QUANTUM RESERVOIR COMPUTING FOR RENEWABLE ENERGY FORECASTING

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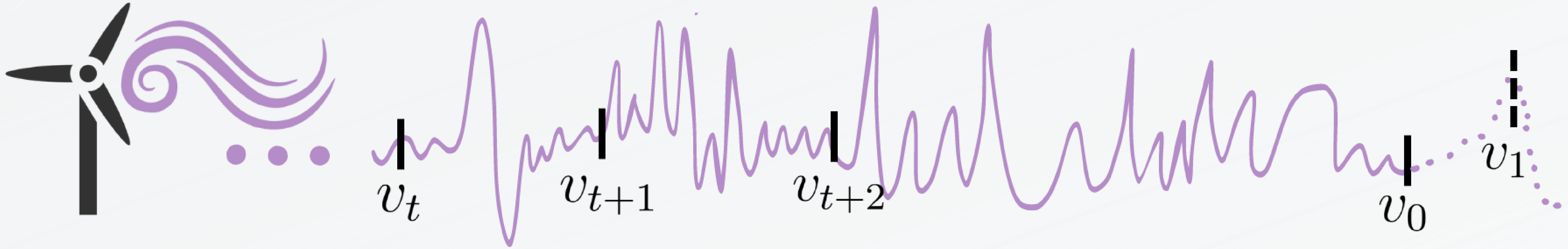
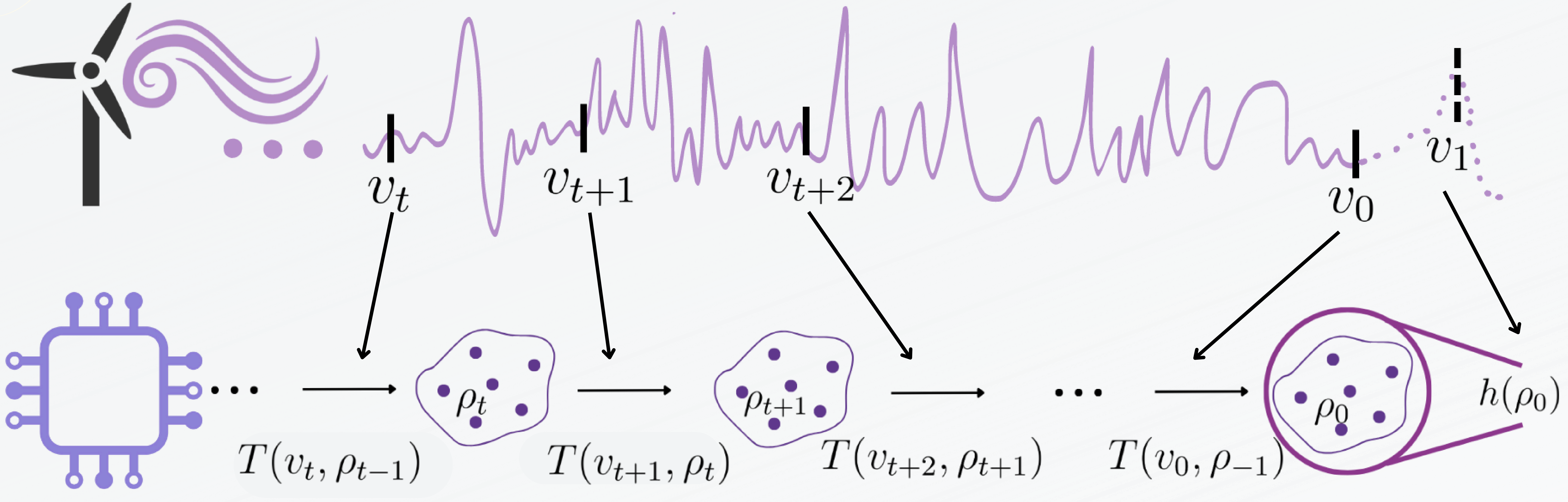
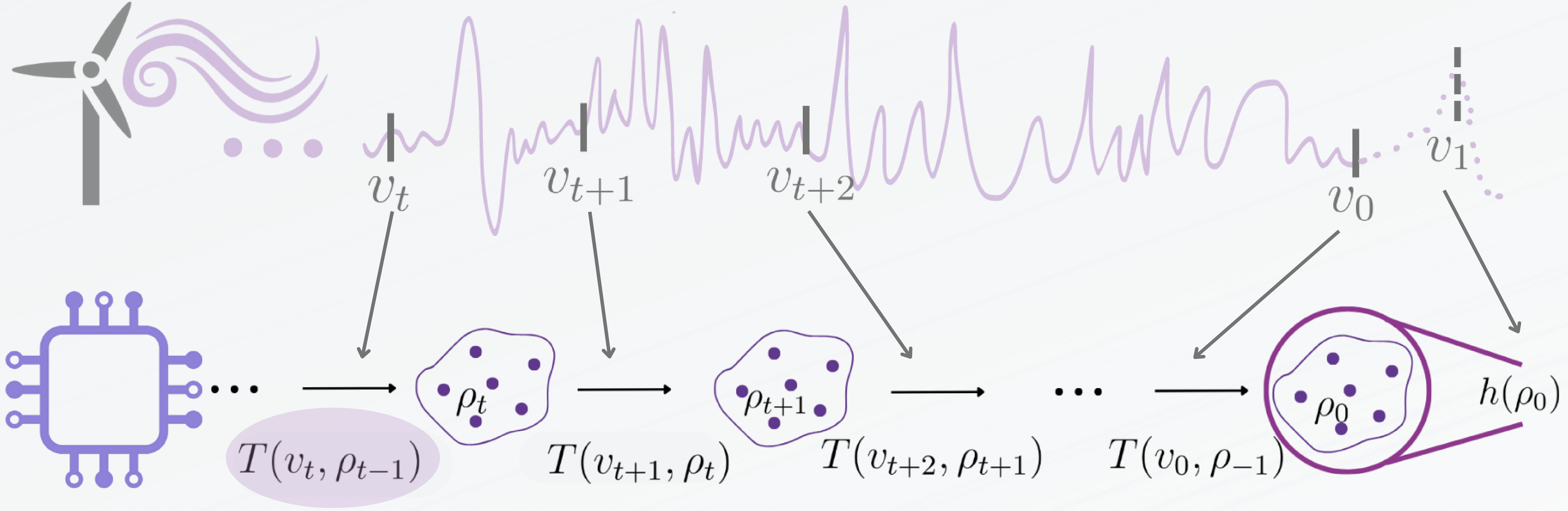
RENEWABLE ENERGY FORECASTING



