

Hybrid Quantum-Classical Benchmarking – Assessing workflows that combine quantum and classical computation

PD Dr. habil. Jeanette Miriam Lorenz
Fraunhofer Institute for Cognitive Systems IKS
& LMU Munich

An example problem in optimization: waste management

Planning of efficient garbage collection routes

Infineon sensors & actuators detect fill level of containers

Task:

- Optimize routes for waste collection vehicles with QC assistance.
- Build on filling level simulation a few days in advance.
- Goal: More efficient (ecologic and economic) routes.

Even 1-2% increase of efficiency would make a great impact.



Infineon
Loudspeaker &
Microfons



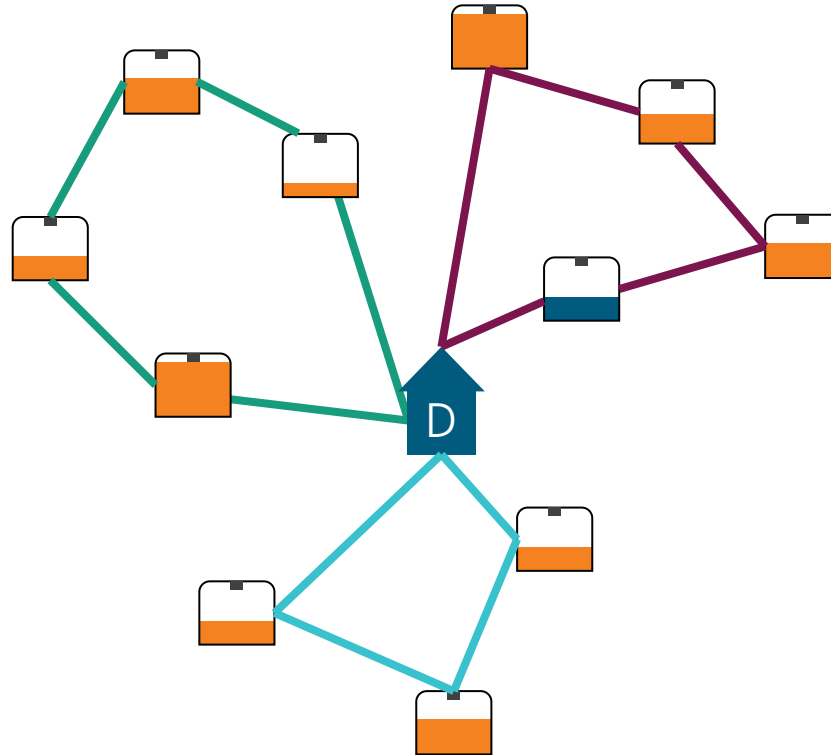
Can quantum computing come to help?

Near-term algorithms for addressing optimization problems:

- Quantum Approximate Optimization Algorithm (QAOA)
- Variational Quantum Eigensolver (VQE)

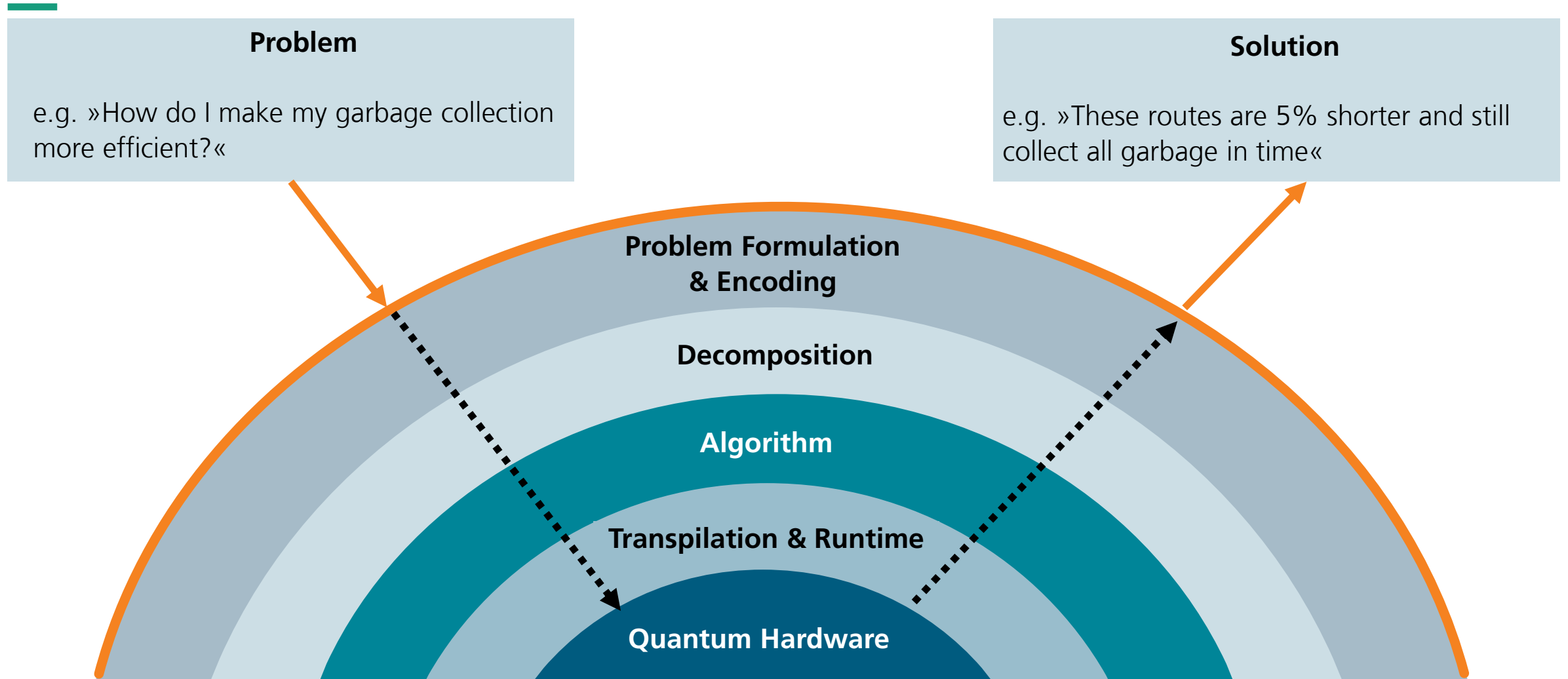
In both cases **variational algorithms**, **(potentially) affected by noise**, require a **classical optimizer** (which?)

Quantum advantage not proven.



Decoherence
Noise
Parameterized
Hardware-Efficient
QAOA Time Pulse
Logical Fidelity
Connectivity Algorithm
Physical Readout
QUBO Trapped VQE Depth
Superconducting Qubit Toffoli
Ions Annealing
Entanglement
Capability Ansatz
Decomposition
Circuit
Encoding

An abstraction layer for QC-assisted solutions of optimization problems



BMWK-project: Quantum-enabling Services and Tools for Industrial applications

QuaST decision tree

Realizing quantum-assisted solutions to optimization problems requires the non-trivial connection of different algorithms & software layers.

E.g.:

- Which (quantum) algorithm to choose to obtain advantage over classical solvers?
- How to control interplay with classical optimizers?

