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Welinq

Distributed Quantum Algorithms: PDEs and MIS problems

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09/10/2025

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Overview

- Intro to Distributed Quantum Computing
  - Distributed Quantum Algorithms
  - Protocols for non-local operations
- Applications:
  - Distributed Quantum Variational PDEs Solver
  - Analog Distributed Quantum Algorithm for MIS on neutral atoms QPU

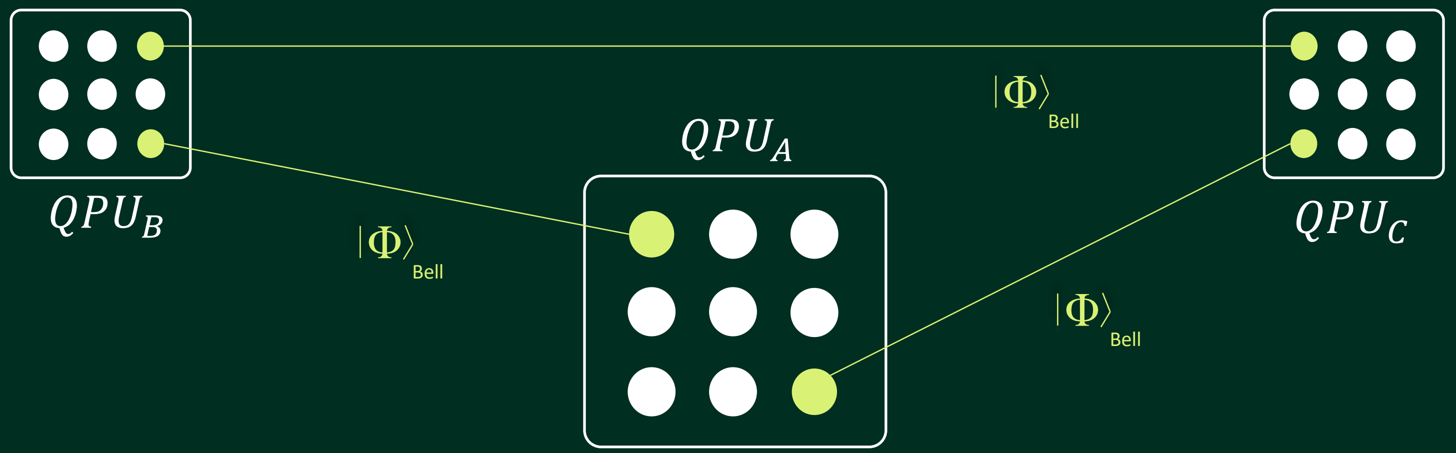
Distributed Quantum Computing (DQC)

Problem

All major quantum computing technologies exhibit hard technological limitations on the number of qubits that can be embedded in a single quantum chip [1].