RESERVOIR COMPUTING



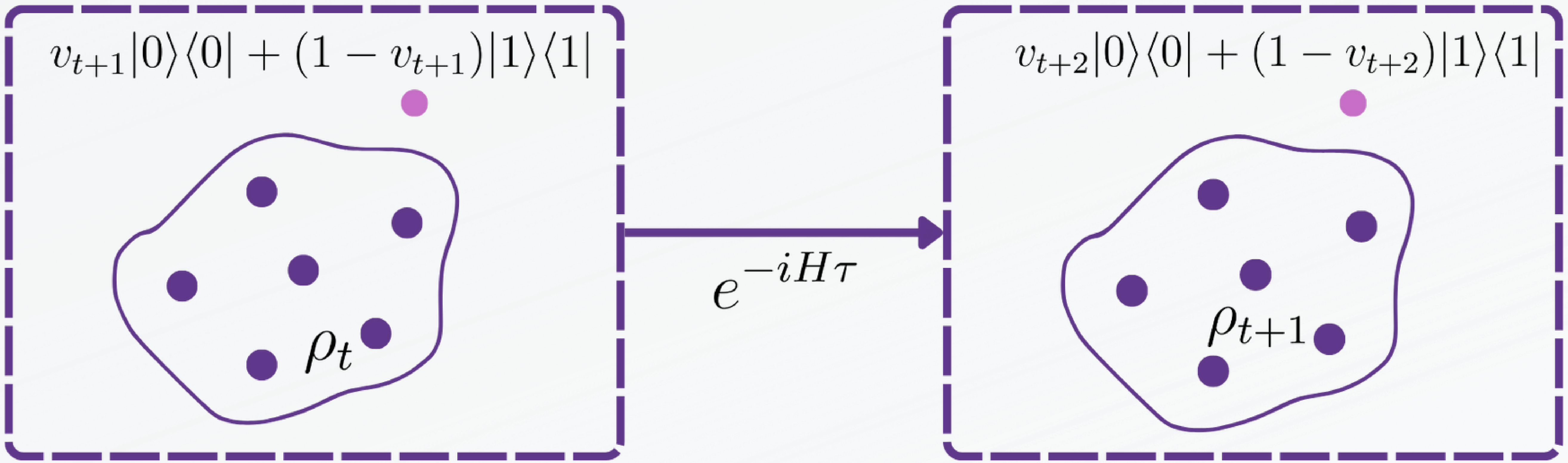
CHOICE OF THE RESERVOIR



EXAMPLE OF A QUANTUM RESERVOIR

“Partial Trace Reservoir (PTR)”

- Set of qubits forms the Reservoir
- additional ancillary qubit carries the input
- apply global dynamics on all qubits
- reinitialise ancillary qubit
- inject new input into ancillary qubit