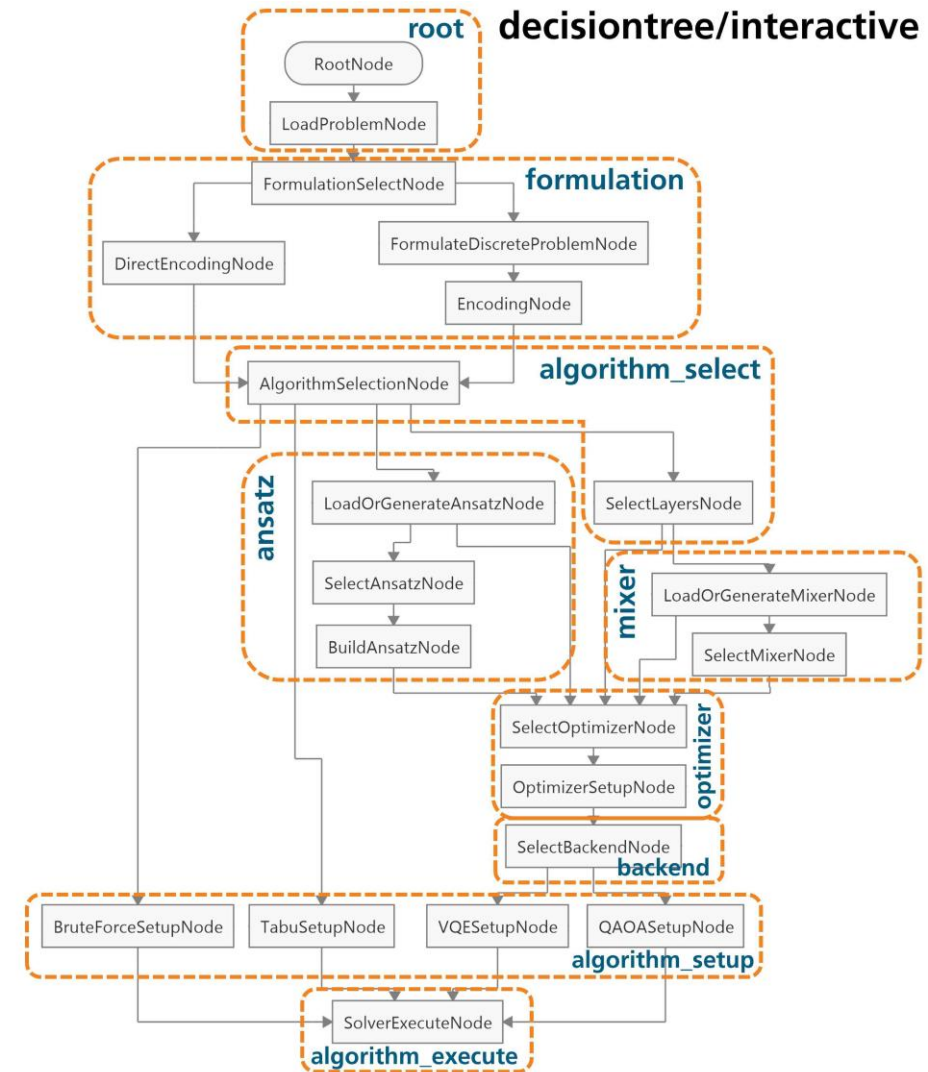
Solution:

Decision tree - an industrial end user can simply specify the application problem and obtains a guide which encoding, algorithm, classical optimizer, quantum hardware etc. to select.

"Recommending Solution Paths for Solving Optimization Problems with Quantum Computing", B. Poggel et al., QSW 2022.

"Quantum-Assisted Solution Paths for the Capacitated Vehicle Routing Problem", L. Palackal et al., QCE 2023.

"Creating Automated Quantum-Assisted Solutions for Optimization Problems", B. Poggel et al., arXiv:2409.20496 [quant-ph]



Supported by:

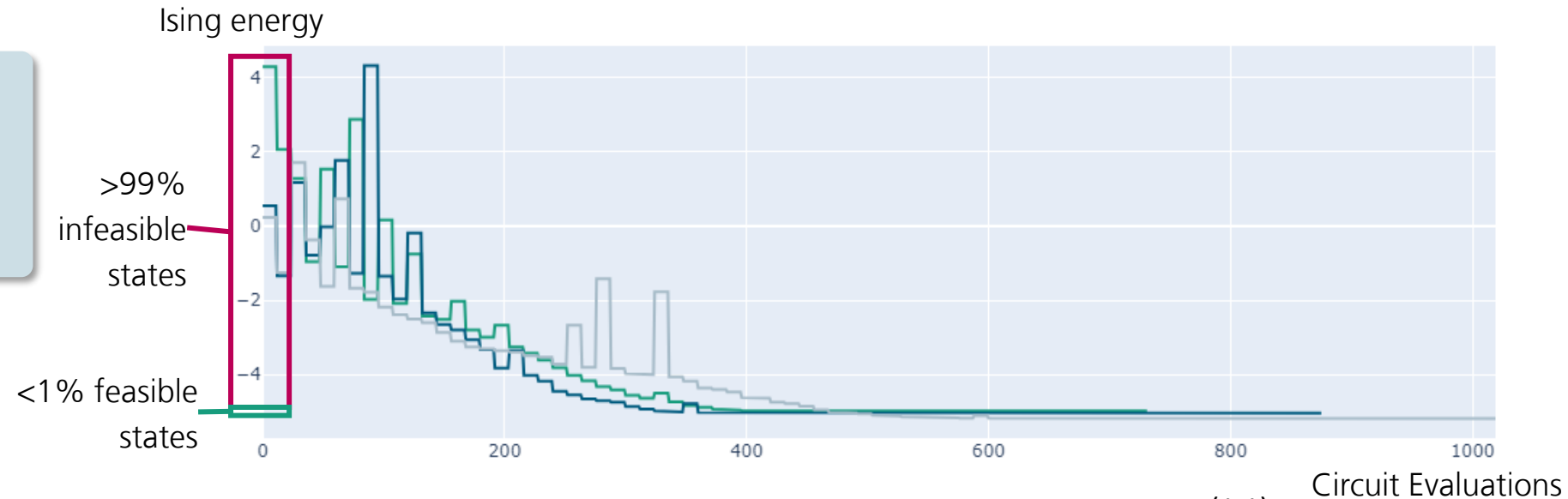


on the basis of a decision by the German Bundestag

How to measure the performance of the resulting algorithm?

From the application perspective

Ising energies are **not the relevant quantity** from an application perspective!