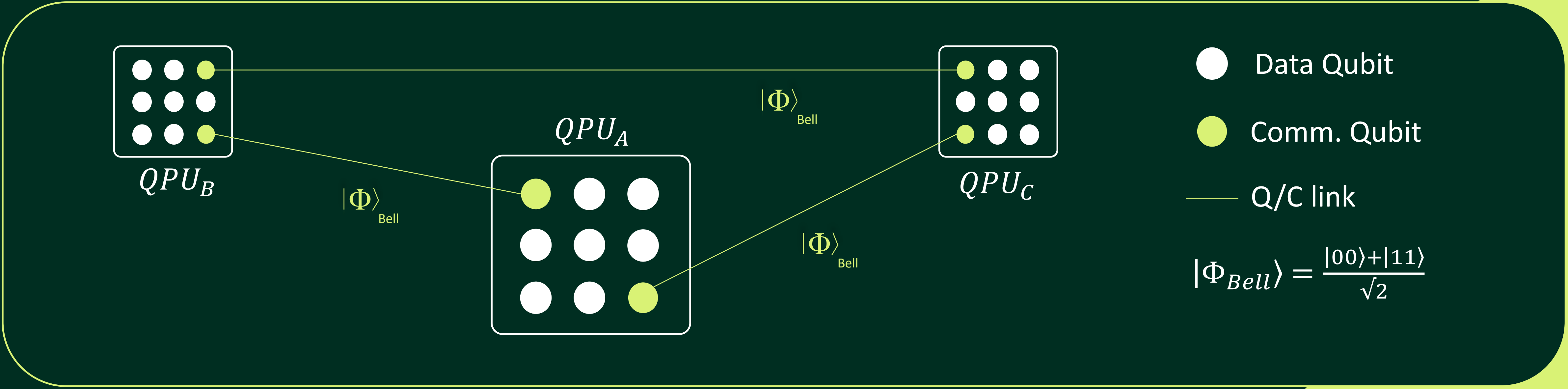
DQC is a viable path to scalability

Individual quantum processors work together to solve computational tasks exceeding the computational resources available within a single processing device



Ref [1]: Review of Distributed Quantum Computing. From single QPU to High Performance Quantum Computing

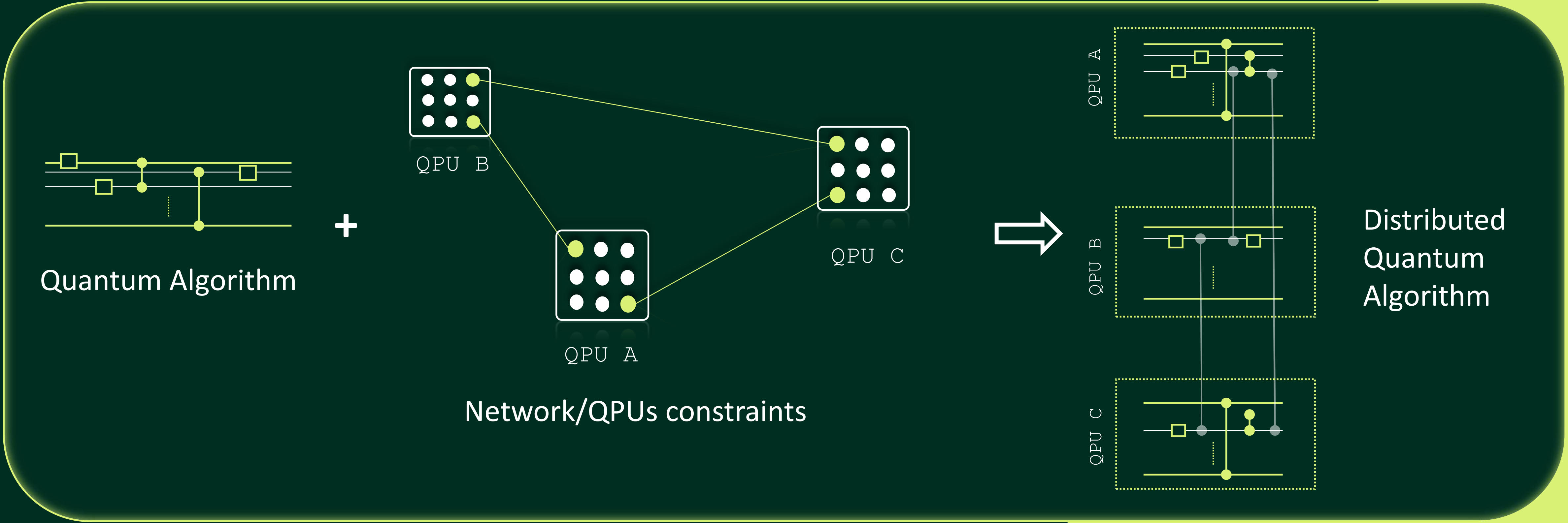
Distributed Quantum Computing (DQC)



- ***Emulating large quantum computer*** as networks of ***interconnected*** smaller quantum computers, allows to ***scale the number of qubits.***
- ***Quantum communication protocols*** (LOCC) are used to allow operations between QPUs (TeleData, TeleGate, Entanglement Swapping) consuming pre-shared Bell pairs between QPUs
- Each QPU is capable of managing ***data qubits*** dedicated to computation and ***communication qubits*** dedicated to implementation of non-local operations

