Sampling rare events on Quantum Computers

Thales Research & Technology
Quantum Computing team

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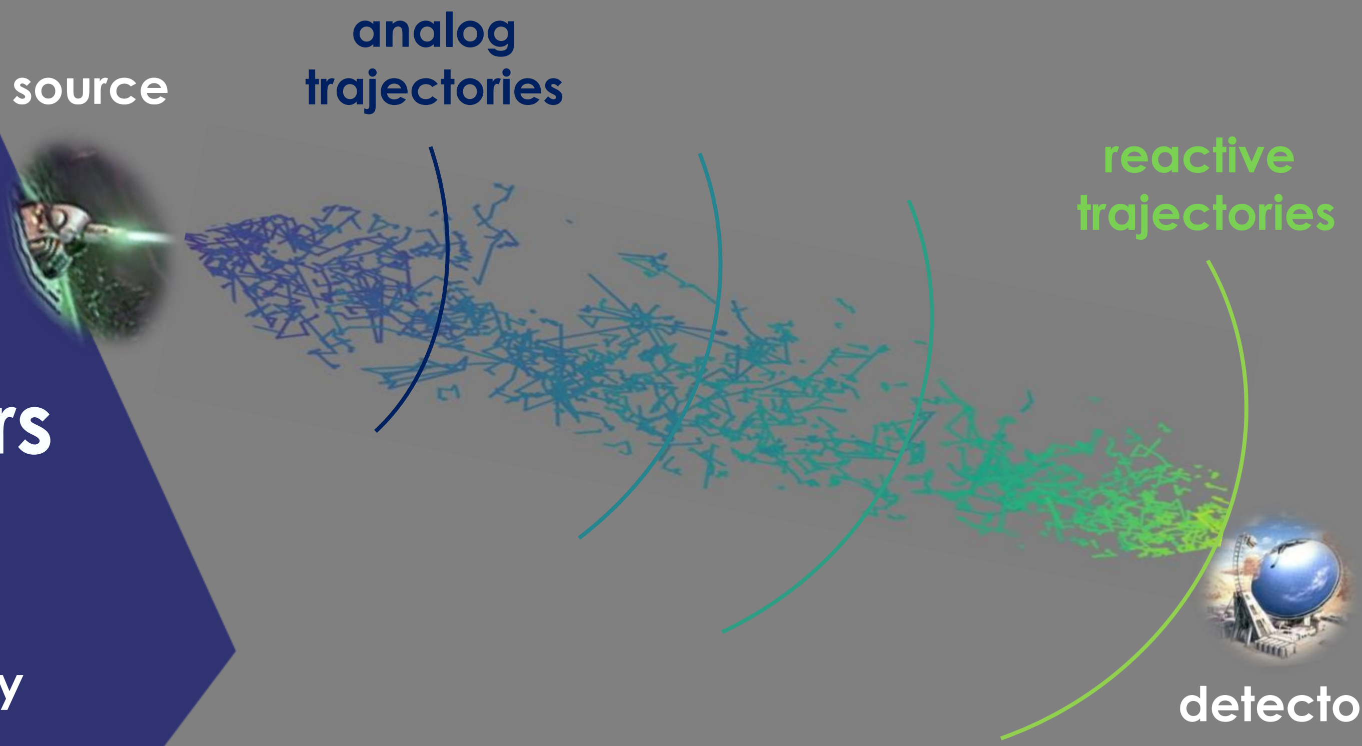


Goal: maximize number of counts in the detector

Monte Carlo on Quantum Computers: overview and motivation

> Montanaro algorithm

Quantum speedup of Monte Carlo methods,
A. Montanaro, 2015

> Amplitude amplitication on a real quantum computer

Low-depth amplitude estimation on a trapped-ion quantum computer
T. Giurgica-Tiron et al., 2022

Benchmarking Amplitude Estimation on a Superconducting Quantum Computer,
S. Certo et al, 2022

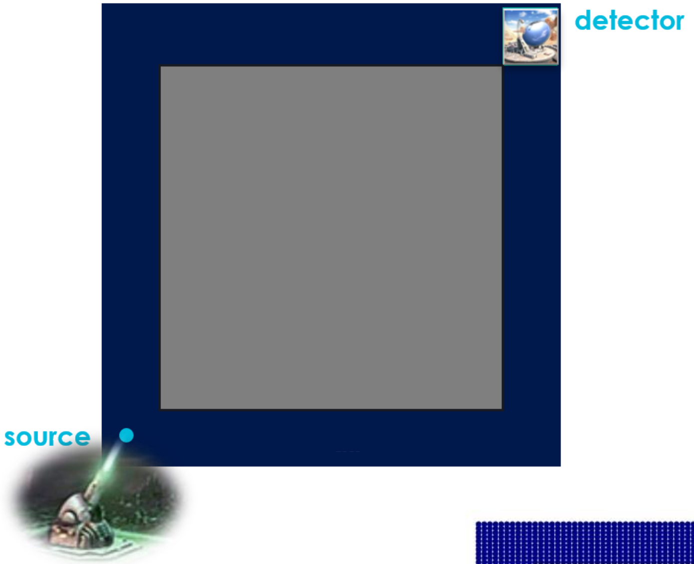
> Quantum random walk

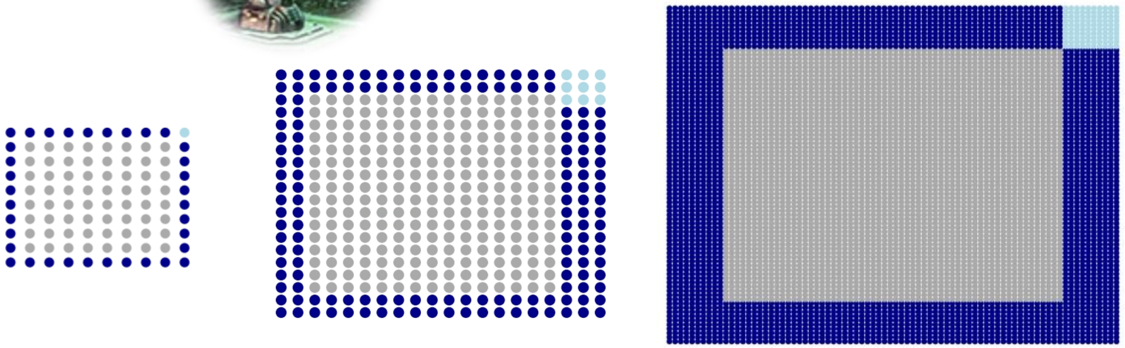
Quantum random walks - an introductory overview,
J. Kempe, 2003