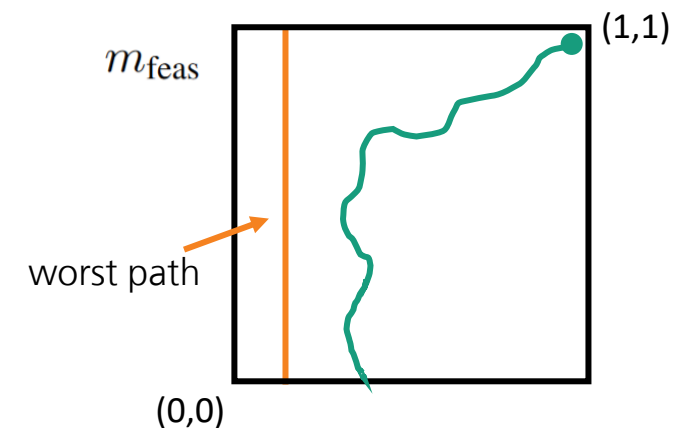
→ we need **application-centric** performance metrics for quantum algorithms

Feasibility ratio:

$$m_{\text{feas}} := \frac{\# \text{ feasible shots}}{\# \text{ shots}}$$

TSP length ratio:

$$m_{\text{len}} := \frac{\text{optimal TSP path length}}{\text{averaged TSP path length for feas. shots}}$$



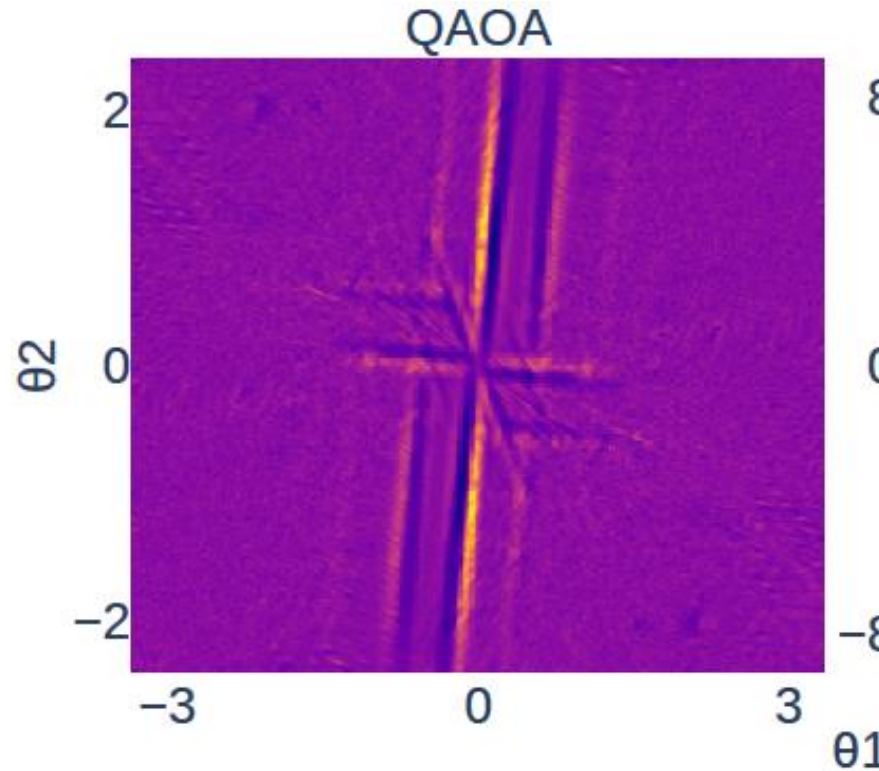
One example challenge coming up

The appearance of the cost landscape for the classical optimizer

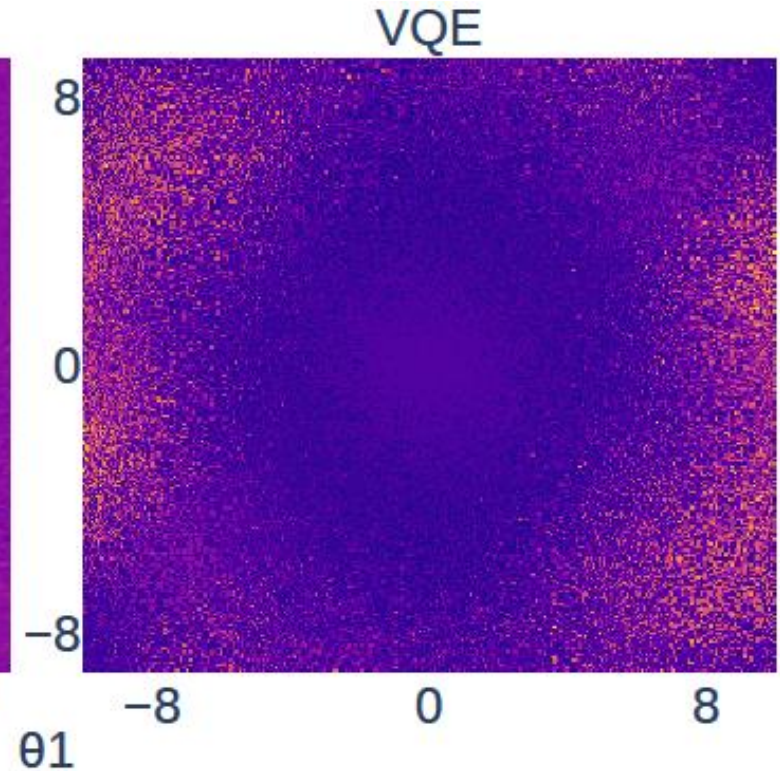
[L. Palackal, B. Poggel, M. Wulff, H. Ehm, J. M. Lorenz, C. B. Mendl, Quantum-Assisted Solution Paths for the Capacitated Vehicle Routing Problem, QCE 2023, arXiv:2304.09629 [quant-ph]]

- Optimizers face challenging **loss landscapes**
- Tradeoff between **number of parameters** and **complexity in a single parameter**
- Influenced by **ansatz** and **encoding**
- Common problem: **barren plateaus** and **high-frequency components/rugged landscapes**