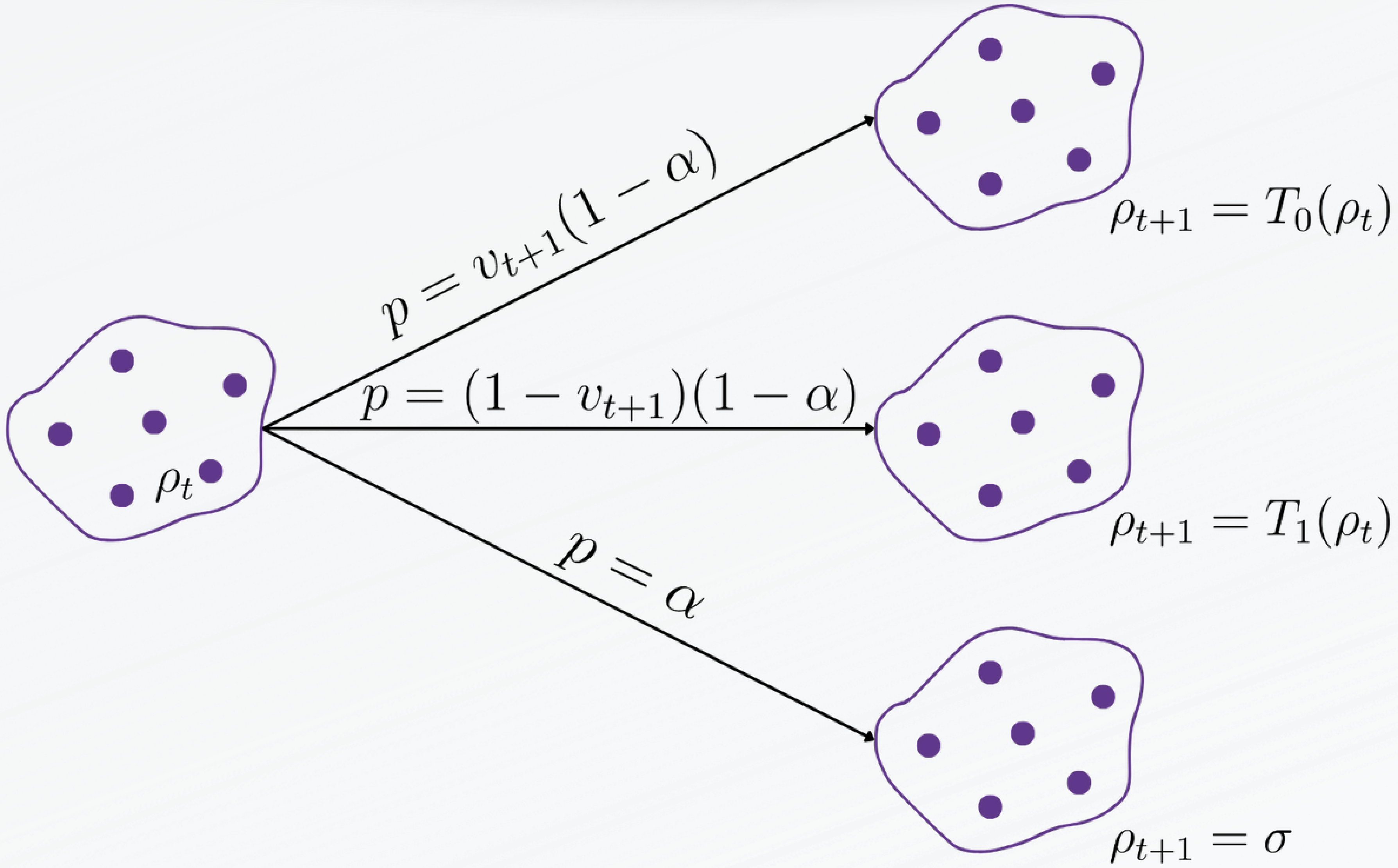


Jiayin Chen and Hendra I. Nurdin *Learning Nonlinear Input-Output Maps with Dissipative Quantum Systems* In: Quantum