Gaussian Boson Sampling

Applications

Jesua EPEQUIN, November 2024

EDF R&D Centre, Beijing, China

01 GBS

Photonic Quantum Model

Gaussian Boson Sampling

Summary

- Gaussian states are a type of quantum states
- They can be described by a covariance **Σ** matrix and a vector of means.
- In GBS, Gaussian states are measured using photon-number-resolving detectors.
- Probability of observing S = (s_1,…,s_m) is

$$\mathcal{P}(S) = \frac{1}{\sqrt{\det(\sigma_Q)}} \frac{\mathrm{Haf}(K_S)}{s_1! \ldots s_m!},$$

- where K and σ_Q is a 2m x 2m symmetric matrix depending on **Σ**

- The Hafnian is defined as

$$\mathrm{Haf}(K) = \sum_{\mu \in \mathrm{PMP}} \prod_{(i,j) \in \mu} K_{i,j},$$

where PMP is the set of perfect matching permutations.

Gaussian Boson Sampling

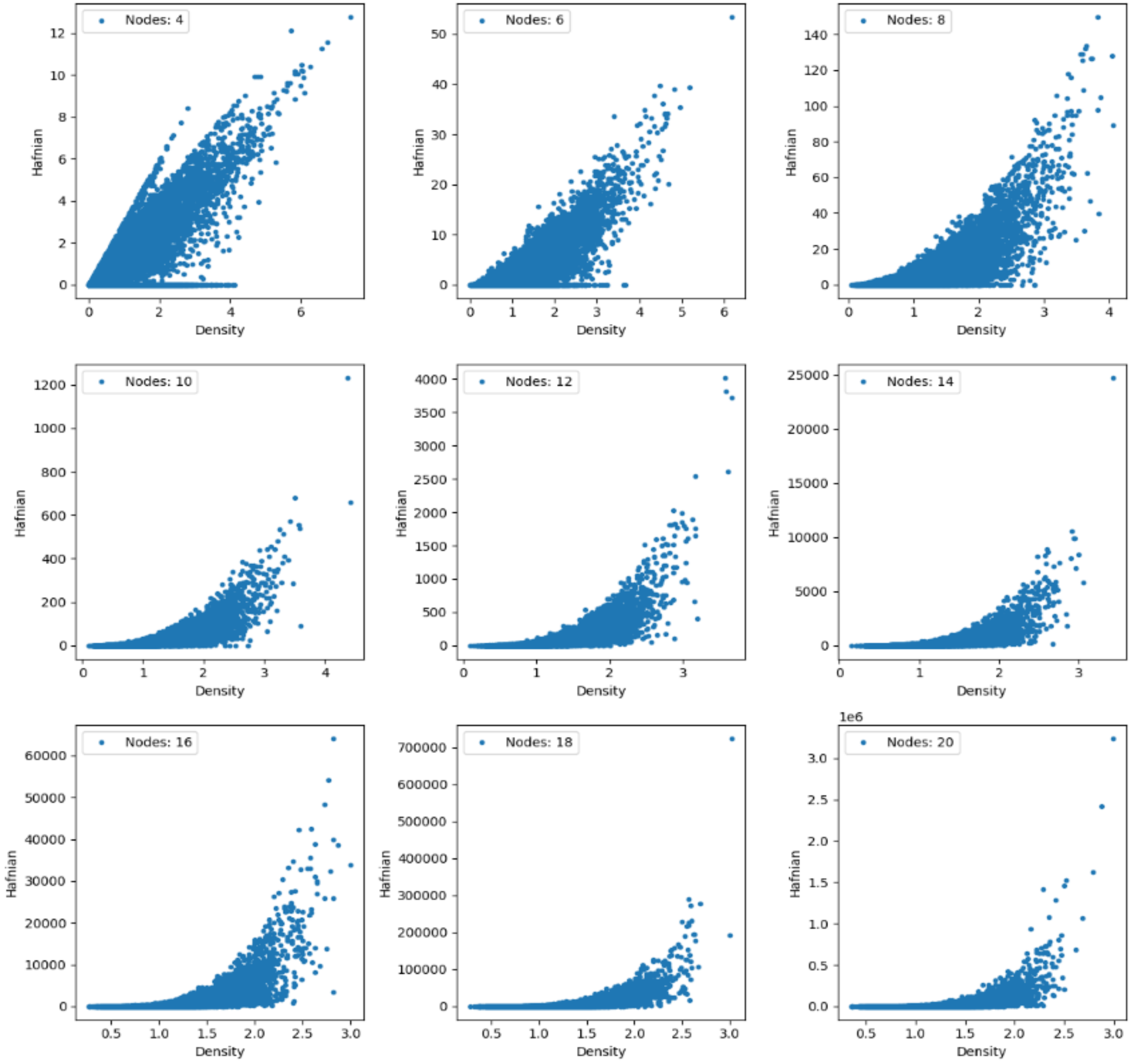
Dense subgraph identification

- If A is adjacency matrix of unweighted graph G, then Haf(A) is number of perfect matchings $N_{PM}(G)$.

- *Average density* defined as :

$$d_{avg}(G) = 2 \sum_{e \in E(S)} w_e \; S|(|S|-1)$$

- If W is weighted adjacency matrix, Haf(A) correlates with Average Density.



02 Feature Selection

First Application

Feature Selection

GBS application

- Feature selection is a combinatorial optimization problem.
- We map database into graph G by considering the entire feature set as the vertex set and using inter-feature *mutual information* to compute edge weights.
- Mutual Information : how much information can be extracted through the knowledge of the other. Low values indicate mutual independence.
- Higher density subgraphs correspond to less redundant features.

Results

Benchmarking on multiple dataset

Dataset	Algorithm	Evaluation metrics								
		Naive Bayes			Tree			SVM		
		F1	AUC	MCC	F1	AUC	MCC	F1	AUC	MCC
WDBC	Baseline	0.908±0.044	0.985±0.017	0.856±0.068	**0.912±0.055**	**0.936±0.034**	**0.841±0.063**	0.968±0.029	0.994±0.010	0.951±0.045
	LSFS	**0.925±0.043**	**0.986±0.019**	**0.881±0.070**	0.895±0.048	0.934±0.037	0.834±0.064	0.961±0.032	0.993±0.010	0.940±0.048
	SPEC	0.904±0.032	0.982±0.019	0.851±0.047	0.886±0.039	0.923±0.035	0.829±0.062	0.933±0.028	0.991±0.012	0.896±0.046