Total parameters: 10



Total parameters: 27



Comparing QAOA and VQE

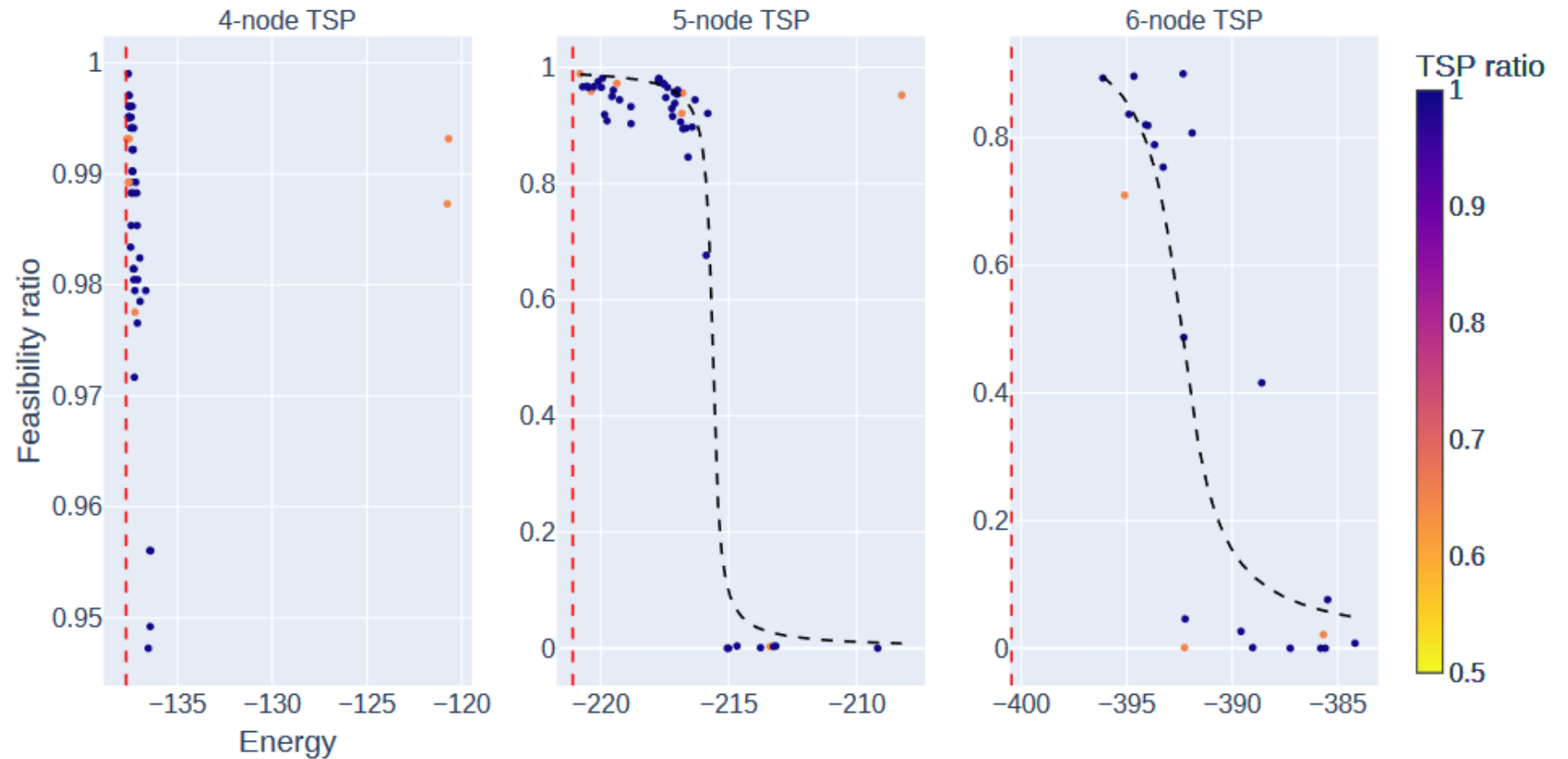


[L. Palackal, B. Poggel, M. Wulff, H. Ehm, J. M. Lorenz, C. B. Mendl, Quantum-Assisted Solution Paths for the Capacitated Vehicle Routing Problem, arXiv:2304.09629 [quant-ph]]

Conclusion:

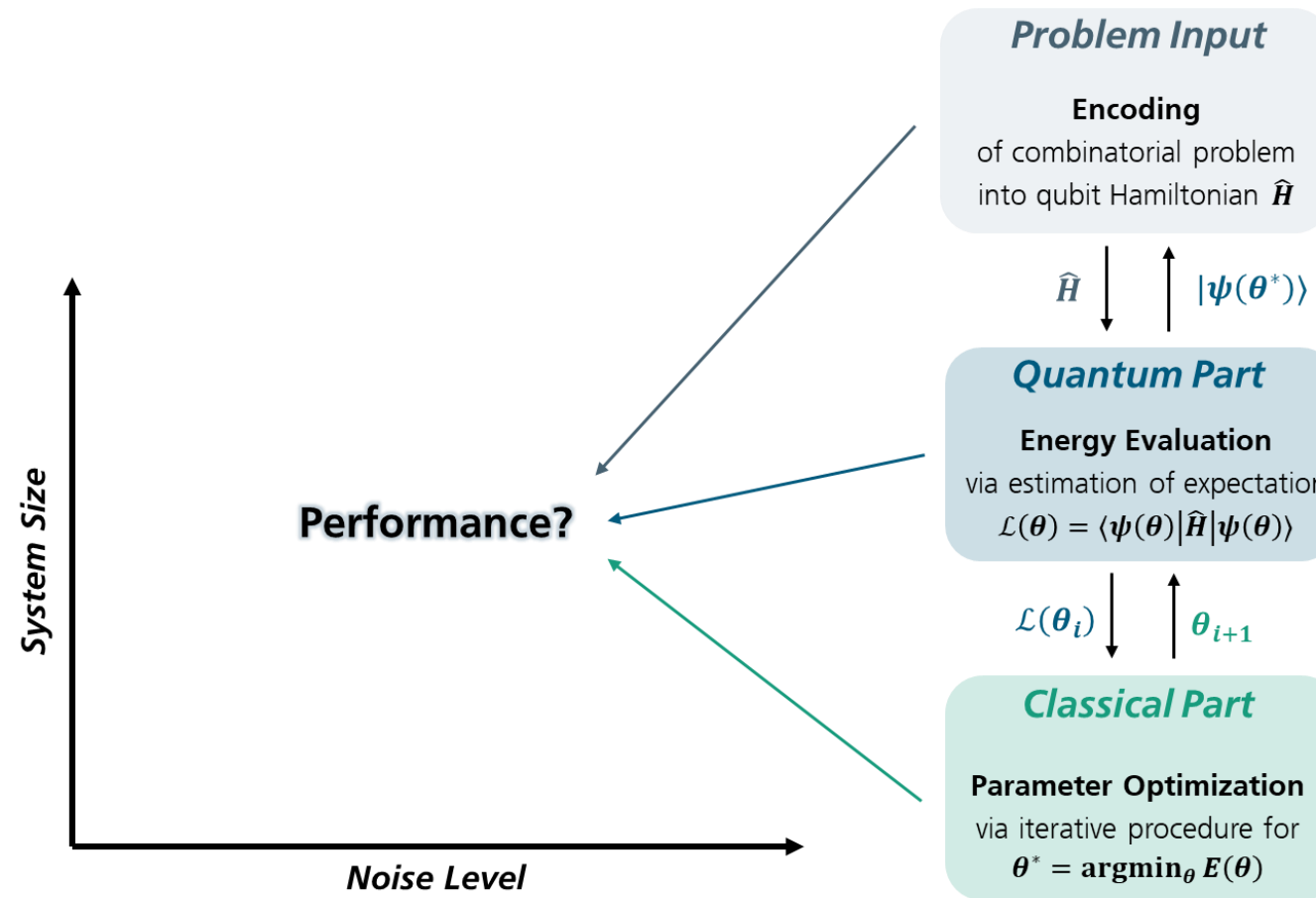
QAOA does not perform well at all, solutions can be obtained via VQE.

For this problem an energy threshold needs to be reached such that the algorithm is able to find feasible solutions.