Particle transport on a grid-graph (1/2)

Monte Carlo particle transport on quantum computers, *Noé Olivier and Michel Nowak, 2024*

> Step 1: Discretize the geometry



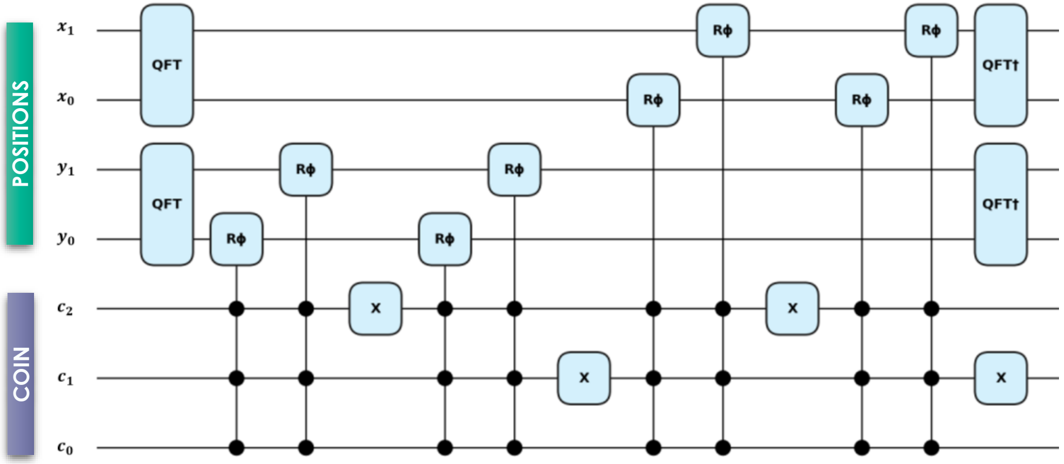


7 qubits

9 qubits

13 qubits

> Step 2: define shift operator



Gate-Based Circuit Designs For Quantum Adder Inspired Quantum Random Walks on Superconducting Qubits, *D. Koch et al., 2021*

Particle transport on a grid-graph (2/2)

Monte Carlo particle transport on quantum computers,
Noé Olivier and Michel Nowak, 2024

> Step 3: particle transport coin operator

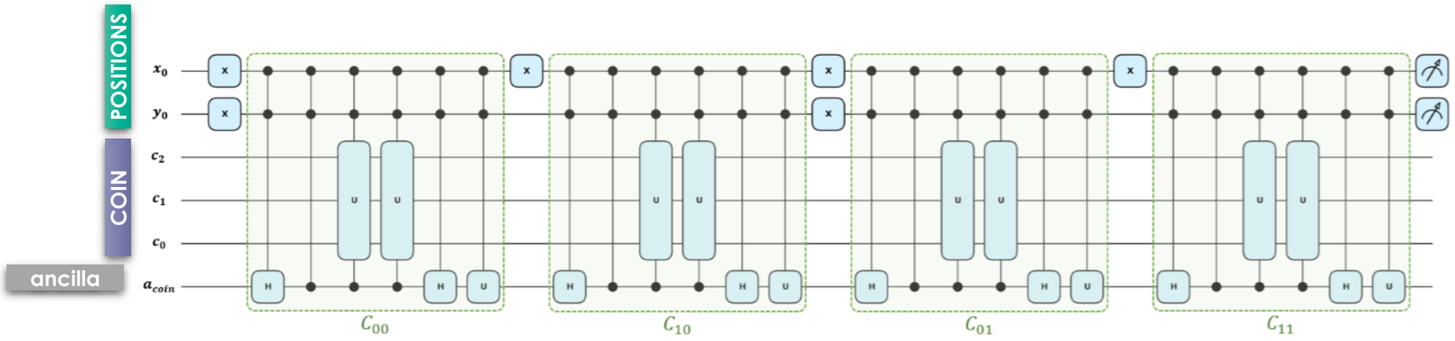
- Geometry/cross section aware coin operator

Coin qubits	Directions
$\|100\rangle$	$\|\rightarrow\rangle$
$\|101\rangle$	$\|\uparrow\rangle$
$\|110\rangle$	$\|\leftarrow\rangle$
$\|111\rangle$	$\|\downarrow\rangle$
$\|0\cdot\cdot\rangle$	$\|\circlearrowright\rangle$

$$C_{|x,y\rangle} = \begin{pmatrix} M_A & 0 \\ 0 & M_{S|x,y\rangle} \end{pmatrix}$$

$$M_A = \mathrm{diag}\left(\frac{p_a}{4}, \frac{p_a}{4}, \frac{p_a}{4}, \frac{p_a}{4}\right)$$

$$M_{S|x,y\rangle} = \mathrm{diag}\left(p^{x,y}_{s,\rightarrow}, p^{x,y}_{s,\leftarrow}, p^{x,y}_{s,\uparrow}, p^{x,y}_{s,\downarrow}\right)$$



Quantum circuits for discrete-time quantum walks with position-dependent coin operator
Ugo Nzongani et al, 2023

