Hybrid Quantum Approaches to Risk Analysis in Complex Industrial Systems

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TQCI seminar 9 october 2025 AQADOC

Probabilistic Risk/Safety Assessment (PSA/PRA)

Hybrid Sequence Processing

Hybrid approach : fault trees to s-t networks

Quantum Inspired Algorithms : Tensor networks

Conclusions and perspectives

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Probabilistic Risk Assessment (PRA)

What is PRA?

- PRA is a systematic and comprehensive methodology to evaluate risks associated with a complex engineered technological entity (such as an airliner or a nuclear power plant).

PRA consists of[1]:

- Determining potential undesirable consequences associated with use of systems and processes
- Identifying ways that such consequences could materialize
- Estimating the likelihood (e.g., probability) of such events
- Providing input to decision makers on optimal strategies
- The term "Probabilistic Safety Assessment" (PSA) is used specifically for nuclear industry in Europe and in IAEA documents

[1] Mosleh 2014. PRA: A perspective on strengths, current limitations, and possible improvements.

Reliability issues

Main problems[2]

- SAT (AllSAT): *Given a Boolean formula $f(\mathbf{x})$ on a set of variables, find one (all) evaluation(s) that renders |f* `TRUE`;
- Reliability: *Assuming that the truths of f are probabilistic, calculate the probability that |f* is `TRUE`;
- Reachability *Given an AEF finite state automaton |M*, identify the *transitions that lead to a failure state*;
- Reliability of an AEF: *Calculate the probability of reaching a failure state in a time t.*

Complexity issues

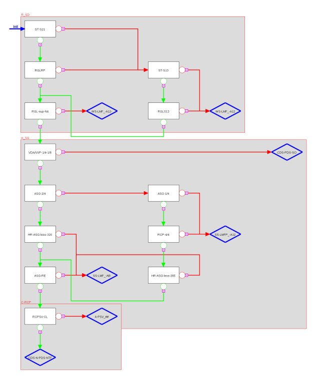
- If it is not NP-Hard it is PSPACE-hard

[2] Rauzy, A. (2018). Notes on Computational Uncertainties in Probabilistic Risk/Safety Assessment. Entropy, 20(3). https://doi.org/10.3390/e20030162

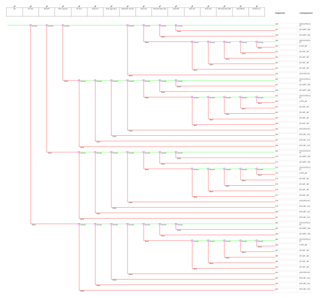
Accident sequences as event trees/sequence diagrams

Event sequence diagram

- It summarizes all the scenarios triggered by some initiator.



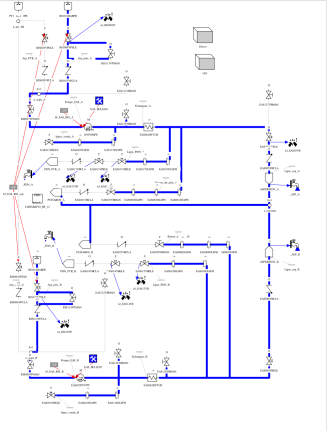
Event tree representation

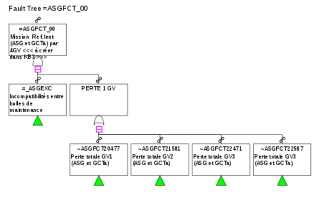


- This is how these scenarios are represented in quantification tools

System analysis

- Objective: model the system failures that are identified in the sequence analysis
- Fault-trees are used to provide a logical failure model





An event tree

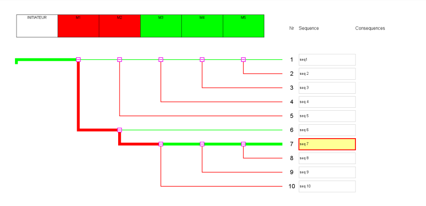


Figure 1: Very small Example

Master fault tree

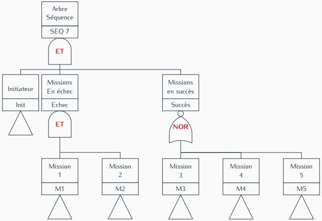


Figure 2: Master fault tree of sequence 7

$$\bigvee_{j=1}^{m} \bigwedge_{k=1}^{sj} E_{jk} = (E_{11} \wedge \cdots \wedge E_{1s_1}) \vee \cdots \vee (E_{m1} \wedge \cdots \wedge E_{ms_m})$$

- The problem : reducing a boolean formula to its normal disjunctive form.