A systematic benchmarking pipeline for variational algorithms

[Scalability Challenges in Variational Quantum Optimization under Stochastic Noise, A. Bärligea, B. Poggel, J.M. Lorenz, arXiv:2503.14696 [quant-ph]]

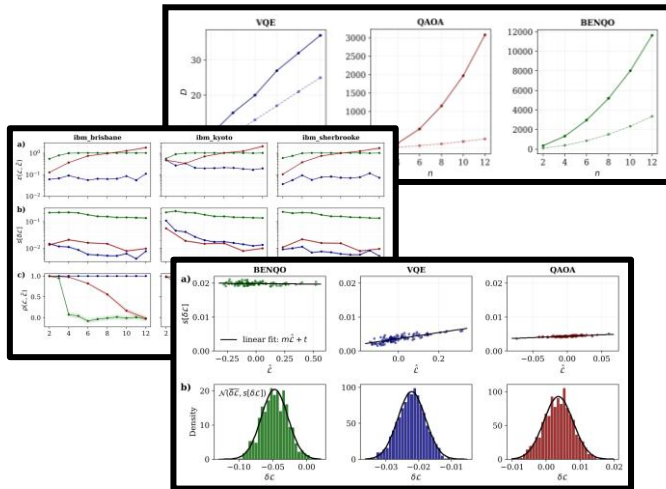


A systematic benchmarking pipeline for variational algorithms

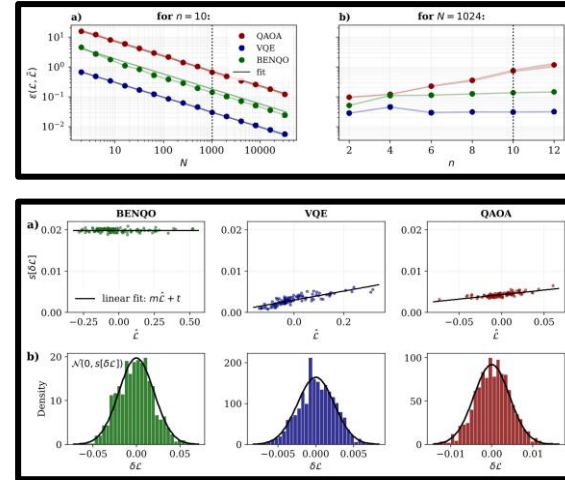
[Scalability Challenges in Variational Quantum Optimization under Stochastic Noise, A. Bärligea, B. Poggel, J.M. Lorenz, arXiv:2503.14696 [quant-ph]]

Analysis of ...