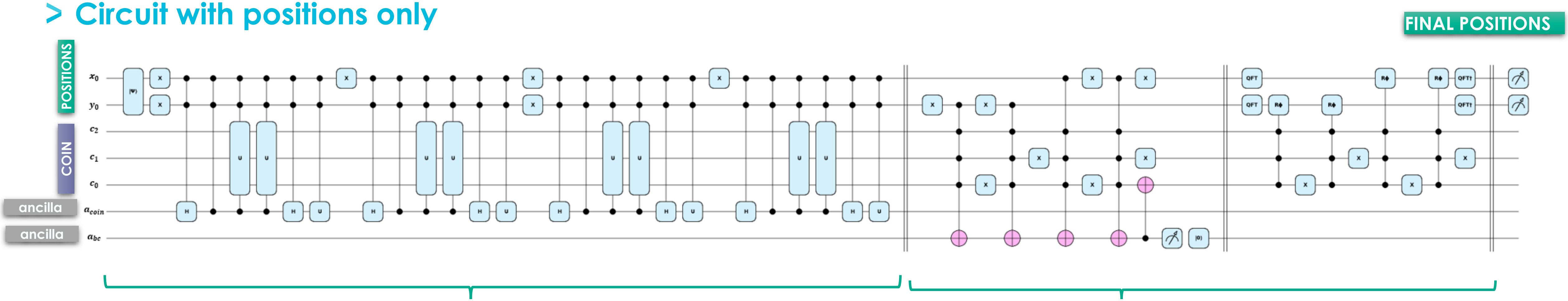
- Boundary conditions
  - set local scattering probability to 0
  - Reset coin at each step

Efficient Quantum Circuits for Non-Unitary and Unitary Diagonal Operators with Space-Time-Accuracy trade-offs
J. Zylberman et al., 2024

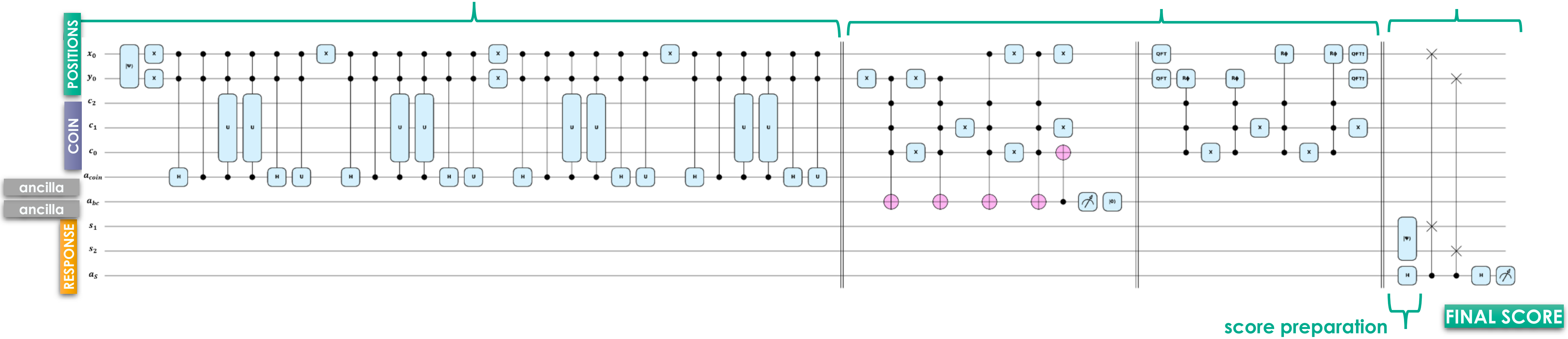
Circuits for particle transport

Monte Carlo particle transport on quantum computers,
Noé Olivier and Michel Nowak, 2024