Distributed Quantum Algorithms

Algorithms demanding large number of qubits are ***partitioned***, with its fragments executed on QPUs of smaller qubit capacity, constituting a ***quantum network***

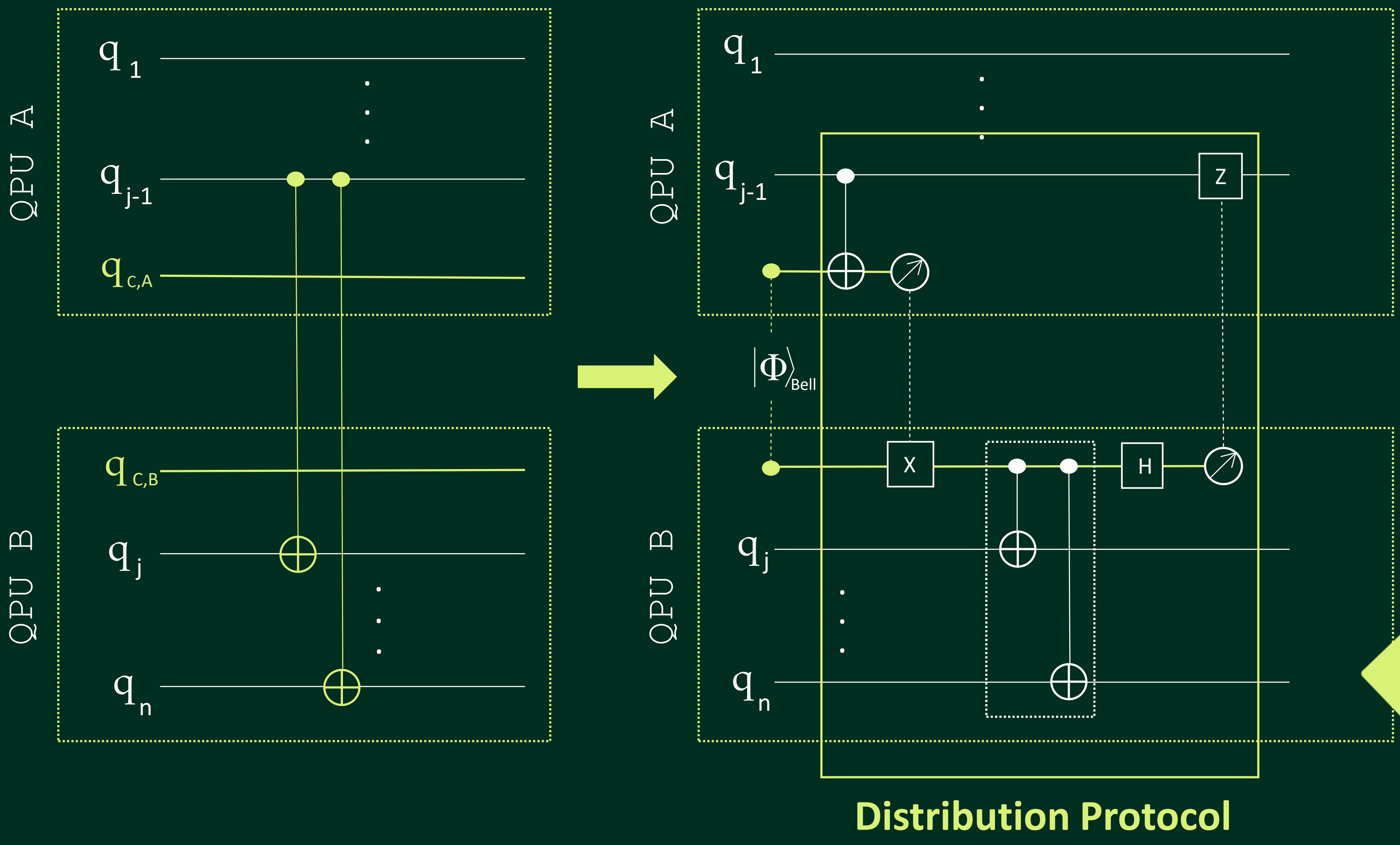


- How to implement non-local operations ? Quantum Protocols

[Ref [2]: Distributed Quantum Computing: a Survey]