	GBSFS	0.886±0.050	0.972±0.019	0.824±0.076	0.887±0.052	0.921±0.046	0.832±0.099	**0.973±0.017**	**0.996±0.007**	**0.959±0.026**
Ionosphere	Baseline	0.911±0.037	0.935±0.048	0.736±0.121	0.901±0.046	0.857±0.069	0.694±0.131	**0.951±0.044**	0.964±0.054	**0.855±0.138**
	LSFS	0.841±0.035	0.832±0.081	0.449±0.191	**0.913±0.053**	0.870±0.074	**0.765±0.156**	0.938±0.048	0.957±0.059	0.818±0.150
	SPEC	0.756±0.108	0.804±0.073	0.431±0.178	0.876±0.054	0.841±0.078	0.669±0.190	0.939±0.038	**0.965±0.046**	0.819±0.119
	GBSFS	**0.914±0.041**	**0.946±0.051**	**0.740±0.125**	0.902±0.050	**0.875±0.046**	0.761±0.104	0.941±0.041	0.954±0.046	0.828±0.124
Shoppers	Baseline	**0.513±0.070**	0.827±0.033	0.420±0.086	**0.505±0.093**	**0.748±0.049**	**0.434±0.106**	**0.507±0.069**	0.857±0.060	**0.482±0.077**
	LSFS	0.333±0.054	0.719±0.050	0.206±0.063	0.193±0.083	0.553±0.037	0.059±0.059	0	0.500±0.101	0
	SPEC	0.333±0.054	0.719±0.050	0.206±0.063	0.190±0.081	0.548±0.037	0.064±0.062	0	0.500±0.101	0
	GBSFS	0.498±0.065	**0.843±0.068**	**0.440±0.080**	0.493±0.096	0.741±0.051	0.419±0.102	0.503±0.070	**0.872±0.076**	0.476±0.077
Spectf	Baseline	**0.757±0.074**	**0.847±0.054**	0.431±0.080	0.828±0.047	0.623±0.104	0.202±0.212	**0.884±0.025**	**0.801±0.069**	**0.136±0.199**
	LSFS	0.735±0.082	**0.847±0.092**	0.433±0.101	0.811±0.073	0.635±0.144	0.209±0.284	0.868±0.043	0.795±0.100	0.053±0.097
	SPEC	0.731±0.072	0.835±0.082	**0.440±0.068**	**0.835±0.071**	0.574±0.108	0.138±0.237	0.877±0.028	0.800±0.108	0.009±0.106
	GBSFS	0.749±0.072	0.827±0.065	0.398±0.094	0.834±0.043	**0.649±0.095**	**0.258±0.168**	0.868±0.031	0.737±0.103	0.021±0.079
Parkinsons	Baseline	0.726±0.129	**0.844±0.180**	0.404±0.348	**0.881±0.054**	**0.759±0.128**	**0.582±0.238**	0.900±0.074	0.827±0.182	0.496±0.370
	LSFS	0.703±0.140	0.819±0.148	0.437±0.282	0.875±0.075	0.715±0.127	0.454±0.270	0.861±0.064	0.727±0.209	0.236±0.351
	SPEC	0.741±0.132	0.840±0.161	**0.453±0.313**	0.862±0.052	0.770±0.115	0.528±0.226	0.893±0.093	**0.906±0.096**	**0.556±0.341**
	GBSFS	**0.780±0.129**	0.820±0.201	0.442±0.370	0.875±0.068	0.736±0.170	0.495±0.293	**0.904±0.067**	0.836±0.187	0.531±0.342