Information Processing 18 (7), 2019

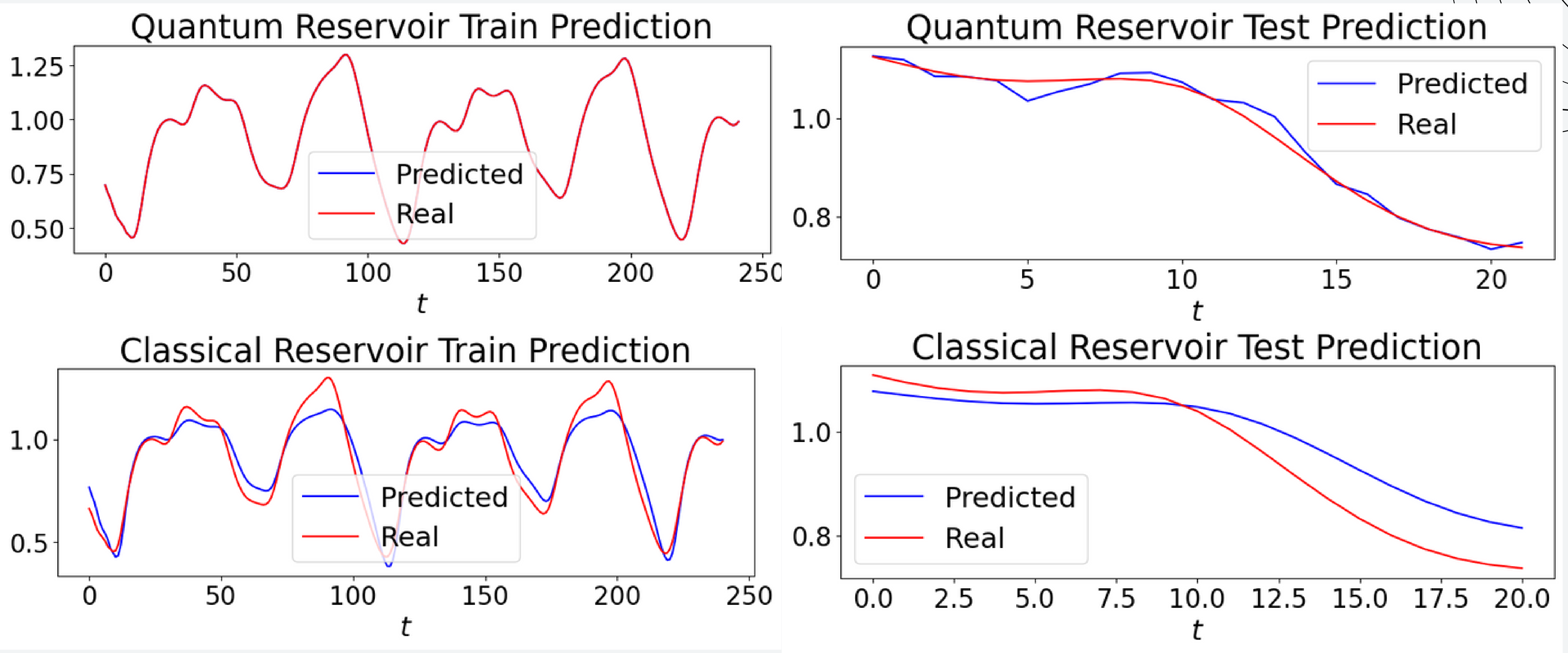
EXAMPLE OF A QUANTUM RESERVOIR

“Random Reinitialisation Reservoir (RRR)”