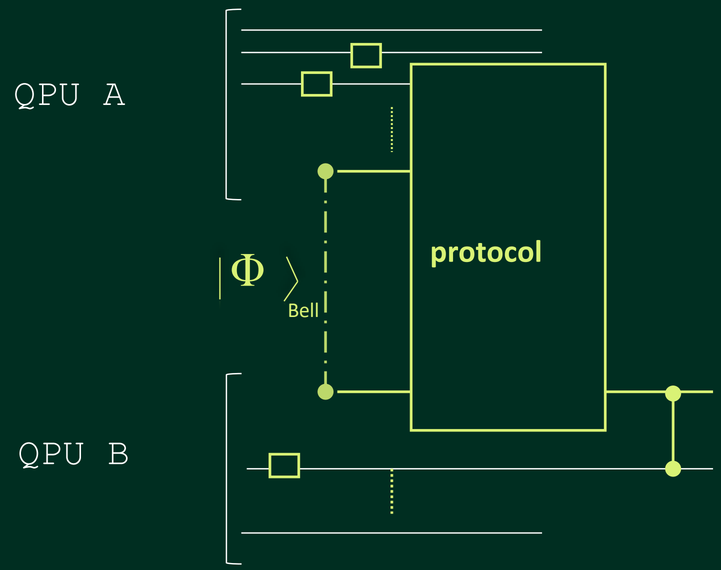
Protocols for non-local operations

Entangled state is consumed to perform non-local operation



Protocols for non-local operations

Introduce **quantum link** to interconnect & use **entanglement**



Entangled state is consumed to perform non-local operation

Alternatively: Circuit-Cutting

Cut into sub-circuits and execute in parallel. ***No physical entanglement used***, but o**verhead in sampling with exponential complexity.**

- **Entanglement is a precious resource, hence it is crucial to minimize the number of inter-QPU operations (Distribution Cost)**

Distributed Quantum Algorithm for Variational PDEs Solver

(Work in Progress)

Welinq + EDF + QUANDELA

Contributors: Welinq Algorithm Group, Kyrylo Kazymyrenko (EDF), Samuel Donachie (EDF)

Distributed Quantum Algorithm for Variational PDEs Solver

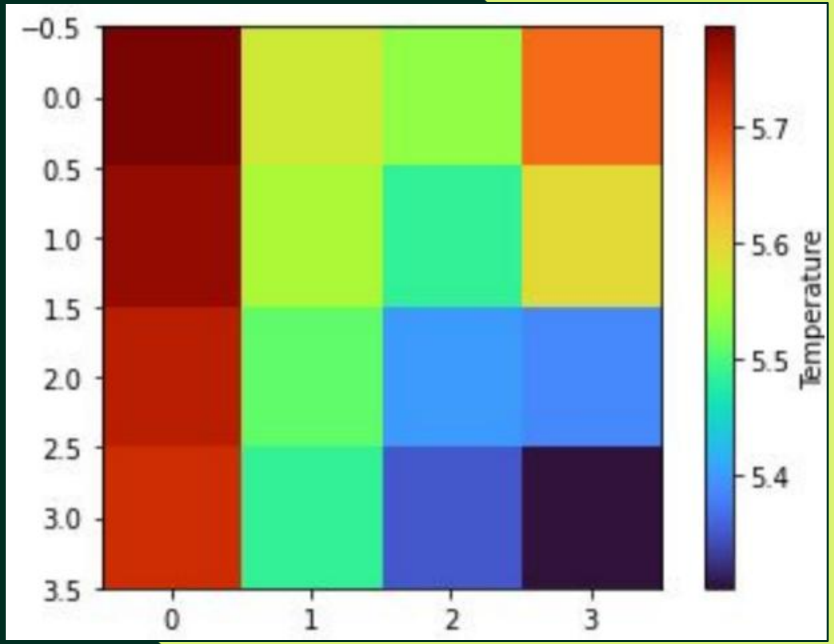
- **The resolution of partial differential equations (PDEs)** has applications in virtually all fields of science. Here we focused on **heat diffusion in materials.**