Results

Win-Draw-Loss

Dataset	Baseline	LSFS	SPEC
Ionosphere	6-0-3	6-0-3	8-0-1
Spectf	5-0-4	4-1-4	4-0-5
WDBC	3-0-6	3-0-6	5-0-4
Shoppers	3-0-6	9-0-0	9-0-0
Parkinsons	5-0-4	8-1-0	3-0-6
Overall	22-0-23	30-2-13	29-0-16

03 | Graph Coloring

Second Application

Graph coloring

Problem and Solution

Problem

- Coloring graph nodes such that no two adjacent nodes have same color

Solution

- Find totally disconnected subgraphs of maximum cardinality (MIS)
- Each subgraph can be assigned to a color

Strategy

- Select number of colors K inferior to graph size
- Construct augmented K graph
- Calculate complement of augmented graph
- Find clique in complement graph ← GBS
- If clique size not equal to graph size, repeat with K+1 colors

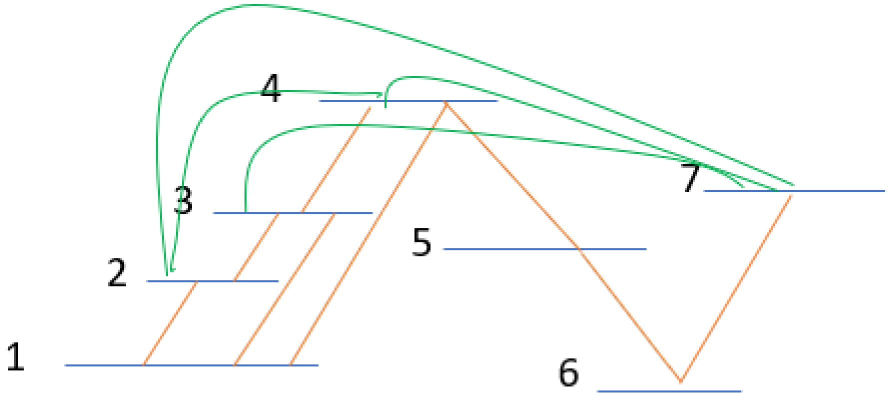
Smart Charging

Introducing use case

- Overlapping Intervals for charging EVs to **one terminal**

Two EV are compatible :

  - if their Charging Intervals (IC) do not overlap
  - if they belong to different groups

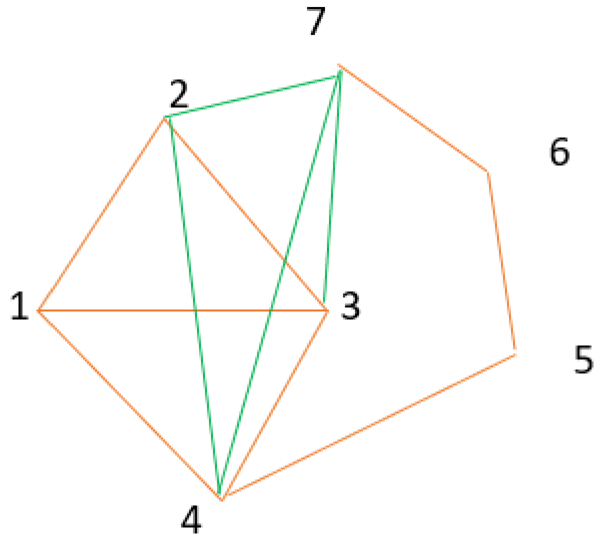


- Conflict graph representation: G(V,E)