> Circuit with positions only

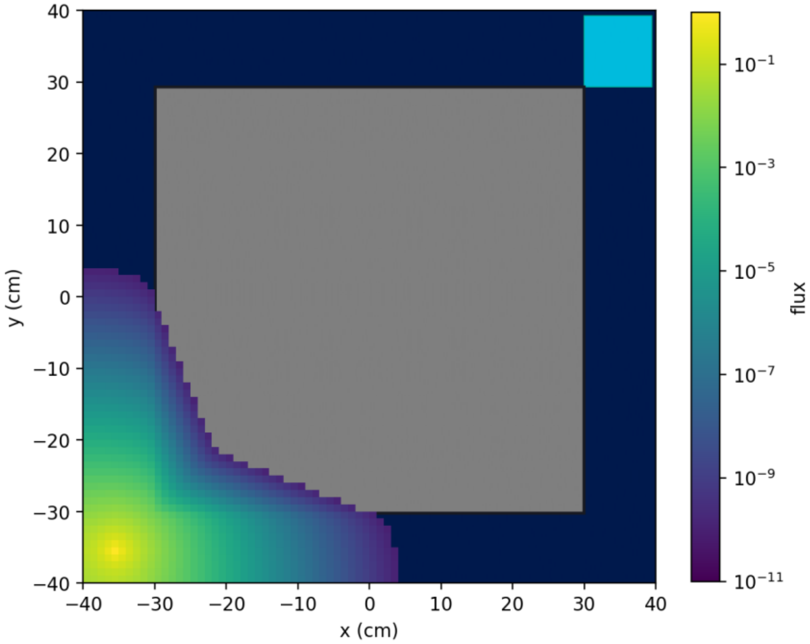


> Circuit with score

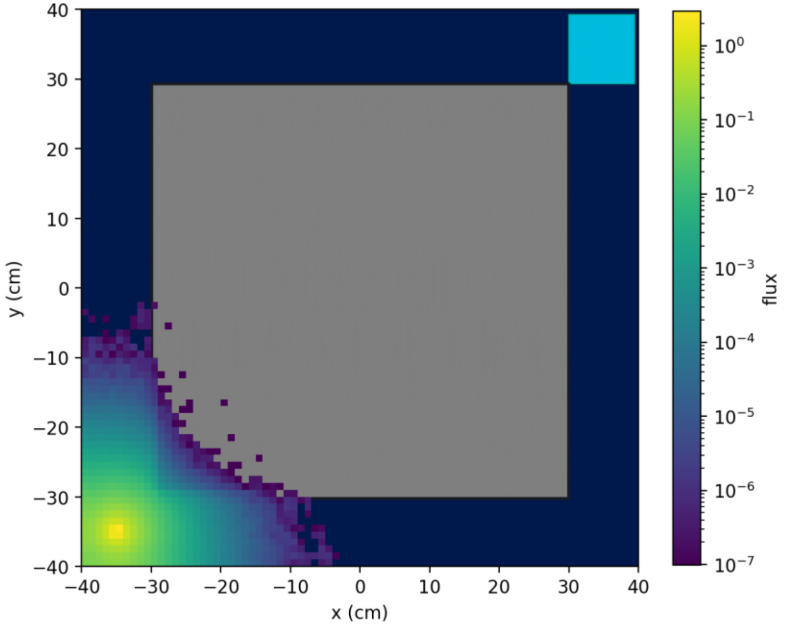


Flux comparison

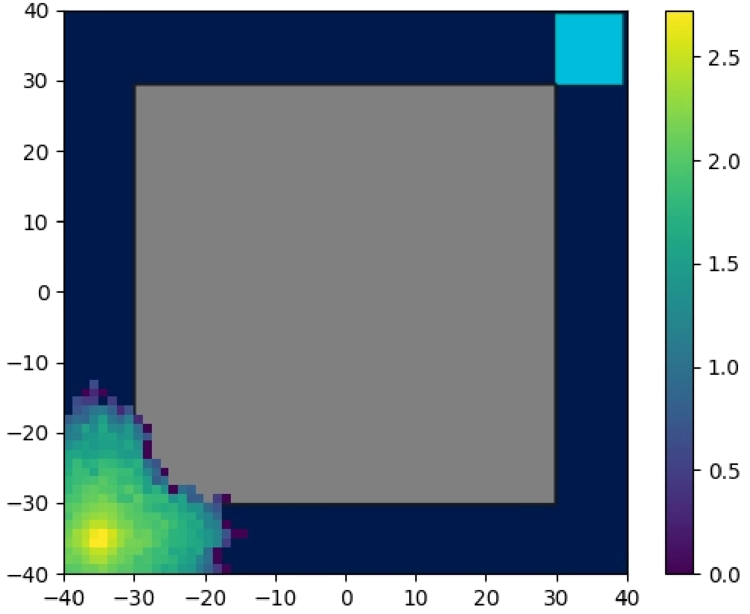
Monte Carlo particle transport on quantum computers,
Noé Olivier and Michel Nowak, 2024