- In a stationary regime, we can place the problem in a discrete form, using a mesh and finite elements methods.

$$K|x\rangle = |f\rangle$$

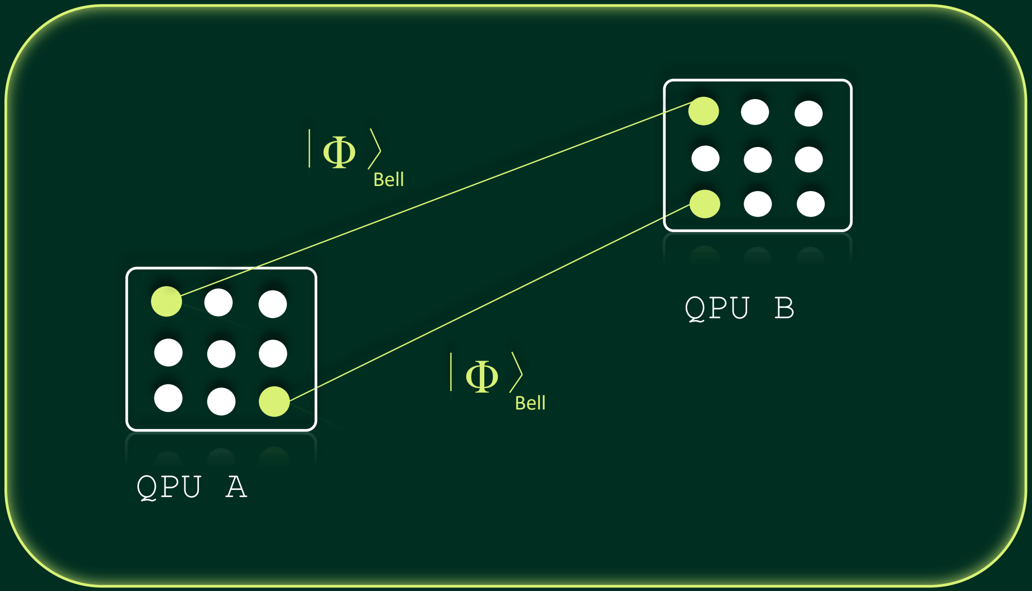
Variational Quantum Algorithm (VQA) is the standard approach to solve PDEs where the **solution vector** $|x\rangle$ **is approximated** on a quantum computer using a **parameterized state** $|\boldsymbol{\psi}(\boldsymbol{\theta})\rangle$ which optimizes the **cost function**

$$\mathrm{C} = -\frac{|\langle f|\psi(\theta)\rangle|^2}{\langle\psi(\theta)|K|\psi(\theta)\rangle}$$



Distributed Quantum Algorithm for Variational PDEs Solver

- Previous works based on a single monolithic algorithm. Here **assumed a network of two interconnected QPUs** with **limited number of quantum links** connecting them



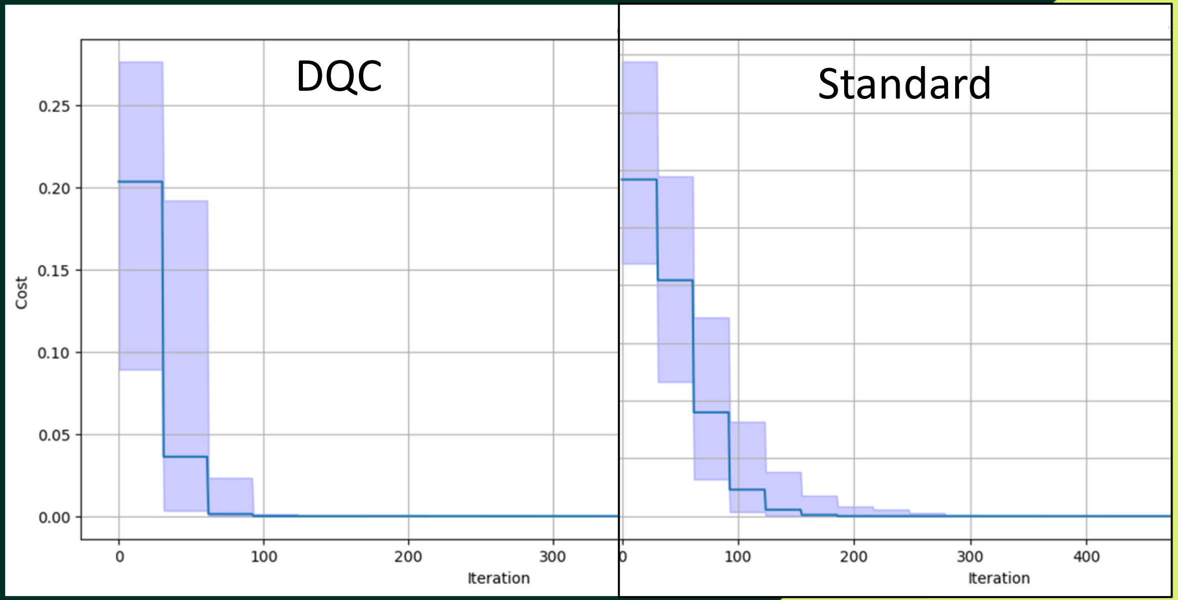
Objective: find a **natively Distributed Quantum Algorithm** able to **minimize the distribution cost** while also **accurately solving the PDEs,** and **compare it with the state of art quantum algo** used in PDE solvers