A vertex for each EV

An edge between two vertices

  - if two IC overlap
  - if two EV belong to the same group



Win Draw Loss

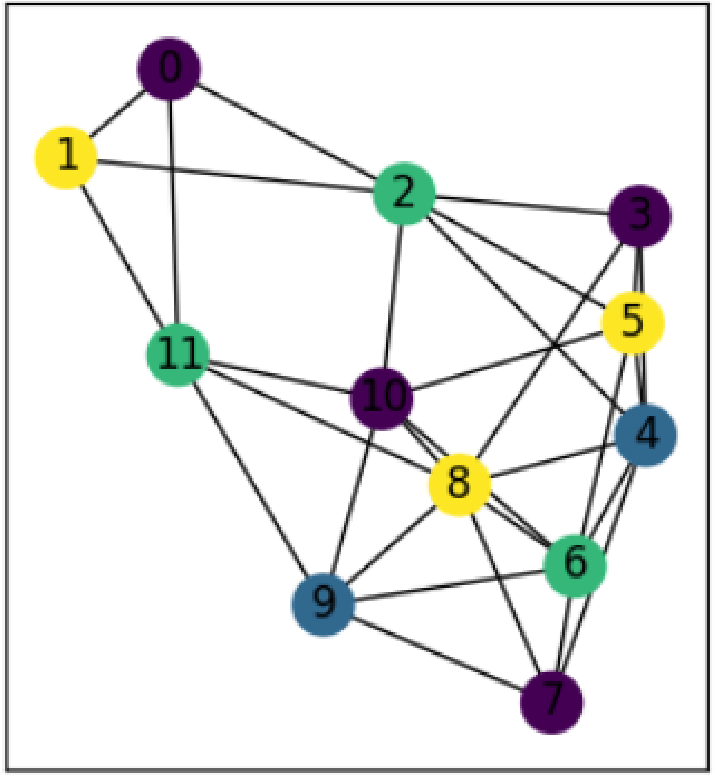
GBSC vs. DSatur

No. Stations	Win	Draw	Loss
4	0	23	0
8	0	24	0
12	1	20	0
16	3	18	0
24	4	10	5
32	2	16	2
Total	**10**	111	**7**