Classical Algorithms

- MOCUS and all variants
  - Top-down algorithm (J.Fussel and W.Vesely in 1972)
  - Approximation algorithms
    (Modularisation → Moduarized cutsets → Demodularized cutsets)
  - Recent variants (Adopted mostly in the Nuclear PSA)

- Binary/Ternary Decision Diagrams like algorithms
  - For small models
  - Bottom-up approach
  - May be useful for some applications (Some days to build a BDD corresponding to the Core Melt Consequence)

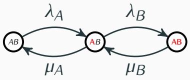
Dynamic approach

Dynamic aspects

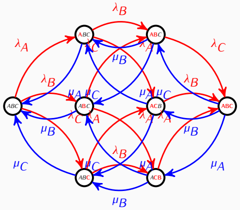
- Credit accident dynamics
  - Events order
  - Consideration of normal-standby modes
- Credit recoveries during mission times
- Instead of prime impliquants (cutsets) search for sequences

Modeling the system state using markov graphs

- 2 components



- 3 components



Related works

- Encoding fault tree circuits for Identification des MCS via amplitude amplification (Silva et al. 2024).[3]
  - Generate an algorithm capable of increasing the sampling probability of states that represent Minimal Cut Sets
- Quantum Algorithm for Path-Edge Sampling[4] (Jeffey et al. 2023)
  - Quantum query algorithms for finding st-cut sets
- PhD Thesis Ahmed Zaiou with LIPN (Y. Bennani and B Matei)[5]

[3] Silva, G. S. M. (2025). Quantum fault trees and minimal cut sets identification. Reliability Engineering and System Safety.

[4] Jeffery, S., Kimmel, S., & Piedrafita, A. (2023). Quantum Algorithm for Path-Edge Sampling. 18th Conference on the Theory of Quantum Computation, Communication and Cryptography (TQC 2023).

[5] Zaiou, A. (2022). Quantum machine learning approaches for graphs and sequences : application to nuclear safety assessment (Doctoral dissertation), Université Paris Nord.

Hybrid Sequence Processing

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Hybrid Sequence Processing

Hybrid approach : fault trees to s-t networks

Quantum Inspired Algorithms : Tensor networks

Conclusions and perspectives

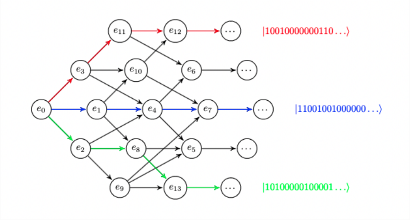
Challenges

1. How can we find all possible failure scenarios of a system from an initial state?
2. If the initial state of the system changes, how we can find the most probable scenarios from the new state pending the search of all possible scenarios? And what are the probabilities/frequencies of these scenarios?

This work was done in Ahmed Zaiou Thesis at LIPN (Y. Bennani and B Matei).

How can we encode paths with quantum states?

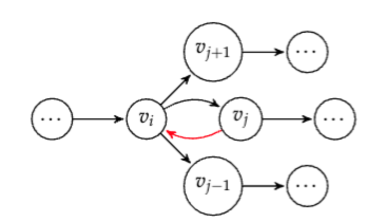
- The paths represent the system evolution with the different failures and recoveries.
- The representation of paths with quantum states Suppose that we have a state $|\lambda\rangle = |v_0 v_1 \dots v_n\rangle$ which represents the path λ.
  - We say that the vertex v_i is in the path λ if and only if the qubit v_i of the state $|\lambda\rangle$ is in the state $|1\rangle$.
  - We say that the edge $e_{i,j} = (v_i, v_j)$ is in the path λ if and only if the two qubits v_i and v_j are in states $|1\rangle$ and $|1\rangle$ respectively and the qubits v_k for $i < k < j$ are all in state $|0\rangle$.

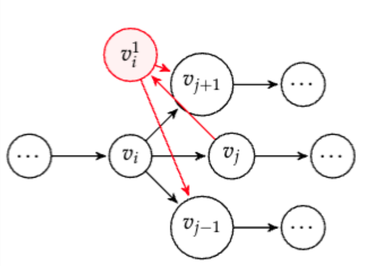


How can we manage loops in the graph?

