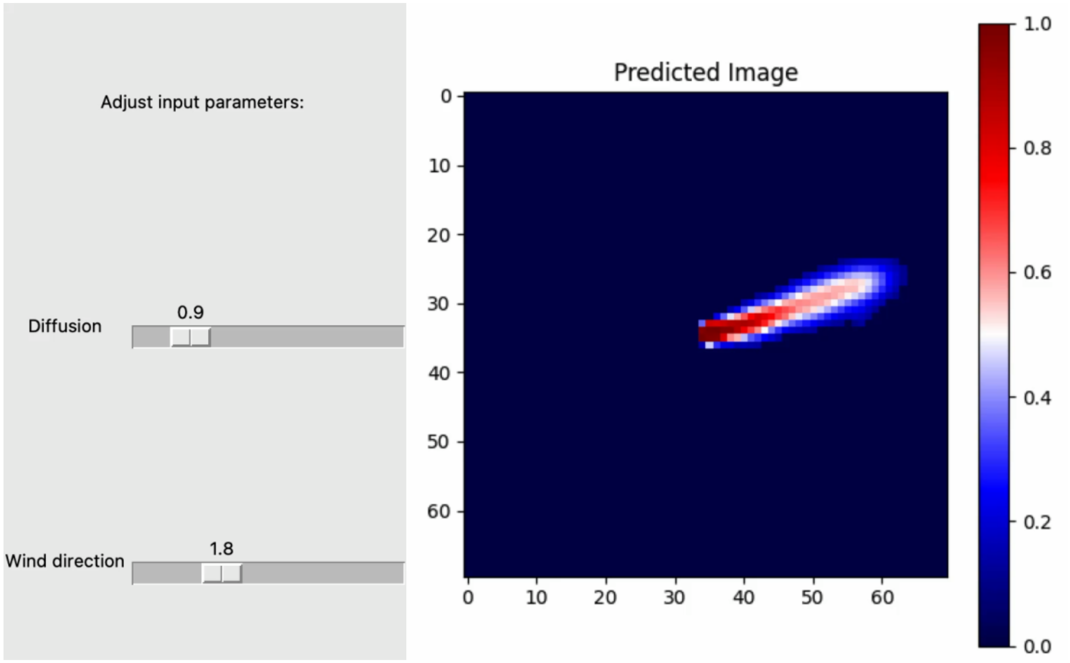
$\overline{\boldsymbol{p}}$	Test
$\overline{E}_{11}$ [MPa]	2504
$\overline{E}_{22}$ [MPa]	2563
$\overline{G}_{12}$ [MPa]	923
$\overline{\nu}_{12}$ [-]	0.384