Win

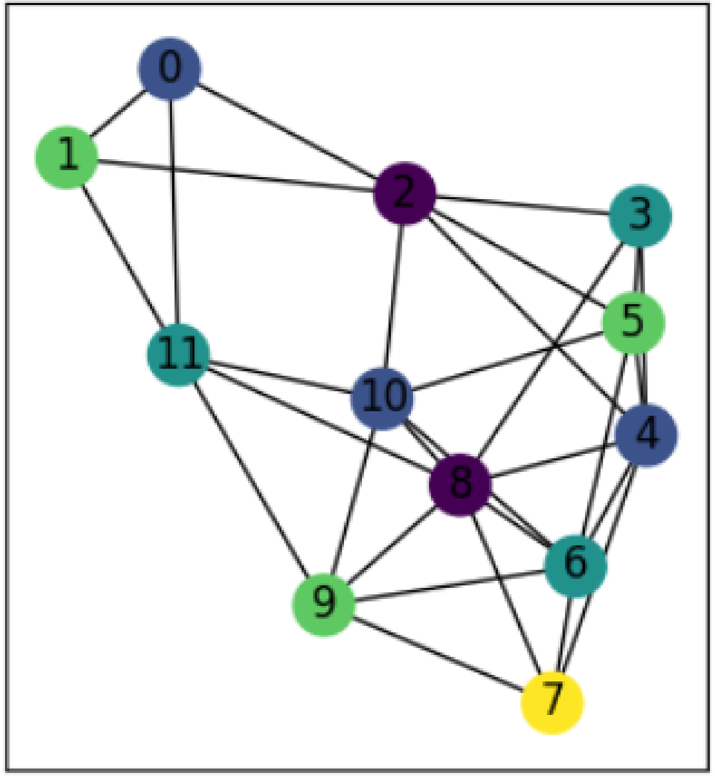
Sample 12 nodes

GBSC



N. Colors : 4

DSat

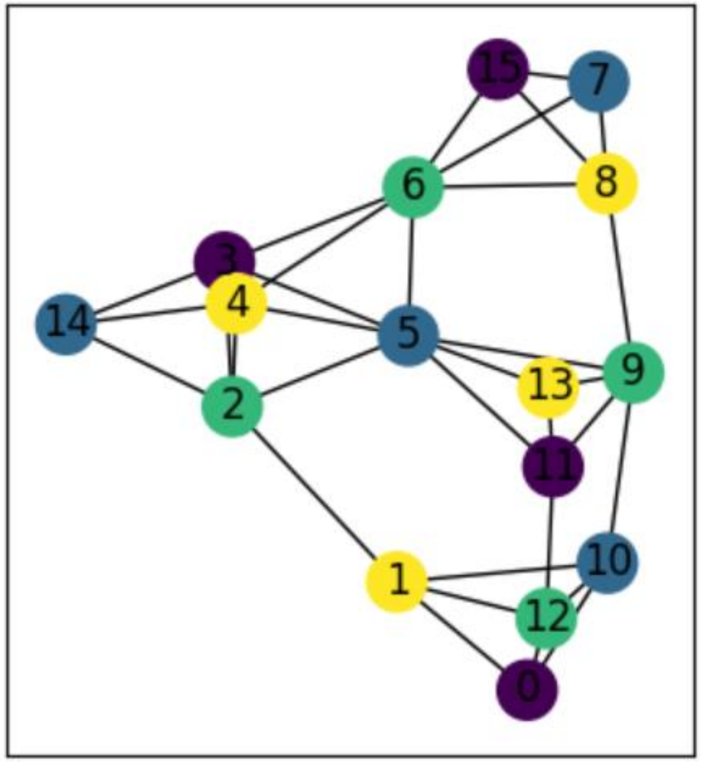


N. Colors : 5

Draw

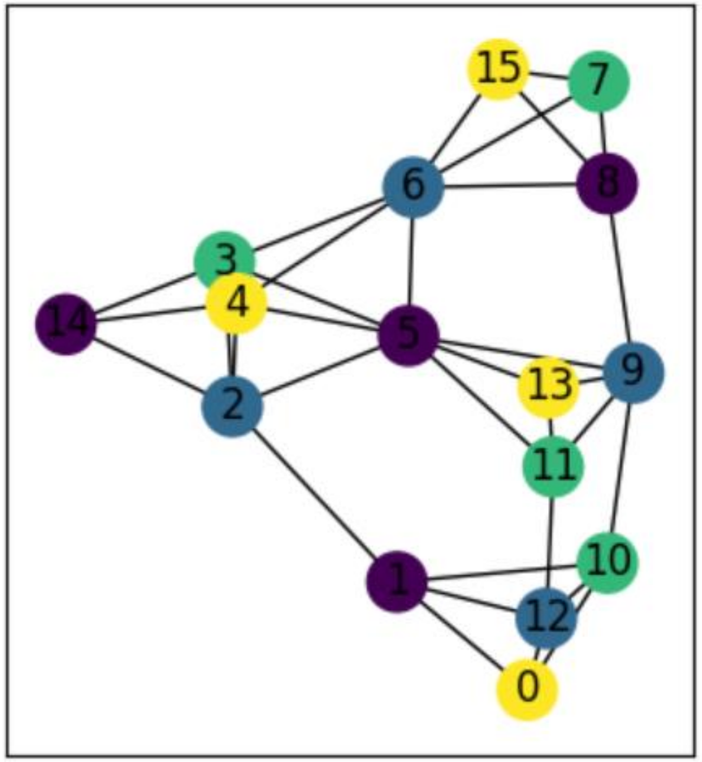