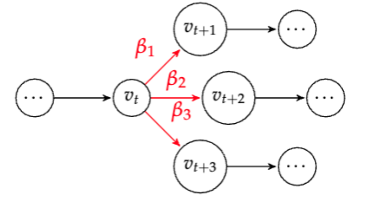
Consider a graph $G = (\mathbf{V}, \mathbb{E}, \mathbf{C})$, if $(v_i, v_j) \in \mathbb{E}$ and $(v_j, v_i) \in \mathbb{E}$ (there is a loop between v_i and v_j).

1. We remove the edge (v_j, v_i) from the set $\mathbb{E}$;
2. we add a new vertex v_i^* in the set of the vertices V;
3. we add the edge (v_j, v_i^*) in E and also we add the edges $(v_i^*, v_l), \forall v_l \in Succ(v_i) \backslash v_j$. Where $Succ(v_i)$ is the set of successors of v_i.





Example of processing a path at time t



Suppose that we have done t steps :

$$\lambda \left| v_0 v_1 \ldots v_t v_{t+1} v_{t+2} v_{t+3} \ldots \right\rangle = \lambda \left| 10 \ldots 1000 \ldots \right\rangle$$

The objective is :

$$\begin{aligned} |\psi\rangle = & \beta_1 \lambda \left| 10 \ldots 1100 \ldots \right\rangle \\ & + \beta_2 \lambda \left| 10 \ldots 1010 \ldots \right\rangle \\ & + \beta_3 \lambda \left| 10 \ldots 1001 \ldots \right\rangle \end{aligned}$$

Sub-circuit to perform the walk at state v_t.

$$U(\theta) = U(\theta, 0, 0) = \begin{pmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$$

General Quantum Oracle

- Given k vertices $\{v_t, \dots v_{t+k}\}$ and each $v_i, i \in \{t, \dots, k\}$ has the successors $v_i^s, s \in N$ and each edge $e = (v_i, v_i^s)$ has a probability $\beta_{(v_i, v_i^s)}$.

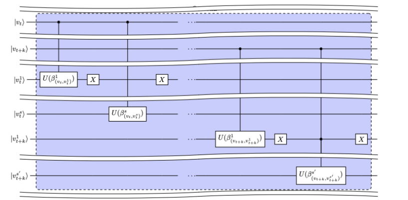


Figure 3: The architecture of our oracle

- This quantum oracle takes k inputs, and it allows to apply the walks to all the successors of each input while keeping the memory of each path.

Quantum algorithm to obtain all paths in a directed a-cyclic graph

Algorithm

Algorithm 1: Quantum algorithm to obtain all paths in a DAG (QAPAG)

Input : Graph $G = (\mathbb{V}, \mathbb{E}, \mathbb{C})$, source v_0.

Initialization

Initialize $n = |V|$ qubits in the state $|0\rangle^{\otimes n}$.

Apply the gate X into the qubit $|v_0\rangle$.

Initialize the set of steps to be handled M with the initial state $M = \{v_0\}$

while $M \neq \emptyset$ **do**

Apply the oracle $O(M, \{Succ(v_t), \forall v_t \in M\})$.

Empty M, $M = \{\}$.

For each $v_i \in Succ(v_t)$ add v_i to M if $Succ(v_i) \neq \emptyset$ and $v_i \notin C$.

end

Measure the circuit and extract the set of paths P_s.

Return: P_s

Complexity analysis

- Classic case: Using the BFS algorithm, the complexity is $\mathcal{O}(n \cdot 2^n)$. With n is the number of vertices.
- Quantum case: For memory complexity, we use n qubits. For the computational complexity, we use at most m quantum gates, where m is the number of arcs.
- Needs scaling

Hybrid approach to find paths in an a-cyclic directed graph

Algorithm 2: Hybrid Quantum algorithm to obtain all paths in an acyclic graph (HQAPAG)

input : A weighted acyclic graph $\mathbb{G} = (\mathbb{V}, \mathbb{E}, \mathbb{C})$, source v_t, number of qubit available in the quantum computer n_q, and the minimal probability of the paths P_{min}.

output: P_g The set of paths from an initial node to marked nodes

init *Initialize the set of walks M to be processed with the source $\{v_t\}$ and the set of paths that arrived at a given marked vertex at $P_g = \varnothing$ and the current paths at $P_t = \varnothing$.*

$M \neq \varnothing$

v_t is the first item of M

Extract a sub-graph $\mathbb{G}_i$ of $\mathbb{G}$ from v_t such that the number of vertices of $\mathbb{G}_i$ is less than N_q