- with probability (1 - α)v apply T_0
- with probability (1-α)(1-v) apply T_1
- with probability α reinitialise the state to σ



Jiayin Chen et al. *Temporal information processing on noisy quantum computers* In: Physical Review Applied 14 (2), 2020



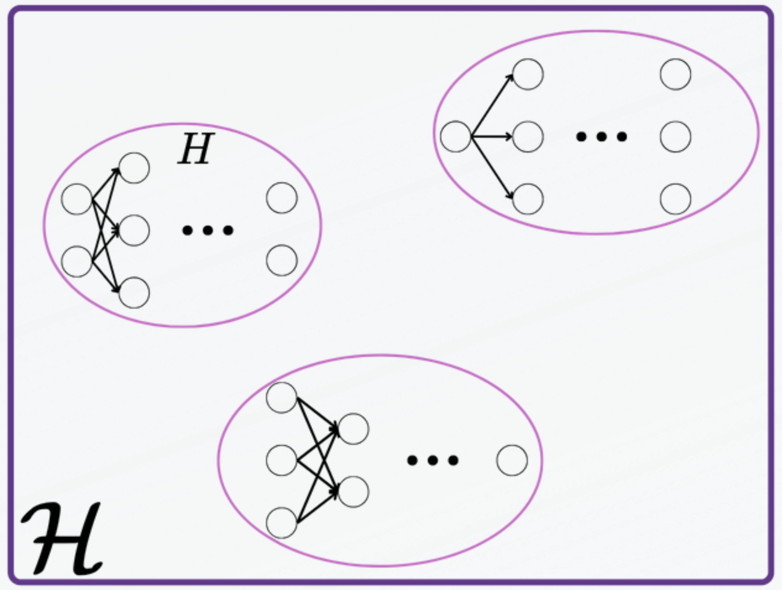
Classical Reservoir performance vs Quantum Simulation

GENERALISATION BOUNDS

Generalisation

$$R(H) := \mathbb{E}_{(V,Y)\sim P}[\ell(H(V), Y_0)]$$

- Consider a hypothesis class of statistical models
- The Generalisation Error is the expected loss of a hypothesis H over a probability distribution P
- Probability distribution is generally unknown



- Upper bound the worst in-class difference between the generalisation error and the empirical risk

$$\hat{R}_m(H) := \frac{1}{m}\sum_{i=1}^{m} \ell(v_i, y_i)$$

Bounds

$$\sup_{H\in\mathcal{H}} \left| R(H) - \hat{R}_m(H) \right|$$

STEPS TO ESTABLISH GENERALISATION BOUND

01

LIPSCHITZ

show that the readout function h is Lipschitz-continuous

02

CONTRACTIVE

find conditions under which the CPTP map is contractive, with explicit contractivity constant

03

RADEMACHER COMPLEXITY

show that the Rademacher Complexity of the QRC class decreases as (1/k)^1/2

Lukas Gonon et al. *Risk Bounds for Reservoir Computing* In: Journal of Machine Learning Research 21 (240), 2020

RADEMACHER COMPLEXITY

Rademacher random variable:
ε_j = 1 or -1 with probability 1/2

$$\mathcal{R}_k(\mathcal{H}) = \frac{1}{k}\mathbb{E}\left[\sup_{H\in\mathcal{H}}\left\|\sum_{j=0}^{k-1}\varepsilon_j H\left(\tilde{V}^{(j)}\right)\right\|_2\right]$$

Class of hypothesis functionals

k identical copies of the infinite input sequence

$$(\ldots, V_{-1}, V_0)$$

RADEMACHER COMPLEXITY

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$\leq 1/\sqrt{k}$?