Sample 16 nodes

GBSC



N. Colors : 4

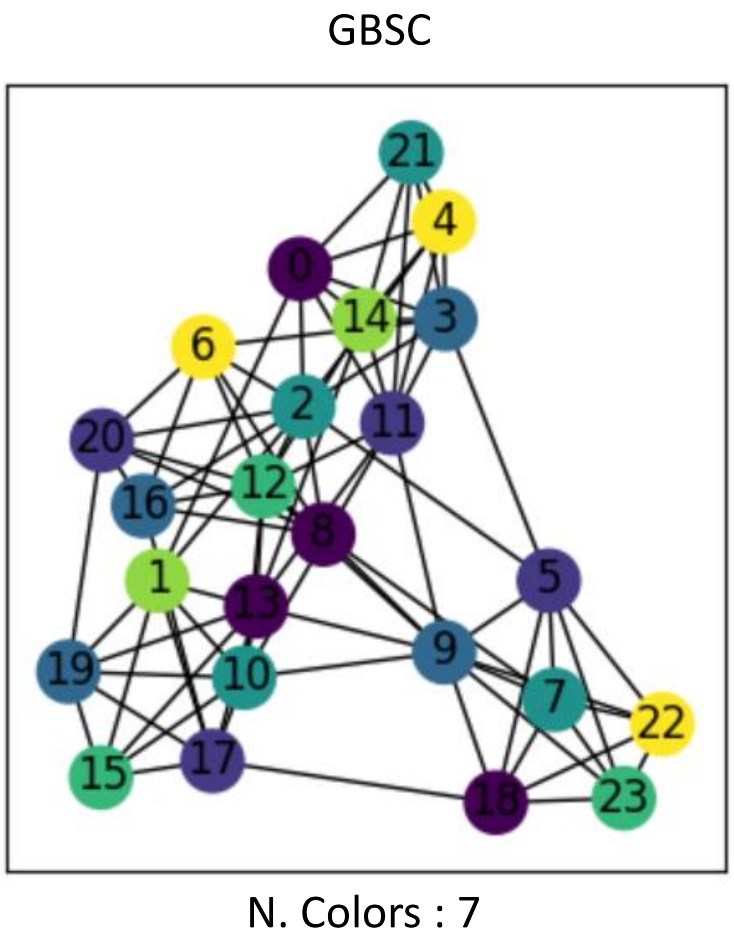
DSat

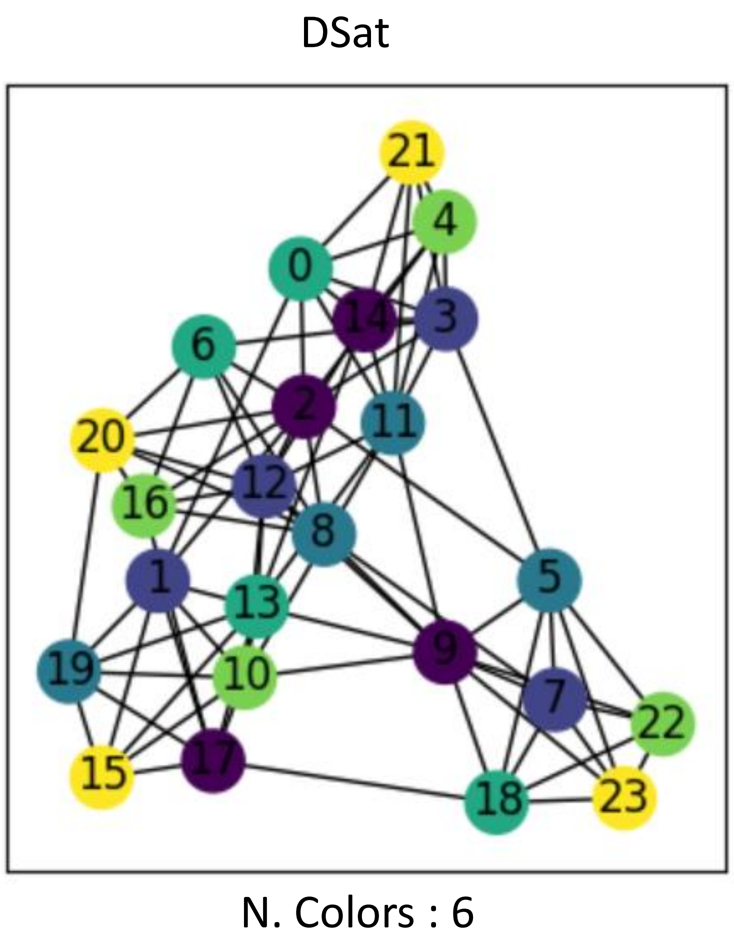


N. Colors : 4

Loss

Sample 24 nodes





Thank you