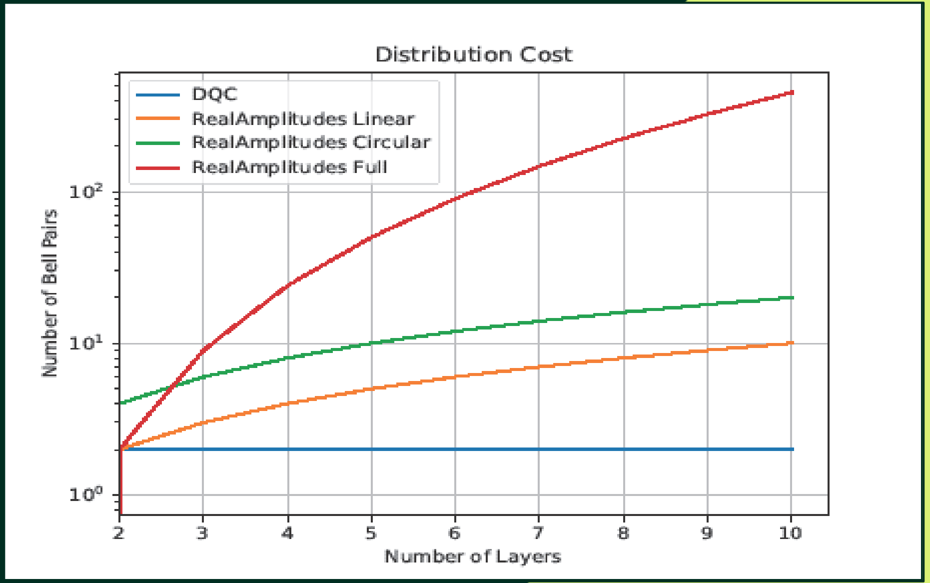
Distributed Quantum Algorithm for Variational PDEs Solver

- Proposed distributed quantum algorithm **converges faster** and achieves **comparable or better solution quality**

- Proposed distributed quantum algorithm has **lower distribution cost than the standard one**

- **Next**: Simulations with increased problem size and hardware specifications in collab. with Quandela





Analog Distributed Quantum Algorithm for MIS on neutral atoms QPU

(Work in Progress)

Welinq + EDF + PASQAL + SORBONNE UNIVERSITÉ

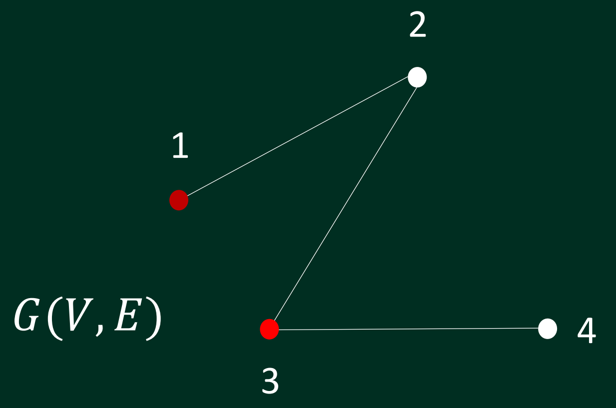
Contributors: Welinq Algorithm Group, Pascale Bendotti (EDF), Alexandra Sheremet (Pasqal), Christoph Durr (Lip6-SU)