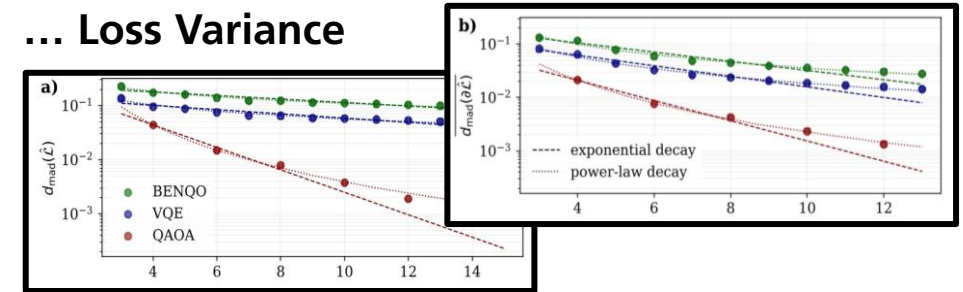
... Hardware Errors



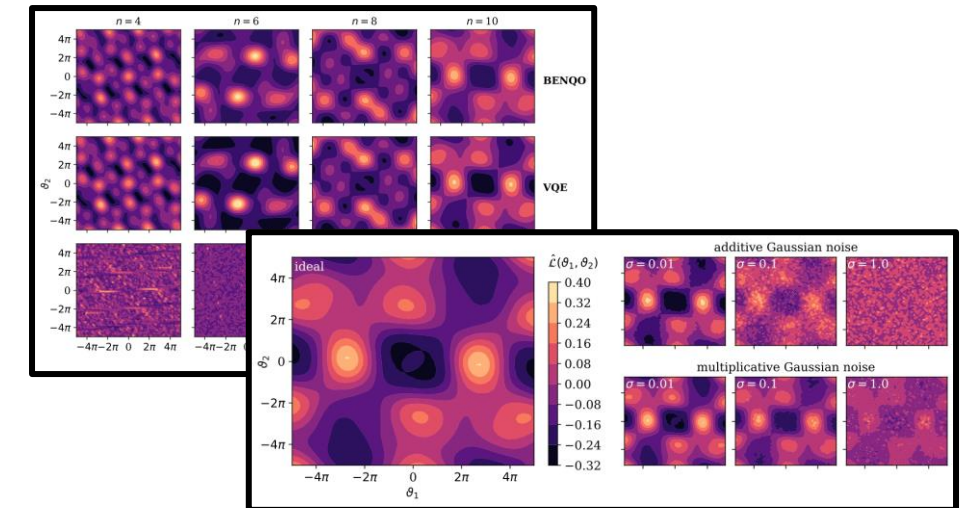
... Finite Sampling Errors



... Loss Variance



... Loss Landscapes

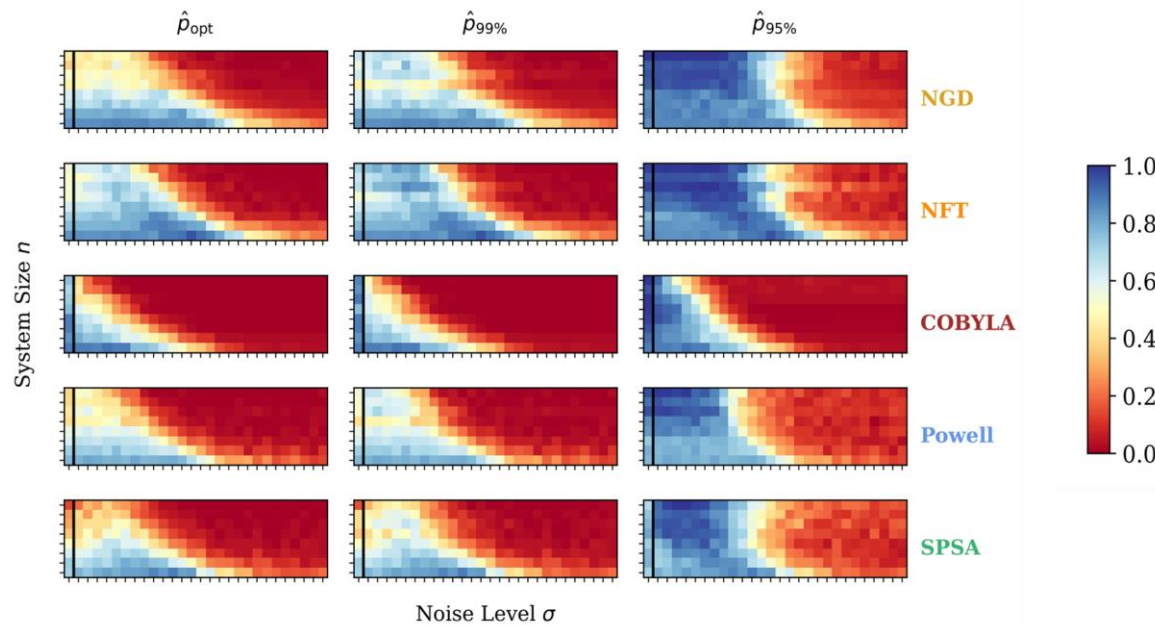


Scaling Behavior
Distribution Behavior

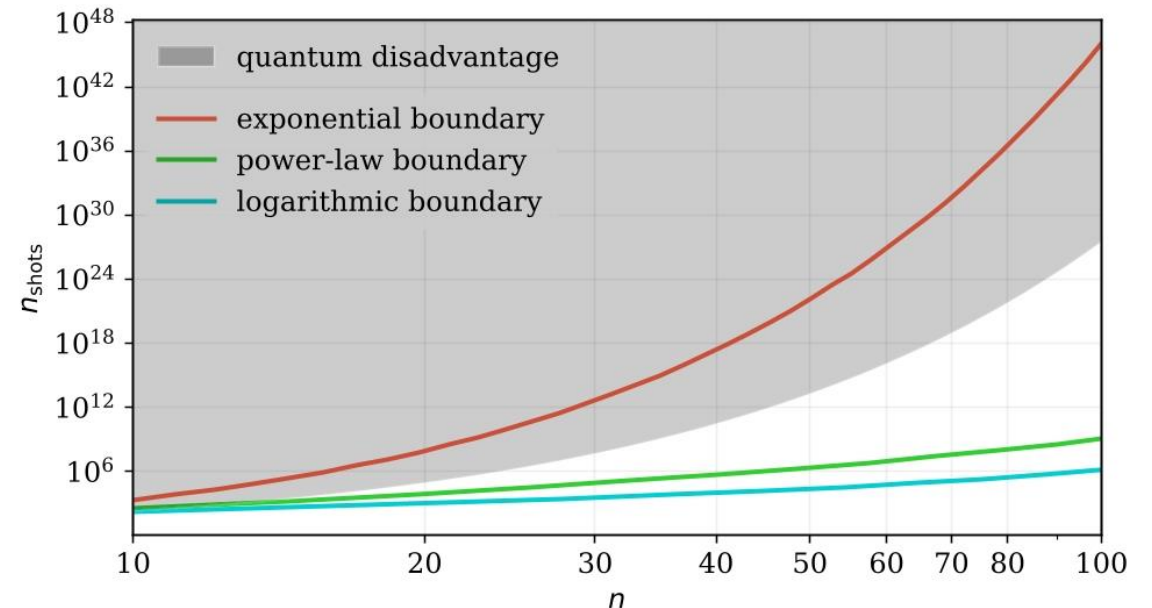
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Sample probability of finding the optimal solution



How many shots do we need for classical optimizers to find useful solutions?



Reliable QC-assisted AI for medical classification tasks

Context: Artificial intelligence increases in importance in the medical diagnosis process (e.g. in imaging).

Challenges: Image data is expensive, complex and only available in small numbers (10^2 - 10^3),

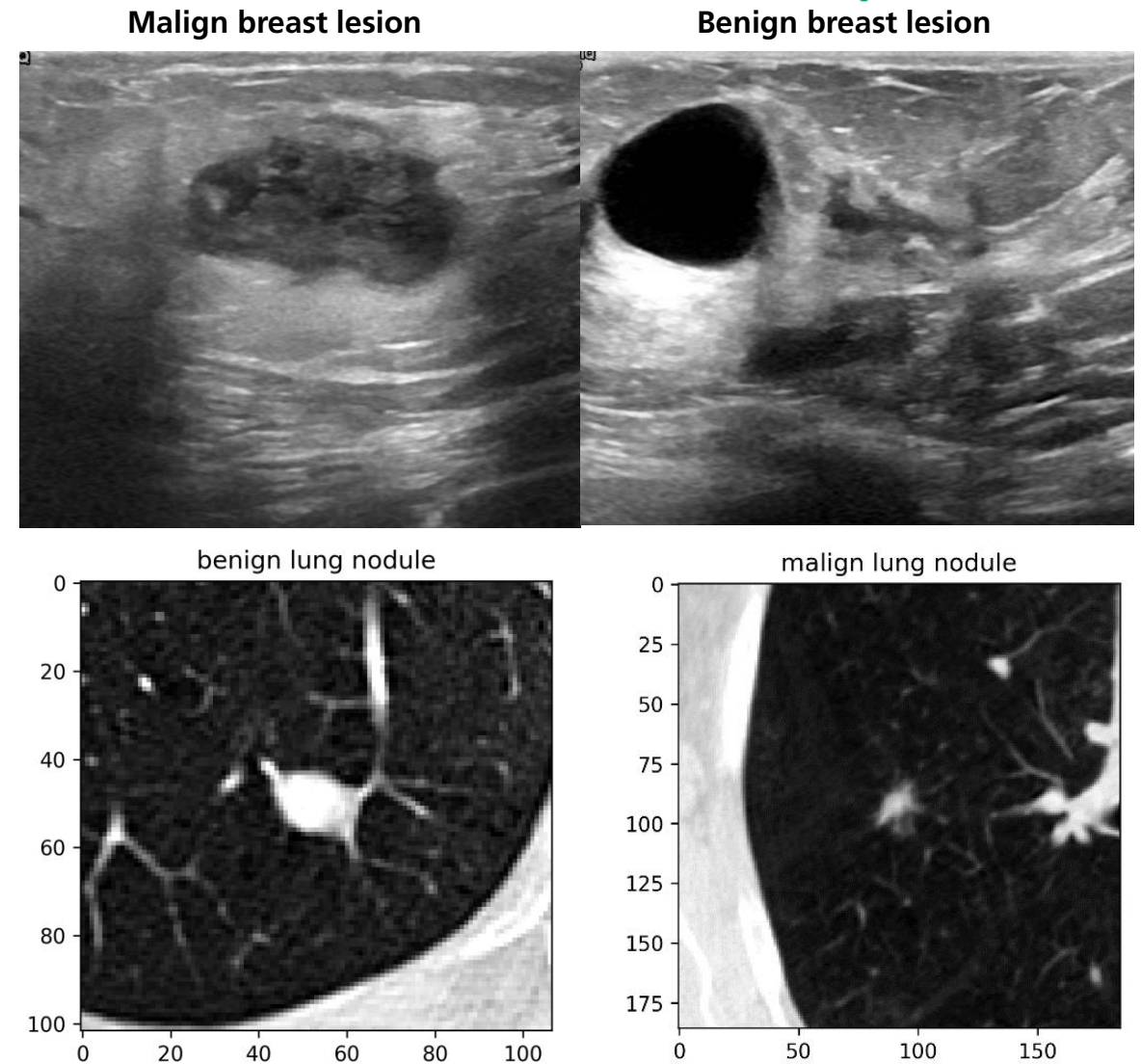
The decision process needs to be comprehensible and reliable.

→ Classical methods need large training datasets.

→ Desirable to propagate uncertainties of data.