Extract the paths $paths_i$ from $\mathbb{G}_i$ using the algorithm 1

Calculate the exact probability of each path

Remove the item v_t from the list M

Update the two path sets P_g,P_t and the set M with the new list of found paths $paths_i$ according to P_{min}

return P_g

- Recursively apply the first algorithm to a sub-graph of successors.
- The probability of the sequences is computed classically

Hybrid approach to find paths in an a-cyclic directed graph

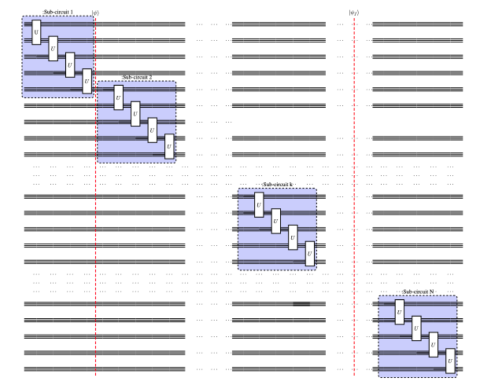
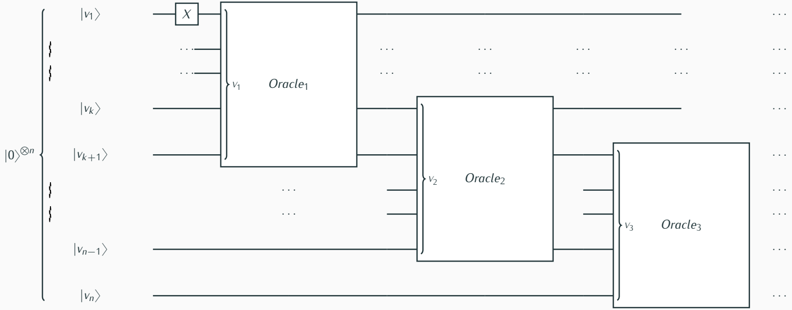


Figure 4: Subdivision of the big circuit for a big graph

Hybrid approach to find paths in an a-cyclic directed graph



Tests and results

Number of found paths (Maximum time: 20s, $P_{min} = 10^{-8}$, Qasm IBM quantum simulator with 32 qubits)

Graph	1	2	3	4	5	6
Number of vertices	50	60	70	80	100	120
Number of edges	235	285	335	385	485	585
The exact number of paths	2463	17134	50989	317324	389869	1475909
NP founded by RW	2060	5182	6682	7726	8762	9405
NP founded by our approach	2463	17134	50989	317324	389869	1475909

Analysis of the running time

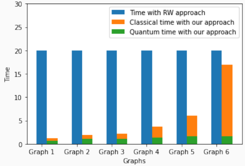
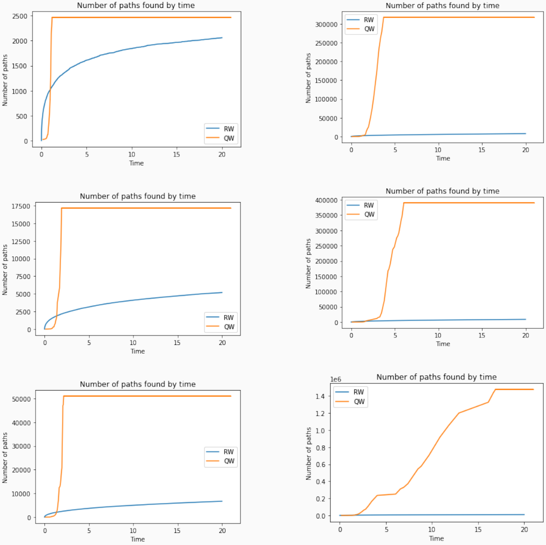
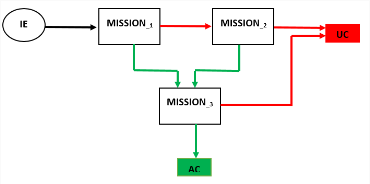


Figure 5: The running time spent for each approach



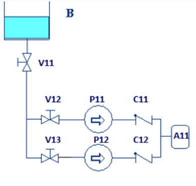
Hybrid approach : fault trees to s-t networks

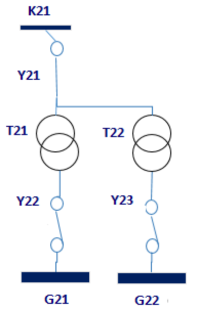
How to transform accident sequences to a network?