Analog Distributed Quantum Algorithm for MIS on neutral atoms QPU

- **Smart Charging**
  Allocate charging requests of EVs so that conflicting requests are avoided

MIS Graph Problem

Smart Charging		***Graph***
Charge request by EV	$\leftrightarrow$	Vertex v_i
Conflict between charge requests	$\leftrightarrow$	Edge $(v_i v_j)$
Max nr. of EVS that can charge the same time	$\leftrightarrow$	MIS



Independent Set (IS): S= [(1,4), (2,4), (1,3)]

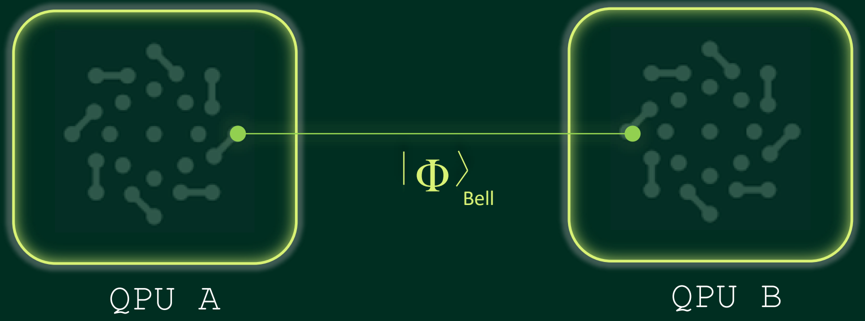
Maximal Independent Set (MIS): IS with maximum number of vertices

MIS problem can be cast **as binary optimization problem** and **solved** using **QAOA or Adiabatic evolution algorithms on a single neutral atom QPU**

$$E(x) = -\sum_{i\in V} \delta_i x_i + \sum_{(i,j)\in E} V_{ij} x_i x_j$$

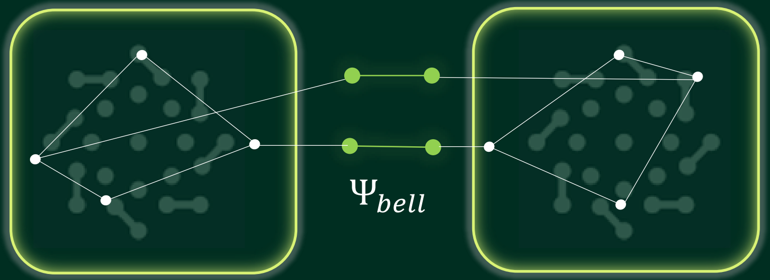
Analog Distributed Quantum Algorithm for MIS on neutral atoms QPU

- Here we assumed a network of **two interconnected neutral atom QPUs** with **limited number of quantum links** connecting them (with **analog evolution,** distribution protocols not possible)

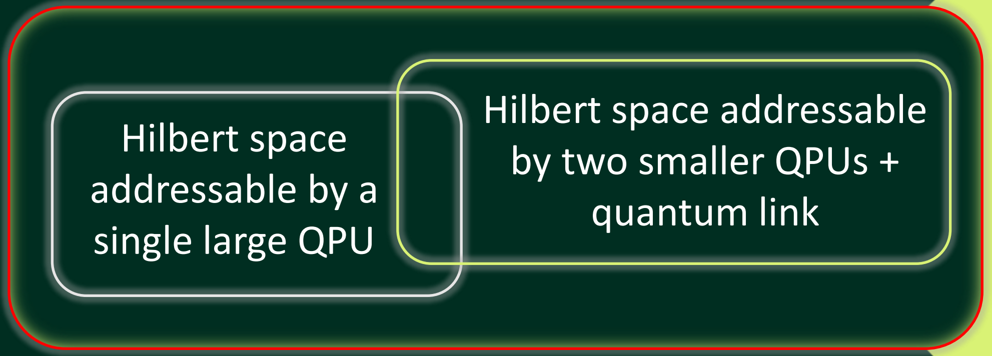


In general:

