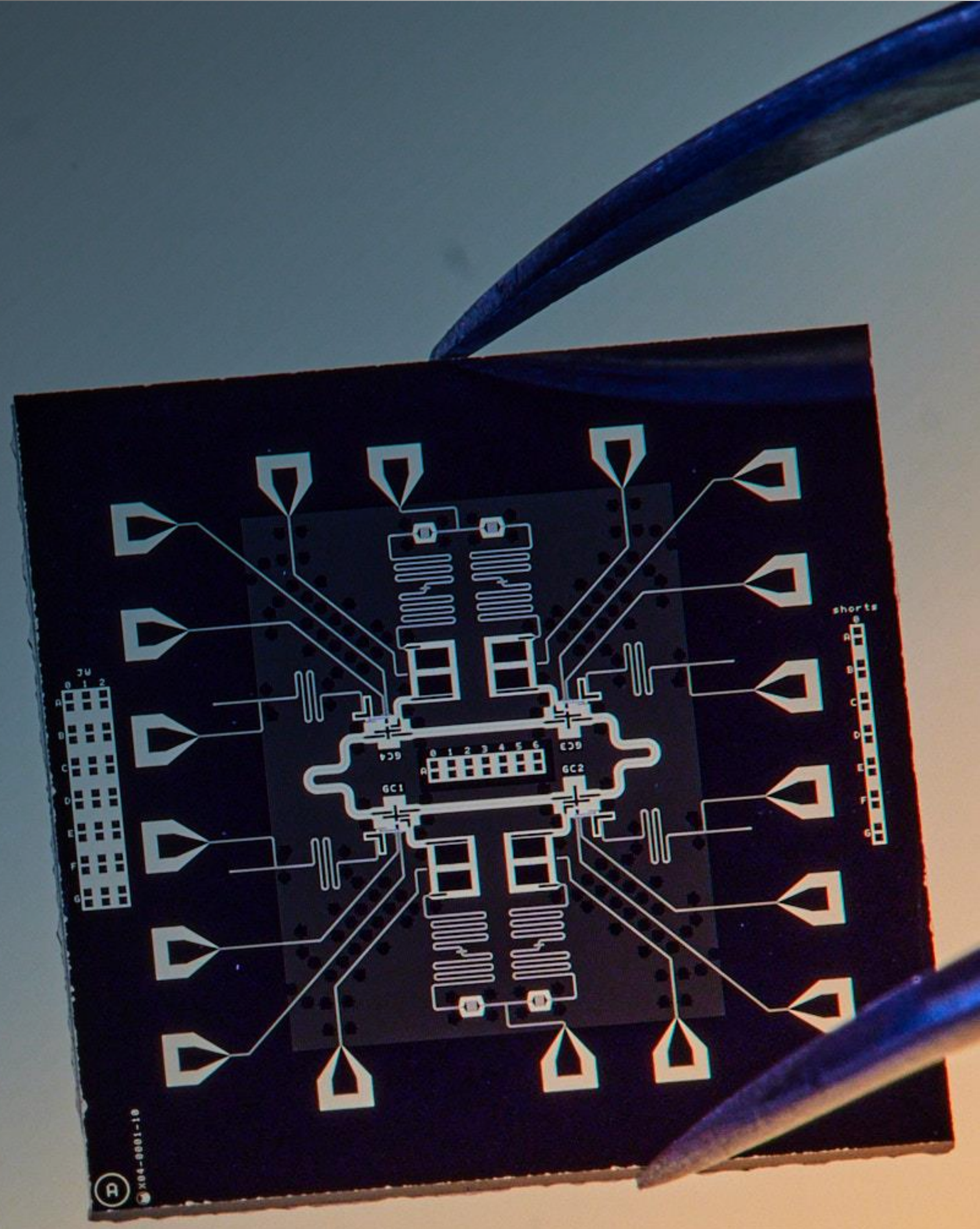
ALICE & BOB

Low-overhead fault-tolerant quantum computing with cat qubits



14 November 2024

TQCI Seminar

Alice&Bob by the numbers

THÉAU PERONNIN
Co-founder & CEO

X - PhD in Quantum Physics from ENS

RAPHAËL LESCANNE
Co-founder & CTO

ENS - PhD in Quantum Physics from ENS

Founded in 2020	18 patents filed	30M€ of VC funding	6 academic partnerships	100 people (incl. 60+ R&D)