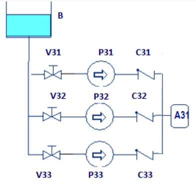


- **IE**: Initiating event
- AC: Acceptable consequence
- UC: Unacceptable consequence
- Mission: {function, conditions, duration }
- →: Mission failure
- ↓ : Mission success

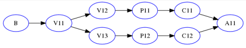
How to transform accident sequences to a network?

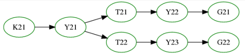


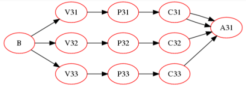




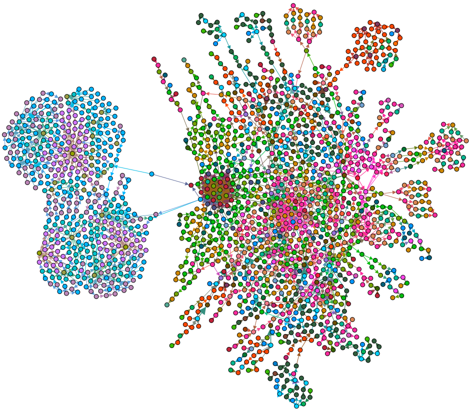
Networks







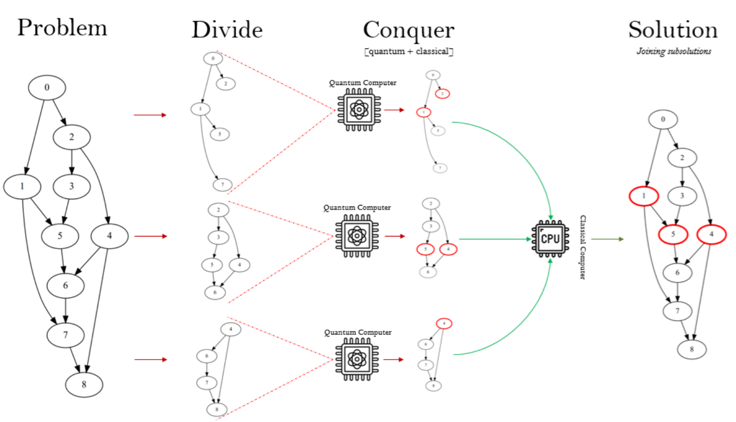
Small real life instance



- Network: directed and attributed (edge types, vertices types, other components properties)
- # Vertices: $N = 1700$
- # Edges: $M = 2700$
- Density: 0.009
- Diameter: 33

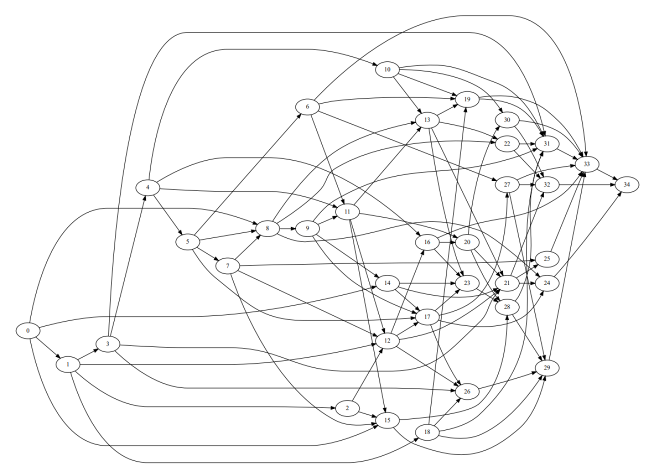
Example with s-t networks

An hybrid version of vertex separator algorithm[6]



[6] with Pedro Henrique Pons Fiorentin and Rola Saidi (Internship EDF R&D)

Generated representative instance



Performances

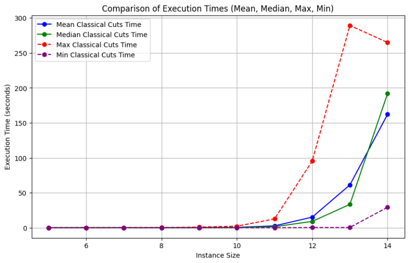


Figure 6: Performance of classical recursive algorithm over increasing instance sizes.