Yes, under some conditions

Class of hypothesis functionals

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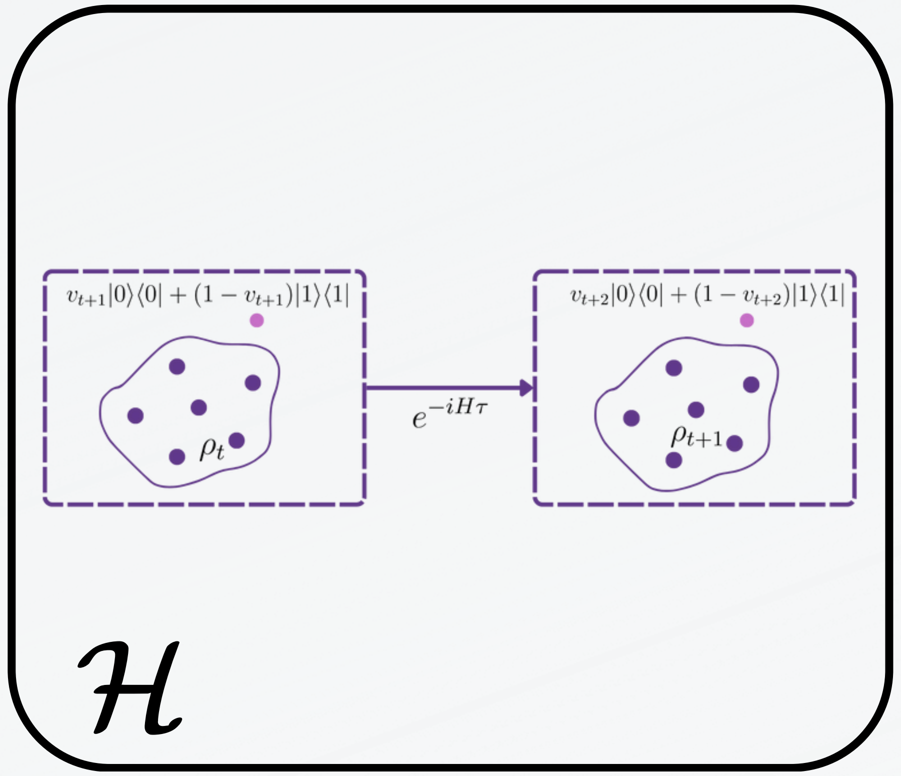
GENERALISATION BOUND FOR QRC

N. M. Chmielewski, N. Amini and J. Mikael. Risk Bounds for Quantum Reservoirs with Polynomial Readout *In phase of submission at Journal of Machine Learning Research (JMLR)*, 2024.



$$\sup_{H\in\mathcal{H}} \left| R(H) - \hat{R}_m(H) \right|$$

$$\leq \quad \mathcal{O}\left(\bar{L}_h \max\left\{ \frac{C_1}{m} \, , \; C_2 \frac{\log m}{m} \, , \; C_3 \sqrt{\frac{\log m}{m}} \, , \; C_4 \sqrt{\frac{\log 4/\delta}{2m}} \right\} \right)$$

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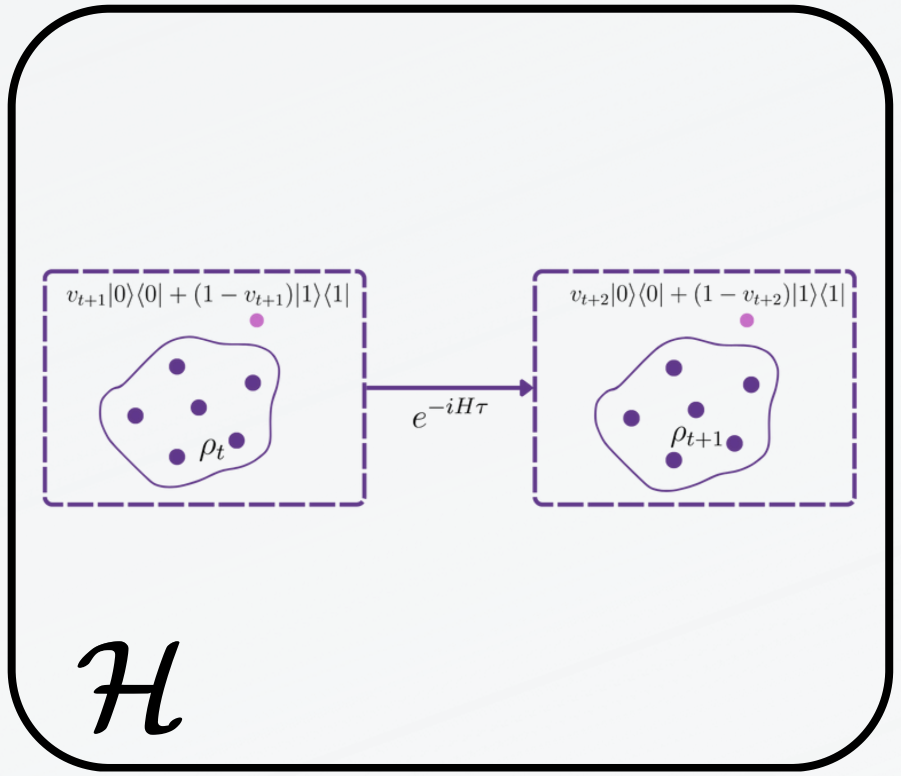