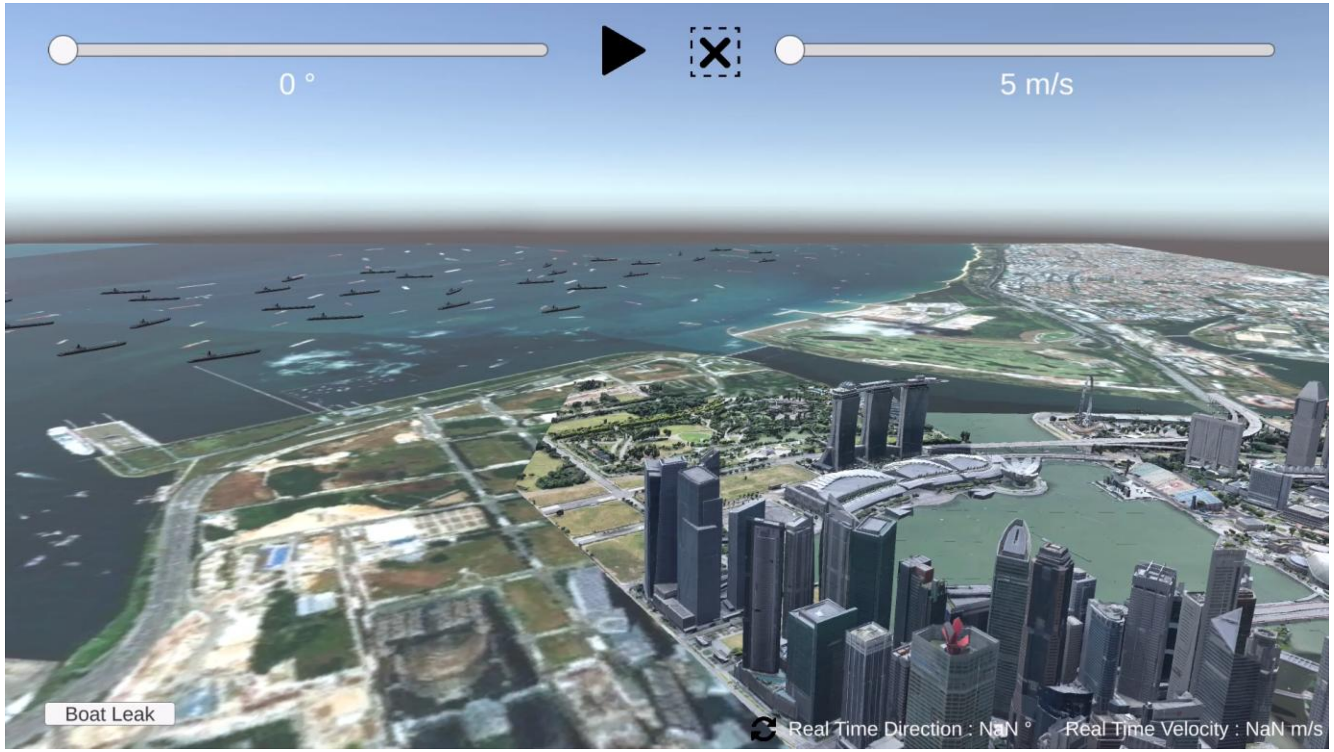


- DIRECT PROBLEM
- INVERSE PROBLEM

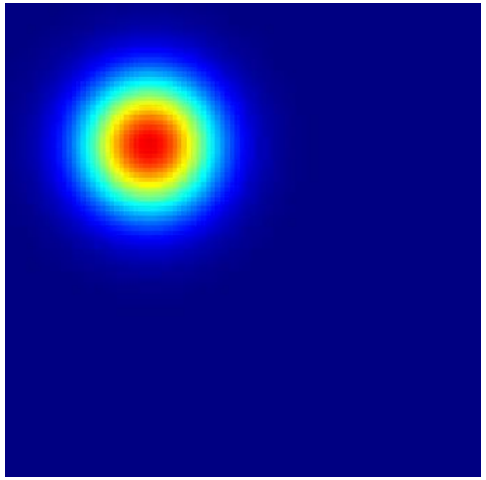


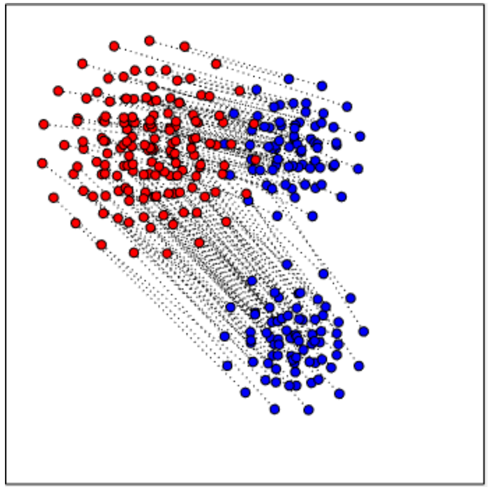
PLUME DISPERSION EMERGENCY MANAGEMENT



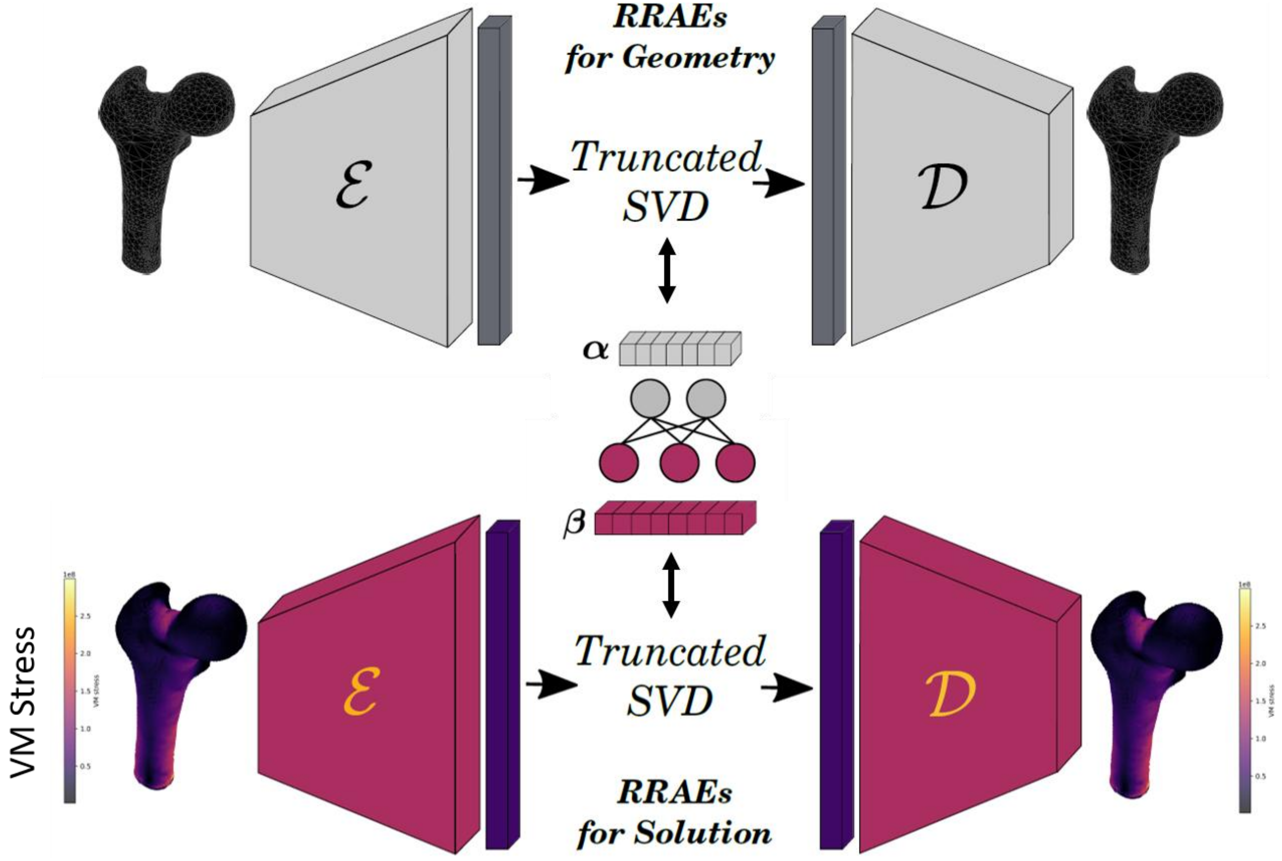


OPTIMAL TRANSPORT





RANK REDUCTION AUTOENCODERS



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- **RRAE:** Jad.MOUNAYER@ensam.eu

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- S. Torregrosa, D. Munoz, H. Navarro, C. Farhat, F. Chinesta, *Towards Generative Design Using Optimal Transport for Shape Exploration and Solution Field Interpolation*, 2025.
- M. El Fallaki Idrissi, I. Ben-Yelun, J. Mounayer, et al., *A new Framework for Generative Design, Real-Time Prediction, and Inverse Design Optimization: Application to Microstructure,* 2025.
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- J. Mounayer, A. Tierz, J. Tomezyk, et al., *Variational Rank Reduction Autoencoder*, 2025.

Codes Repo

Thank you for listening

Q&A