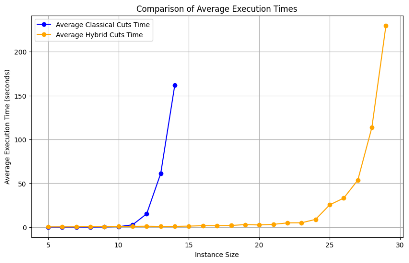
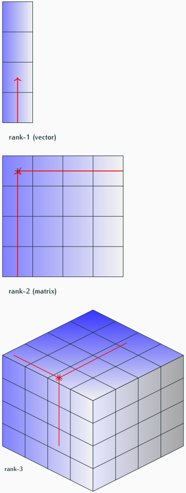


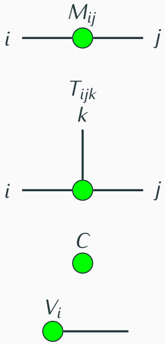
Figure 7: Comparison of the hybrid algorithm running time against a recursive classical method.

Quantum Inspired Algorithms : Tensor networks

Tensor networks for PSA

- A scalar is a 0-tensor (tensor of rank 0) node with no edge.
- A vector is a 1-tensor (tensor of rank 1) node with one edge.
- A matrix is a 2-tensor node with 2 edges where the first represent its line's index and the second its column's index.
- A 3-tensor is a node with 3 edges.





Tensor networks can be seen as

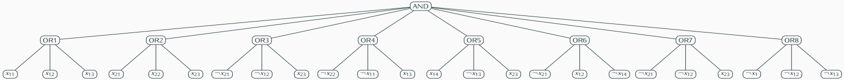
- An intermediate step to Grover resolution
- Compression involves counting
- Counting allows for efficient Grover calibration
- A classic and efficient Grover simulation[7]

[7] Stoudenmire, E. M., & Waintal, X. (2024). Opening the black box inside grover's algorithm. Physical Review X, 14(4), . http://dx.doi.org/10.1103/physrevx.14.041029.

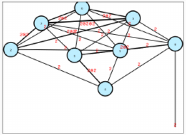
Contraction example

Tensor network of a fault tree

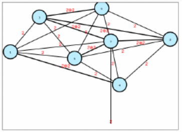
$$f = (x11 \lor x12 \lor x13) \land (x21 \lor x22 \lor x23) \land (\neg x21 \lor \neg x12 \lor x23) \land (\neg x22 \lor \neg x11 \lor x13) \land$$
$$(x14 \lor \neg x13 \lor x23) \land (\neg x21 \lor x12 \lor \neg x14) \land (\neg x21 \lor \neg x12 \lor x23) \land (\neg x1 \lor \neg x12 \lor \neg x13)$$



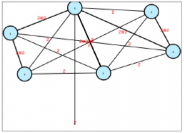
Initial network



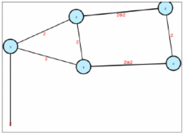
Contraction ord=3



Contraction ord=4



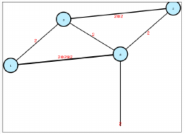
Contraction ord=5



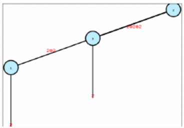
Contraction example

Last contraction steps and collapse to a scaler

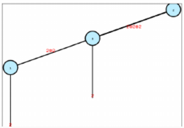
Contraction ord=2



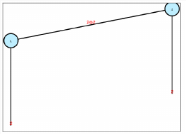
Contraction ord=5



Contraction ord=5



Contraction ord=6



Collapse à un scalaire

```
Summing out index:  (dim=2|id=736|"x1")

Summing out index:  (dim=2|id=708|"x12")

Number of satisfying assignments:  2550.0
```

First results and limitations

- Implementation using Itensor Julia library[8]
- Up to some order (14) it is easy to find all the cutsets
- Beyond Tensor orders 15, only partial solutions were obtained.

[8] Fishman, M., White, S. R., & Stoudenmire, E. M. (2022). The ITensor software library for tensor network calculations. SciPost Phys. Codebases, (4), http://dx.doi.org/10.21468/SciPostPhysCodeb.4.

Conclusions and perspectives

Conclusion

- Different algorithms to deal with static and dynamic approaches of safety assessment
- The general algorithms are qubits demanding and should be tested on state of the art quantum computers
- The hybrid approaches seem promizing and showed some advantage over classical approaches.

Perspectives

- New technologies/formalisms have been developed with spin-offs about our disciplines
  - graphical languages (Tensor networks open new directions)
    - Use tensor networks for splitting big instances
    - Adapt for parallelism/distributed computation.
- The hardware is progressing and being diversified (cold atoms, trapped ions, photonic, electrons superconducting and spin)
- A very important new field to explore
- Exploring other frameworks
  - Ongoing PhD (Rola Saidi Lille/Louvain university)