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Reservoir-Dependent Constants

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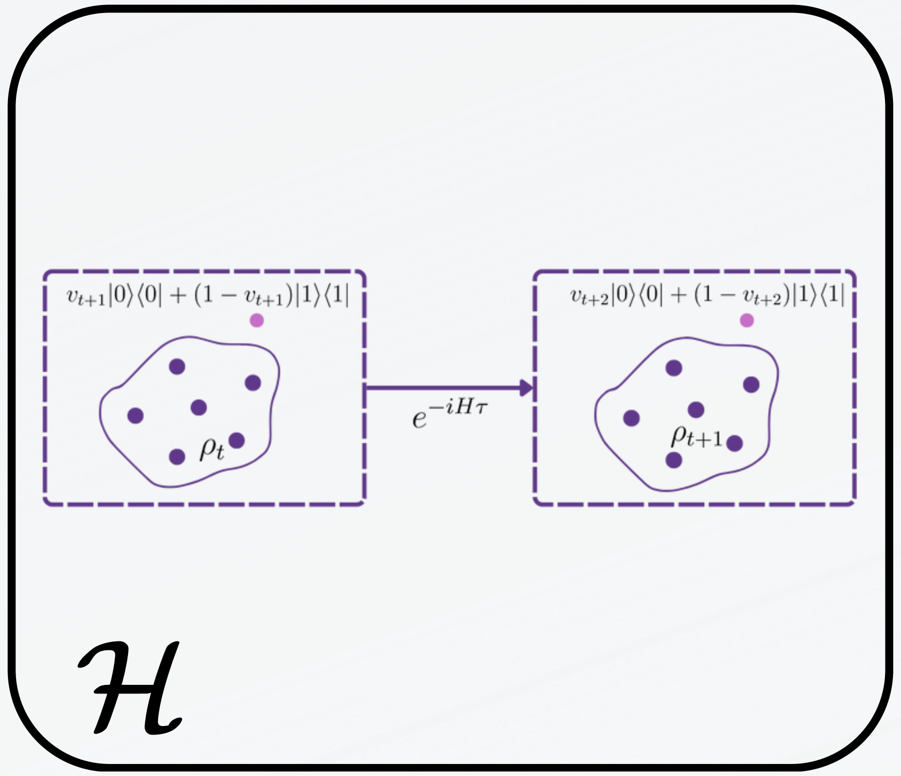

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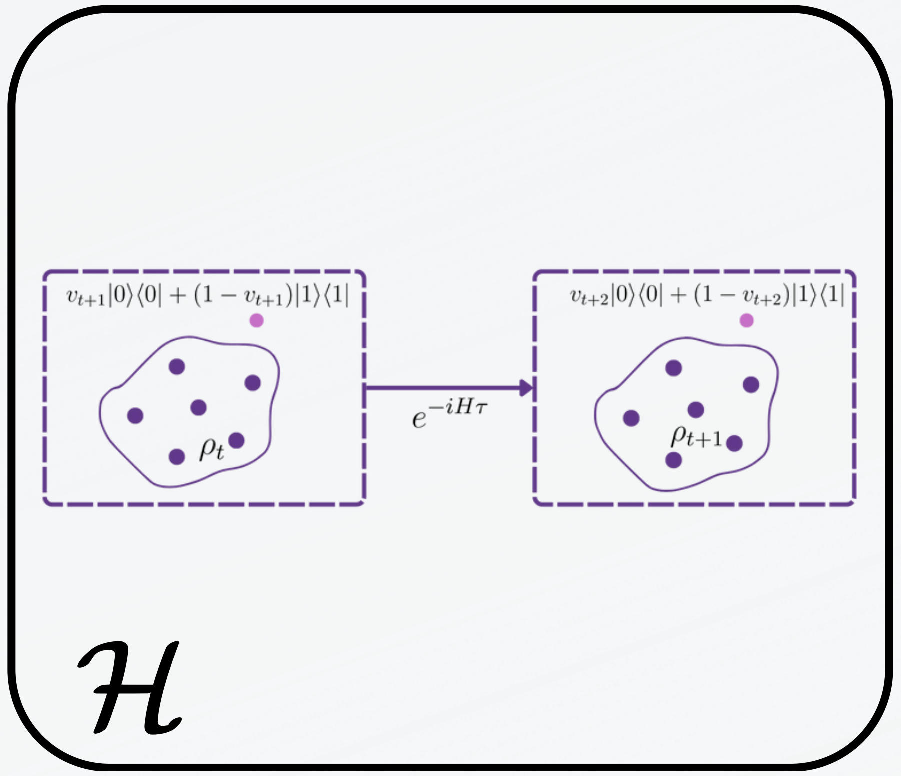