Two possibilities on the quantum side to tackle these challenges:

- Quantum Convolutional Neural Networks
- Quantum Bayesian Neural Networks



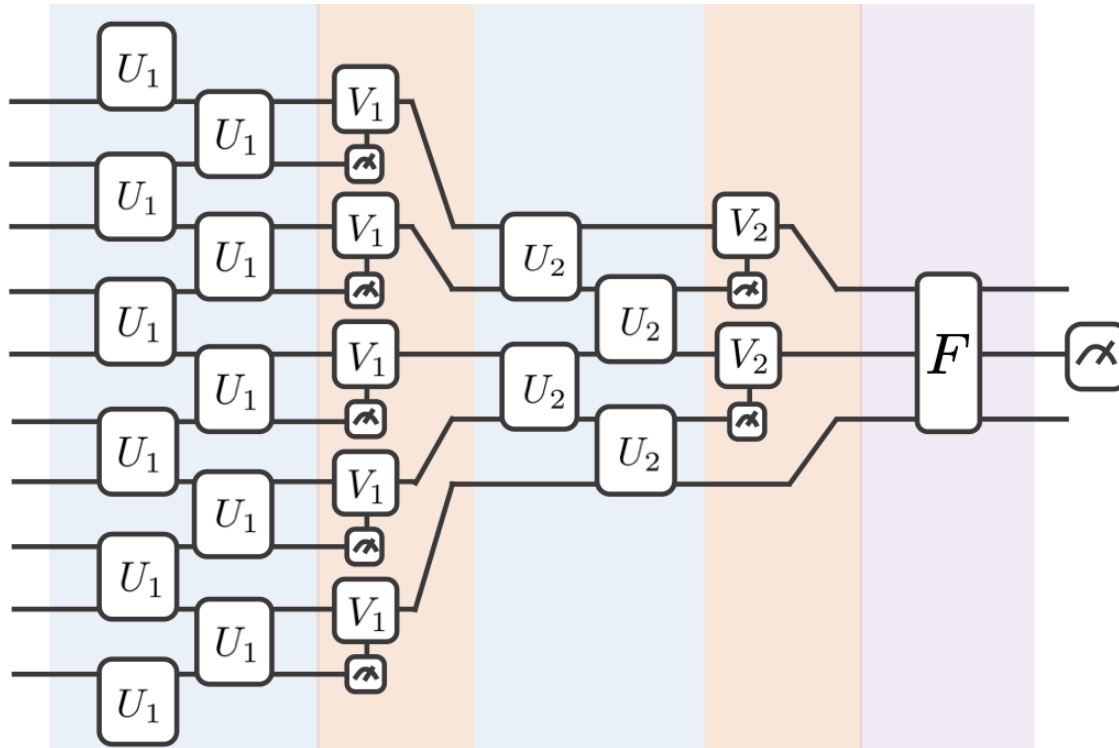
Source: W. Al-Dhabyani, et al, "Dataset of breast ultrasound images". Data Brief, vol 28, pp 104863, 2020

Quantum convolutional neural networks

Generalization with less data

Sources:

- Iris Cong, Soonwon Choi, Mikhail D. Lukin. "Quantum Convolutional Neural Networks" arxiv:1810.03787, 2018.
- Matthias C. Caro, Hsin-Yuan Huang, M. Cerezo, Kunal Sharma, Andrew Sornborger, Lukasz Cincio, Patrick J. Coles. "Generalization in quantum machine learning from few training data" arxiv:2111.05292, 2021.



Architecture of a QCNN as proposed by I. Cong et al.

Studies by M. Caro et al. have shown good generalization properties

The **generalization error** is given by the difference between the expected true loss of a (Q)ML model and the average loss over the training dataset: $gen(\alpha) = R(\alpha) - \hat{R}_S(\alpha)$

Specifically for QCNN particularly favorable: $gen(\alpha) \sim O\left(\sqrt{\frac{T \log MT}{N}}\right)$

for T parametrized local quantum channels, M gates and N the training set size.

Hybrid quantum-classical convolutional neural networks

Collaboration with the LMU hospital, Munich

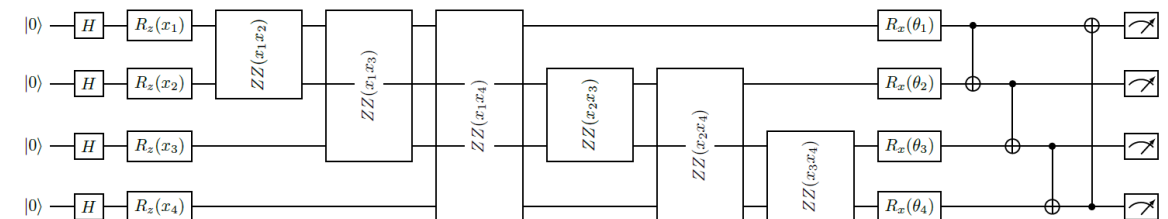
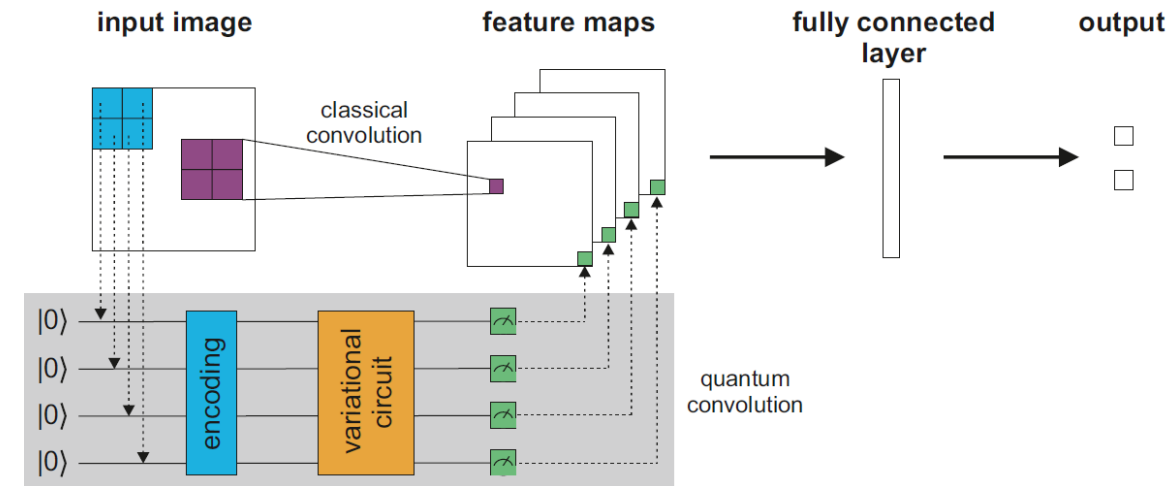


Convolutional neural networks very successful in image classification tasks.

Improvement: Hybrid quantum(-classical) convolutional neural networks promise to be better suited for situations with little training data, potentially leading to a more precise and faster training convergence.

Idea: Replace some of the classical convolutional layers by quantum convolutional layers – also replacement of pooling layers would be possible.

Hybrid ansatz, since only a few classical layers replaced → also possible to execute on the current or soon-available NISQ quantum computers (theoretically).