classical finite differences

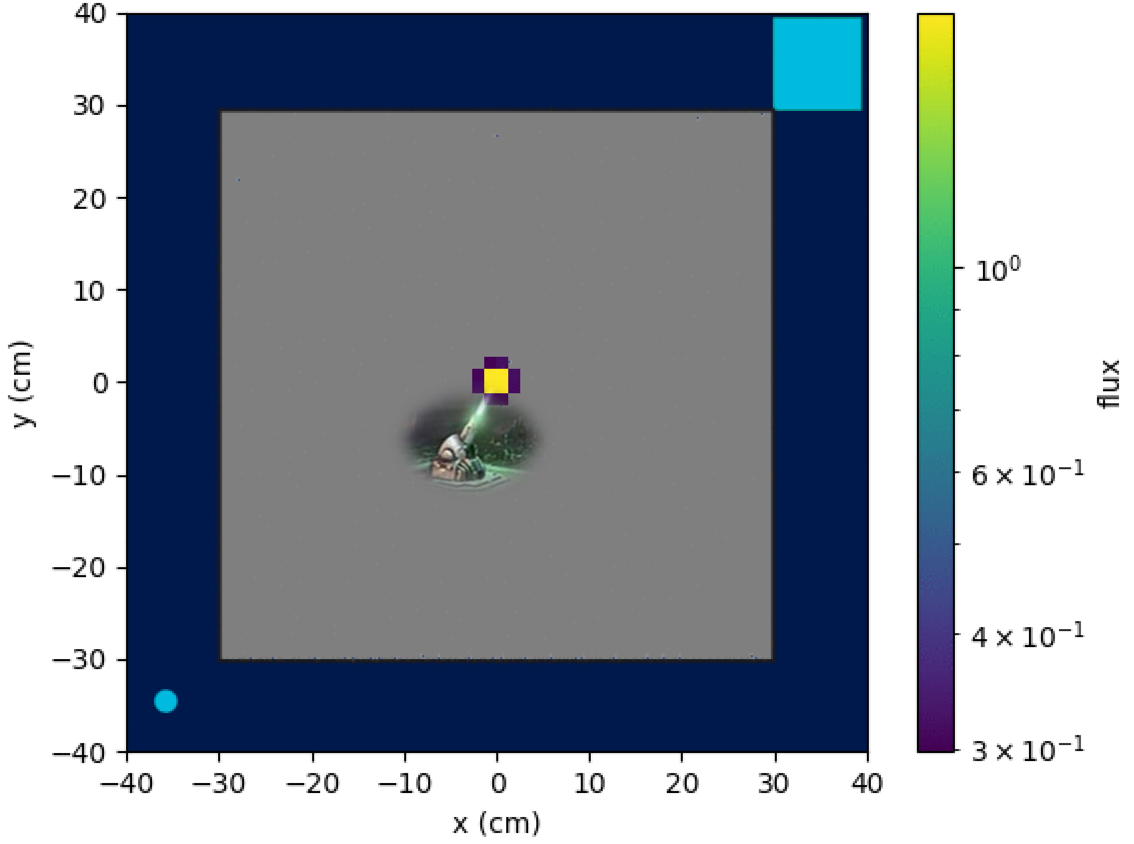


classical Monte Carlo



measured quantum

Quantum walk failure in the analog regime



> Guarantee of convergence

Quantum walks can find a marked element on any graph, *Hari Krovi et al, 2010*

> OK, but not always in a reasonable amount of time

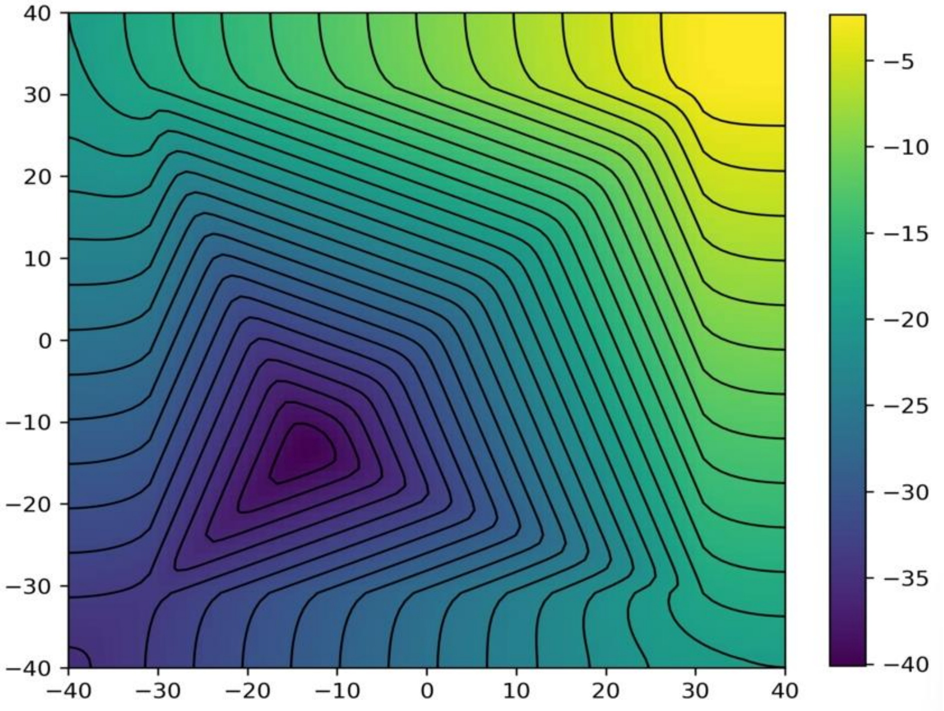
→ Especially when **starting from a localized state**

→ Especially when they are **absorbing conditions**

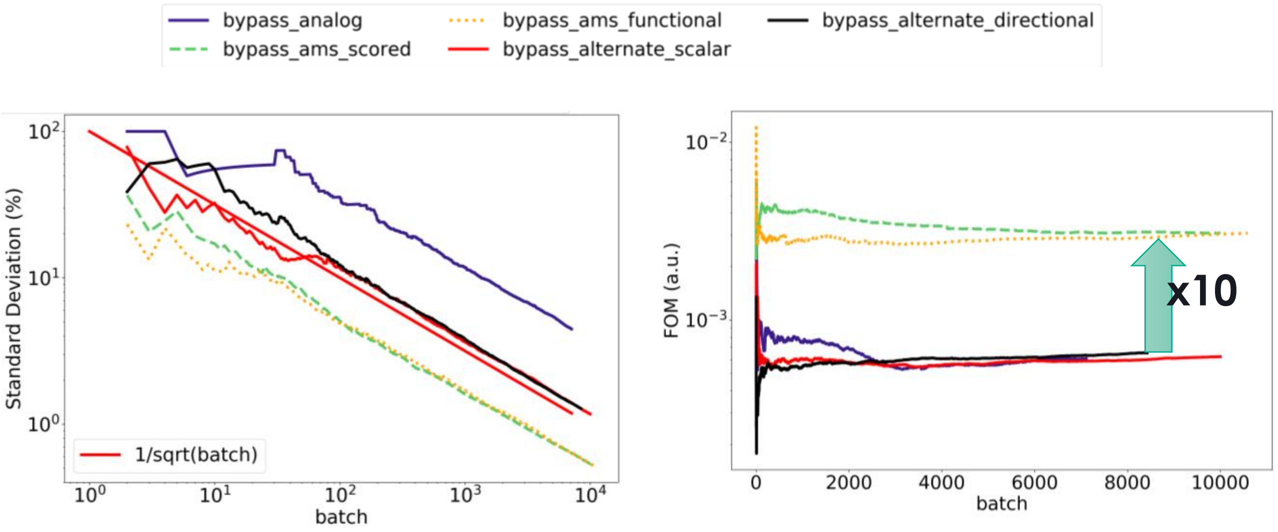
The need for an importance map

> Scalar importance map

- Computed with a **finite difference solver**
- Gives the **expected contribution** for 1 particle starting from **x** to the detector response



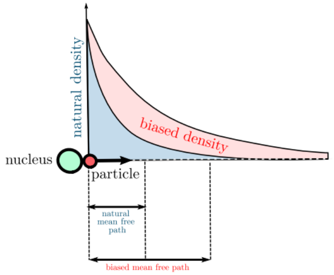
> Classical results with variance reduction



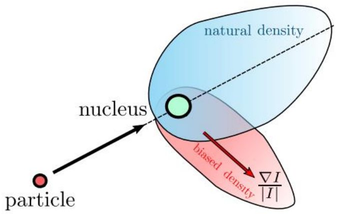
same convergence rates **improved performance measure**

Importance Sampling on quantum computers (1/2)