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Reservoir-Dependent Constants

Lipschitz-constant of readout function

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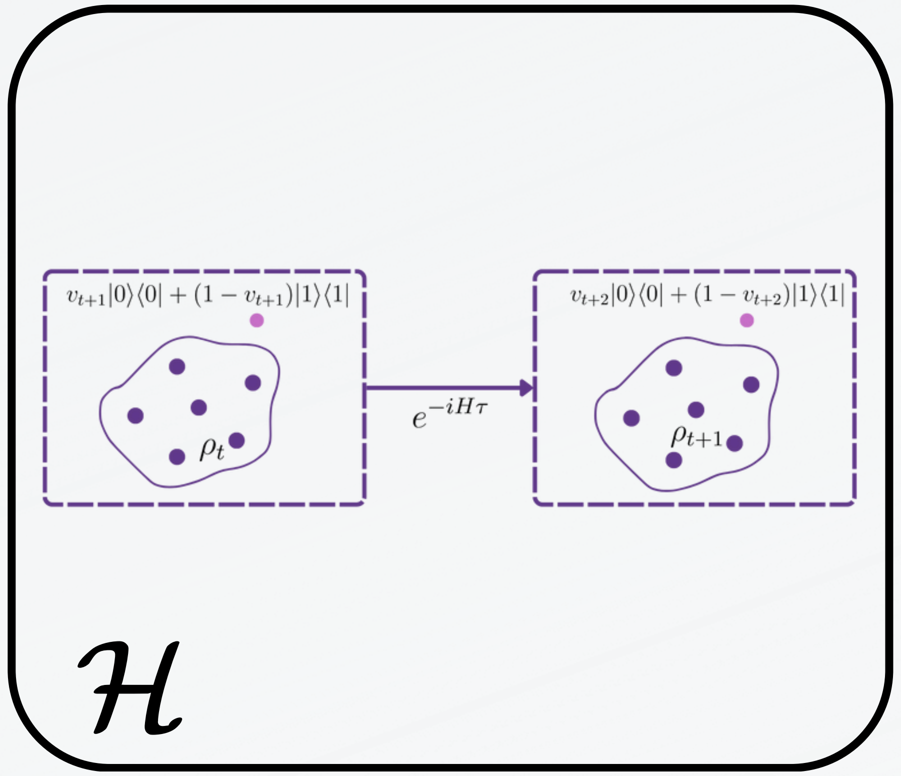

$$\mathcal{O}\left(\bar{L}_h \max\left\{\frac{C_1}{m}\ ,\ C_2\frac{\log m}{m}\ ,\ C_3\sqrt{\frac{\log m}{m}}\ ,\ C_4\sqrt{\frac{\log 4/\delta}{2m}}\right\}\right)$$

Lipschitz-constant of readout function $\sim n\sqrt{2^n}n!$

CONCLUSION

We showed that the generalisation error of the QRC class converges in $\sqrt{\frac{\log m}{m}}$ but we need to improve the Lipschitz-constant.

Use different readout functions (e.g. linear)

Can we still have universality?

Model and implement a hardware efficient QRC

Estimate Information Processing Capacity of different models