- Possible to embed graph topologies beyond the limits of a single device



- This allows to address different portions of the Hilbert Space with the same algorithm

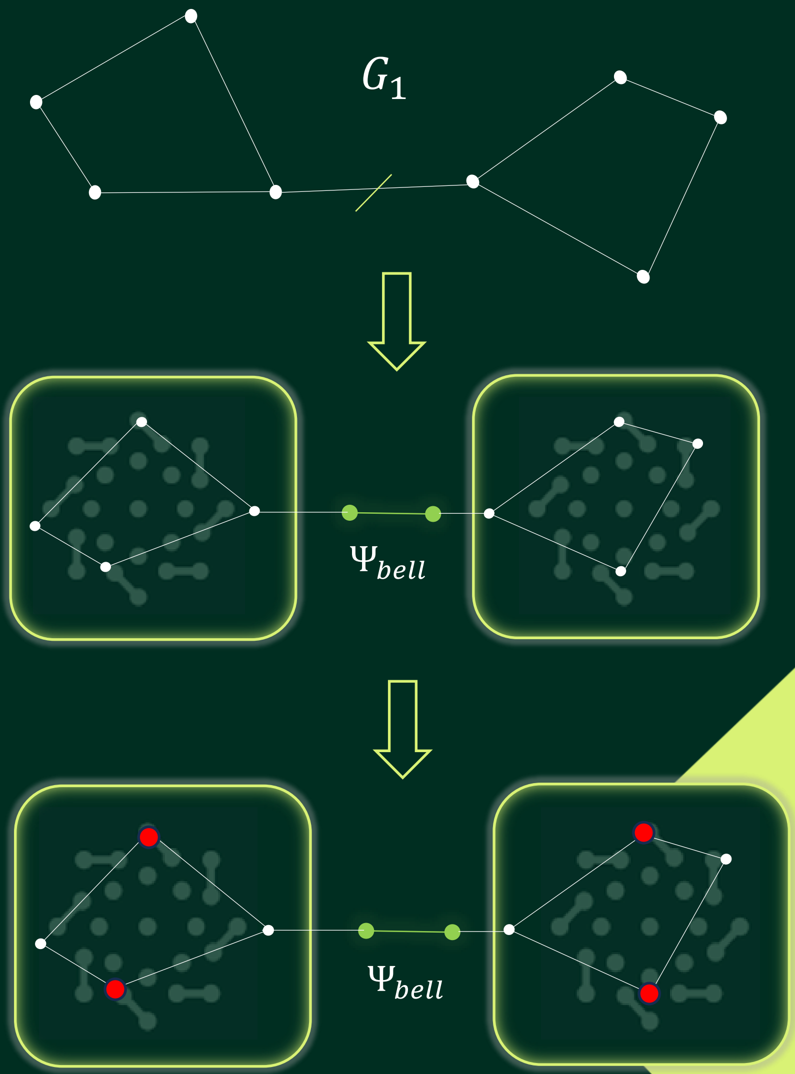


Full Hilbert Space

Analog Distributed Quantum Algorithm for MIS on neutral atoms QPU

For MIS:

- **Exploit property of MIS**: Long edges could be broken into smaller edges with the use of additional nodes
- **Bell pair** provided by the link could be **used to physically implement such MIS property**
- **Next**: find an optimal analog distributed algorithm to recover the global MIS



Conclusion

- **Distributed quantum computing (DQC)** offer a viable **path** to **scalability via interconnecting smaller QPUs with quantum links providing entanglement**
- **Minimize consumption of entanglement used for distribution is key to avoid bottlenecks**
- **In the near term, it is crucial to study natively Distributed Quantum Algorithms *to reduce the distribution cost and extend the range of possible applications***
- **Some use-cases may show a natural fit to DQC and benefit from a DQC approach**

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