> Exponential transform



> Collision biasing



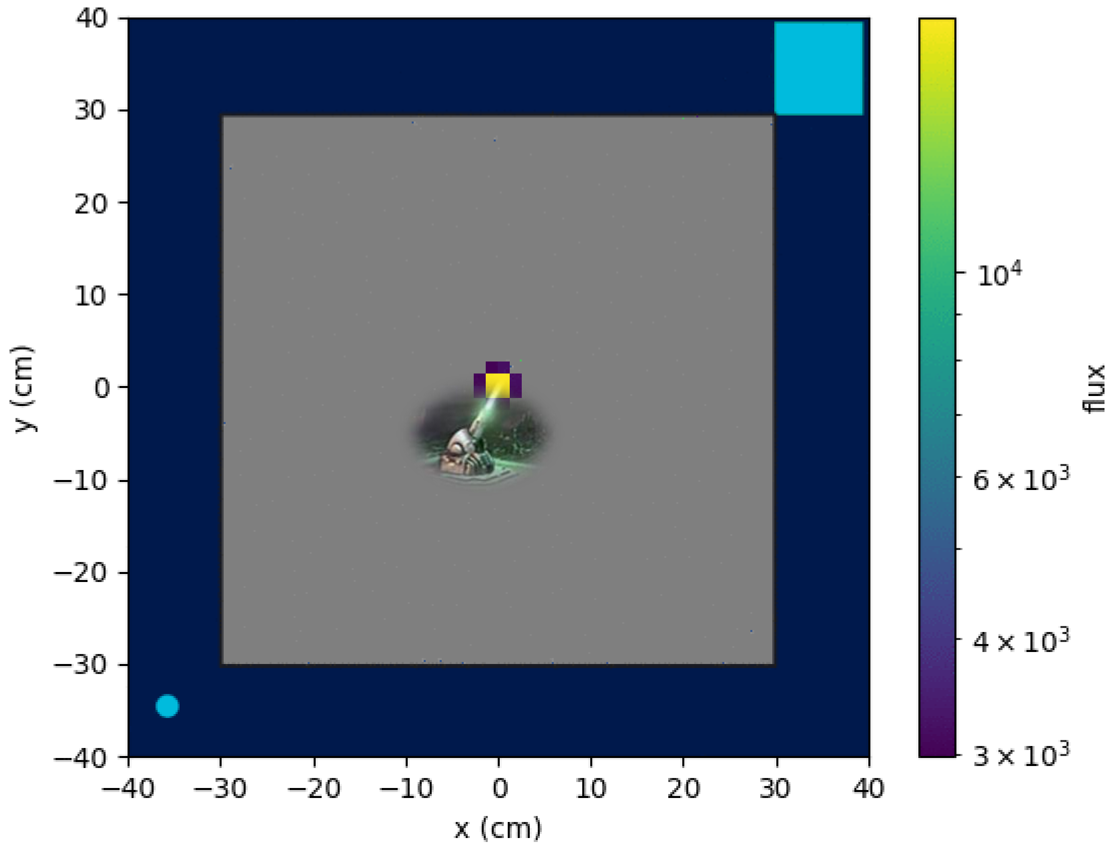
> Population control

- Splitting
- Roulette

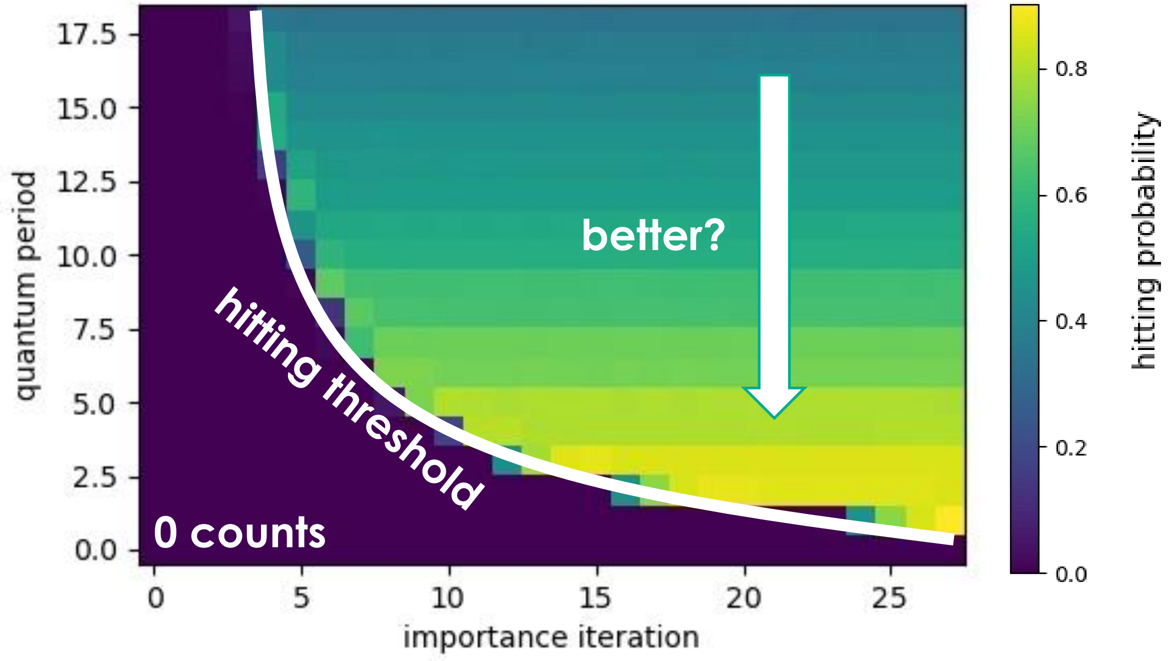
Variance reduction techniques
CEA, 2018

> Extension to quantum formalism

- **Renormalize** sampled distribution by **importance map**
- **Restart quantum walk** for **one shift** operator call



Importance Sampling on quantum computers (2/2)