Hybrid quantum-classical convolutional neural networks

Collaboration with the LMU hospital, Munich

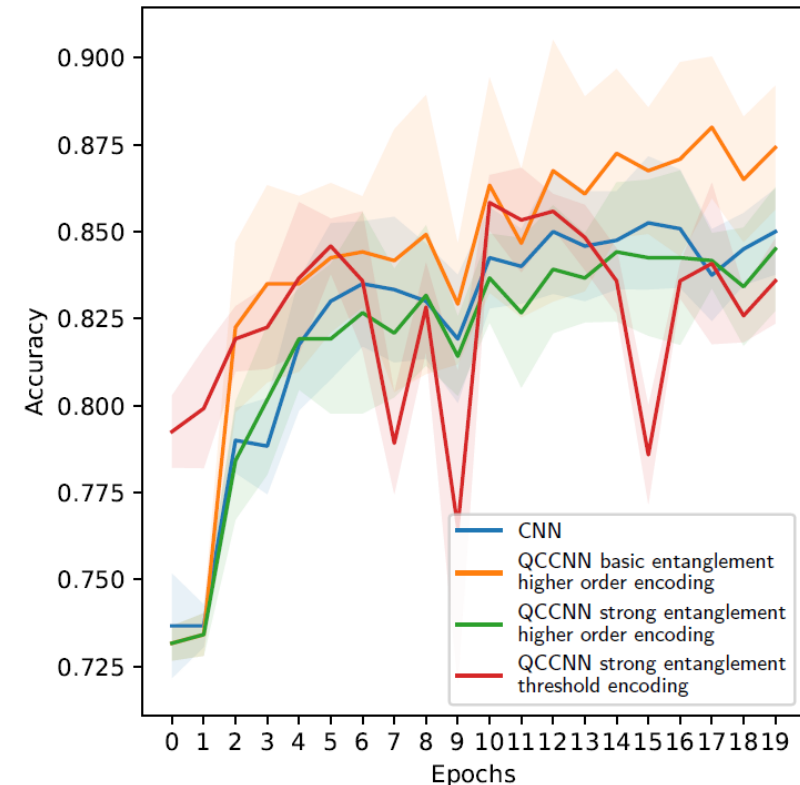


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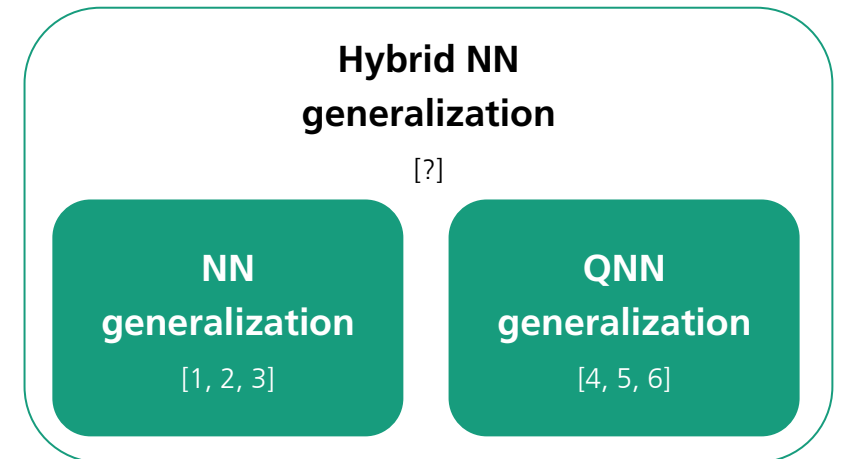
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„Quantum-classical convolutional neural networks in radiological image classification“, A. Matic, M. Monnet, J.M. Lorenz, B. Schachtner, T. Messerer, QCE 2022, arXiv:2204.12390, 2022

Generalization Bounds in Hybrid Quantum-Classical Machine Learning Models?

- **Generalization** is one of the most sought-after ML metrics.
- **Hybrid algorithms** are the only viable solution on the current hardware, and as the technology matures it is expected that hybrid algorithms will continue to evolve and remain relevant beyond NISQ.
- Despite the growing popularity of hybrid models, the conditions under which they generalize accurately remain **largely unexplored**.



[1] Fengxiang He and Dacheng Tao. "Recent advances in deep learning theory". 2021. arXiv: 2012.10931

[2] Amira Abbas et al. "Effective Dimension of Machine Learning Models". arXiv: 2112.04807

[3] Chiyuan Zhang et al. "Understanding deep learning (still) requires rethinking generalization". In: Commun. ACM 64.3 (Feb. 2021), pp. 107–115. ISSN: 0001-0782.

[4] Matthias C. Caro et al. "Generalization in quantum machine learning from few training data". In: Nature Communications 13.1 (2022), p. 4919. DOI: 10.1038/s41467-022-32550-3.

[5] Amira Abbas et al. "The power of quantum neural networks". In: Nature Computational Science 1.6 (June 2021), pp. 403–409. ISSN: 2662-8457. DOI: 10.1038/s43588-021-00084-1.

[6] Hsin-Yuan Huang et al. "Power of data in quantum machine learning". In: Nature Communications 12.1 (May 2021). ISSN: 2041-1723. DOI: 10.1038/s41467-021-22539-9.

Generalization Bounds in Hybrid Quantum-Classical Machine Learning Models

[T. Wu, A. Bentellis, A. Sakhnenko, J.M. Lorenz, Generalization Bounds in Hybrid Quantum-Classical Machine Learning Models, arXiv:2504.08456 [quant-ph]]

Our derivation uses **covering numbers** [1] to quantify the complexity of the quantum and classical components separately. We then combine these complexity measures for the overall hybrid hypothesis class:

The diagram shows the equation $R(h) - \hat{R}(h) \in \tilde{O} \left(\underbrace{\sqrt{\frac{T \log(T)}{N}}}_{\text{Quantum contribution}} + \underbrace{\frac{\alpha}{\sqrt{N}}}_{\text{Classical contribution}} \right)$. Annotations include: 'Generalization error' pointing to the left side of the equation; 'Number of trainable quantum gates' pointing to T ; 'Bound on classical layers' pointing to α ; and 'Number of training data' pointing to N . The terms 'Quantum contribution' and 'Classical contribution' are written below the square root and fraction terms, respectively.

$$R(h) - \hat{R}(h) \in \tilde{O} \left(\underbrace{\sqrt{\frac{T \log(T)}{N}}}_{\text{Quantum contribution}} + \underbrace{\frac{\alpha}{\sqrt{N}}}_{\text{Classical contribution}} \right)$$

This bound decomposes cleanly into quantum and classical contributions, allowing to analyze their interaction.

[1] Matthias C. Caro et al. "Generalization in quantum machine learning from few training data". In: Nature Communications 13.1 (2022), p. 4919. DOI: 10.1038/s41467-022-32550-3.

Generalization Bounds in Hybrid Quantum-Classical Machine Learning Models

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- Introducing classical layers into the architecture **does not introduce significant generalization disadvantage** compared to a fully quantum model;
- A well-designed hybrid model can **match the generalization capabilities** of a purely quantum model **while offering practical advantages** such as reduced quantum circuit depth and better noise tolerance;
- Classical NNs often show unfavorable exponential scaling with layer depth, while hybrid model benefits from a quantum component that introduces a polylogarithmic factor, **potentially improving scaling**;
- **This provides a theoretical justification for hybridization as a strategy for improving scalability without sacrificing learning performance.**

Summary

- The understanding increases that quantum and classical algorithmic parts will work together. It turns out, they are also connected.
- Variational algorithms quickly result in too much sampling overhead – the area, in which they can be useful is relatively small.
- Instead, good generalization properties in the quantum part of algorithms also seem to apply to the whole algorithm, featuring quantum and classical parts.

Contact

PD Dr. habil. Jeanette Miriam Lorenz
Head of Department

Assocociate Professor @ LMU Munich
Speaker of QACI @ Munich Quantum Valley

Tel. +49 89 547088-334

Jeanette.miriam.lorenz@iks.fraunhofer.de

Fraunhofer IKS
Hansastr. 32
80686 München
www.iks.fraunhofer.de



Fraunhofer Institute for Cognitive
Systems IKS