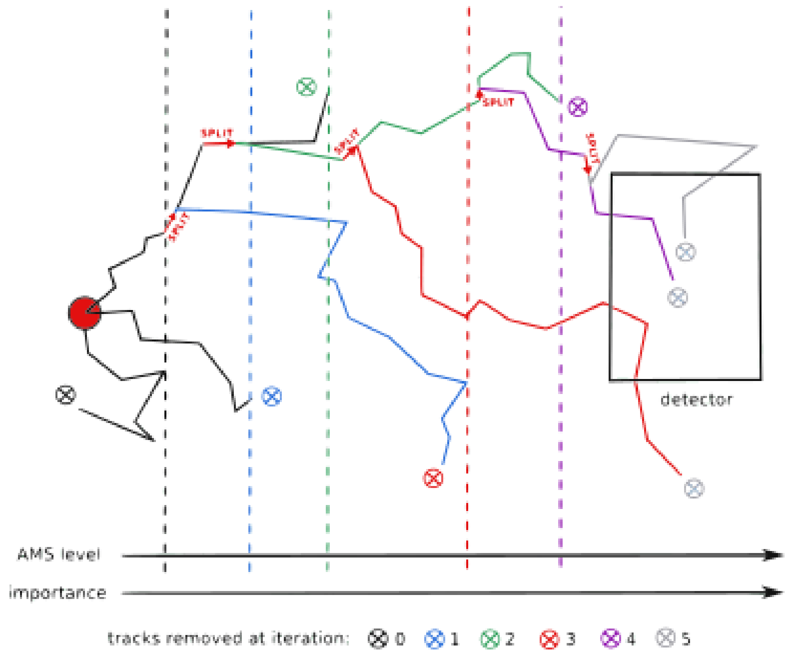
> Discussion

- **Tradoff between querying**
  - The shift operator
  - the importance map
- **Constant hitting probability**
  - With respect to importance map queries
- **Increasing hitting probability**
  - By minimizing the number of quantum shift queries
- **Warning**
  - Periodic boundary conditions applied
  - Reflective too expensive

Adaptive Multilevel Splitting on quantum computers

> Classical AMS

- Restart **K particles** at each **AMS iteration**



Adaptive multilevel splitting for Monte Carlo particle transport, *H. Louvin et al, 2017*

Accelerating Monte Carlo particle transport with adaptively generated importance maps, *M. Nowak, 2018*

> Extension to a quantum formalism?

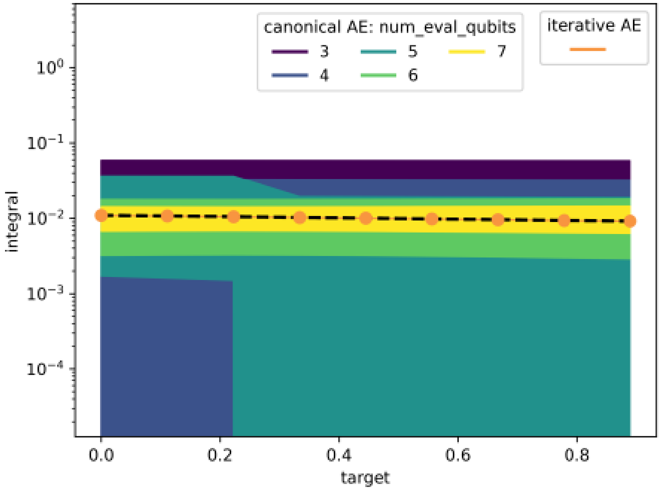
- No need to **follow weights**
  - Because the weight is global for the restarted particles
- **Selection & Splitting** needs to be classically implemented
- **Study tradoff** between
  - Classical queries to the importance map
  - Quantum queries to the shift operator

Amplitude estimation failure in the analog regime

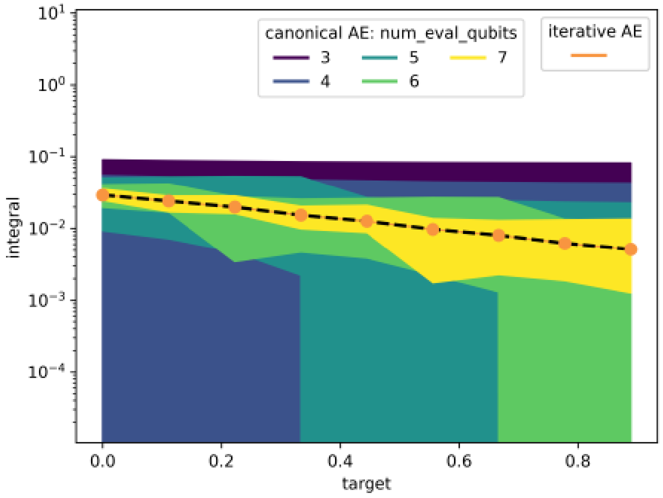
Quantum amplitude amplification and estimation,
Brassard et al, 2000

Iterative Quantum Amplitude Estimation
D. Grinko et al, 2021

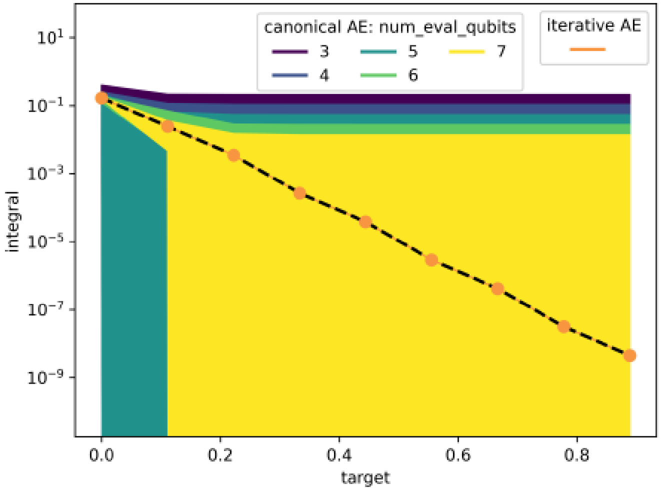
> Exponential integration with attenuation



(a) $\lambda = 0.1$



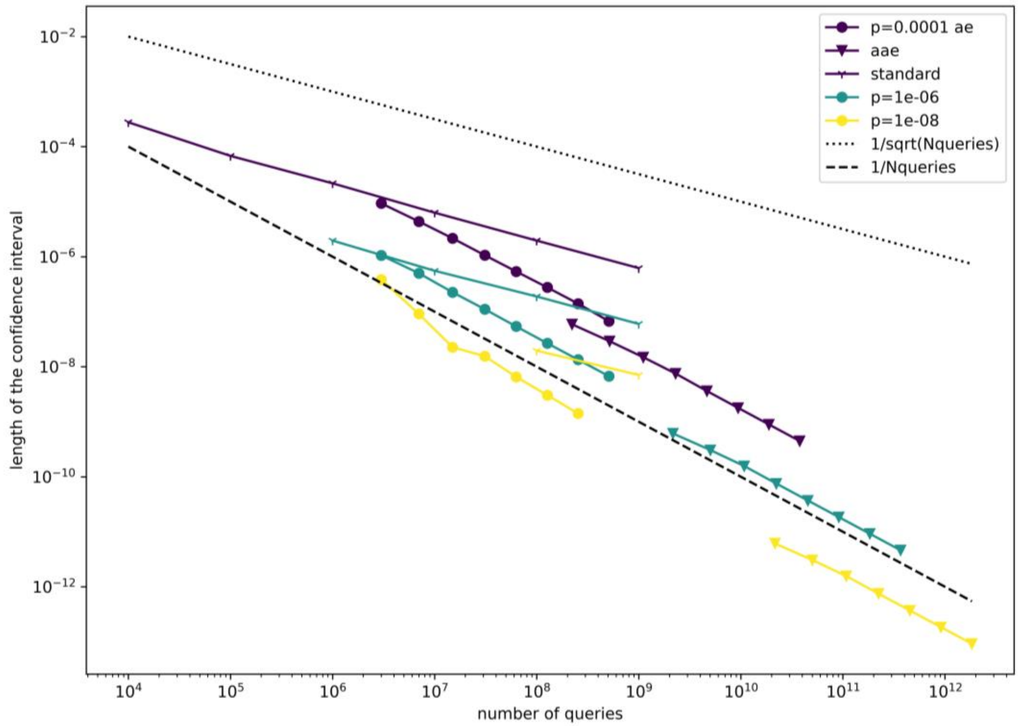
(b) $\lambda = 1$



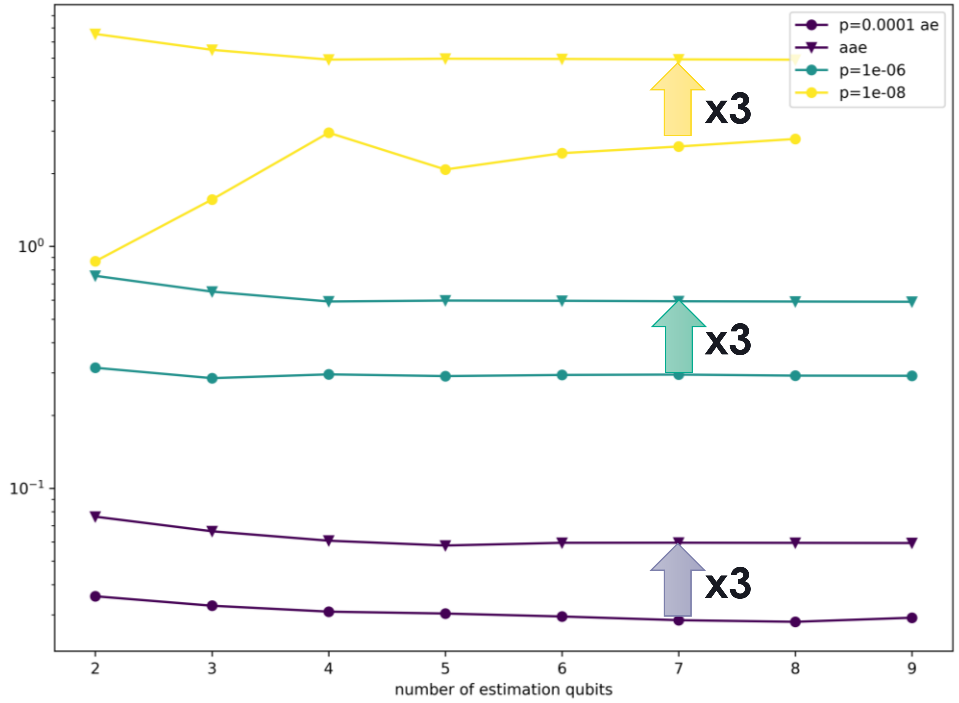
(c) $\lambda = 10$

Implementation and convergence of

Amplified Amplitude Estimation: Exploiting Prior Knowledge to Improve Estimates of Expectation Values, *S. Simon et al. 2024*



faster convergence rates



improved performance measure

Conclusion and perspectives

> Results

- **Monte Carlo proposal on quantum computers**
  - With quantum walks
  - Local probabilities of interactions
  - Reflective boundary conditions
  - « Absorbing » approximation

Monte Carlo particle transport on quantum computers,
Noé Olivier and Michel Nowak, 2024

- **Failure of analog quantum walk**
  - Classical importance sampling applied to quantum walks
  - Cannot follow weights of individual particles
  - Renormalization proposal
  - Tradoff between quantum & classical
- **Failure of analog amplitude estimation**
  - X3 speedup with a priori estimate of the response (implementation of paper)

> Perspectives

- **Adaptive Multilevel Splitting on quantum computers**
  - Coming soon
- **Merge variance reduction techniques with amplitude amplification**
  - Extract hitting probabilities
- **Preprint soon**

The team

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