A spin-off from the French cQED community

Zaki Leghtas

Mazyar Mirrahimi

Philippe Campagne-Ibarcq

Benjamin Huard

Emmanuel Flurin

MINES ParisTech

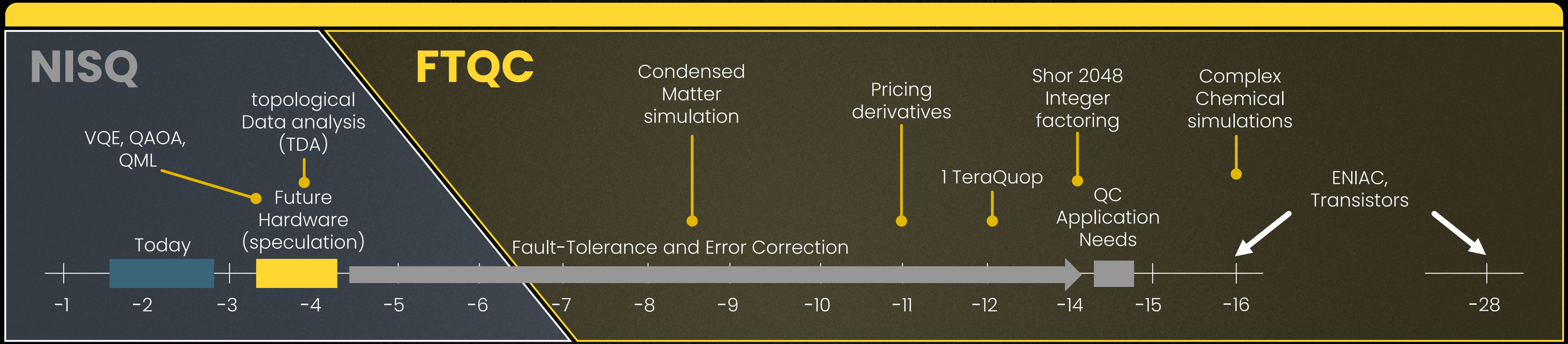
Inria

Inria

ENS DE LYON

cea

Quantum computers are not yet reliable enough

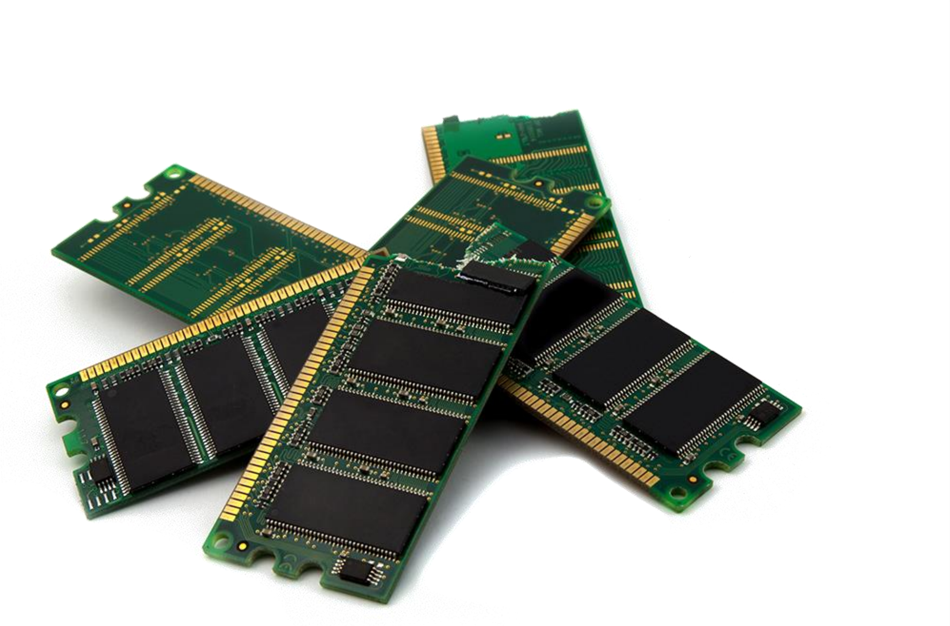


Quantum operations error probabilities ($\log_{10}$)

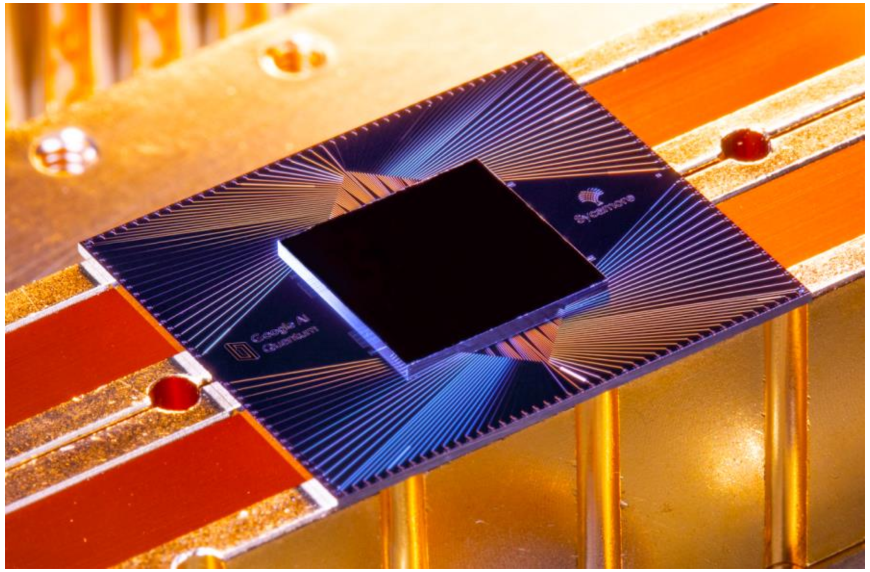
	Chemistry	Finance	Cryptography
# of perfect qb	100 – 1 000	10 000	1 000 – 10 000
# of gates	10^{14} – 10^{16}	10^{10} – 10^{11}	10^{11}
Error per gate	10^{-17}	10^{-13}	10^{-14}
	(Mathias Troyer, Microsoft)	(Will Zeng, Goldman Sachs)	(Banegas, Chalmers)

Understanding quantum technologies, Olivier Ezratty, 2023.

Quantum hardware is too noisy



Classical RAM (Random Access Memory)
~ 10^{-25} errors per bit per operation



Quantum processor (Google Sycamore)
~ 10^{-3} – 10^{-4} errors per bit per operation

Large-scale QC requires ~ 10^{-10} – 10^{-15}

Resource estimation for full-scale FTQC

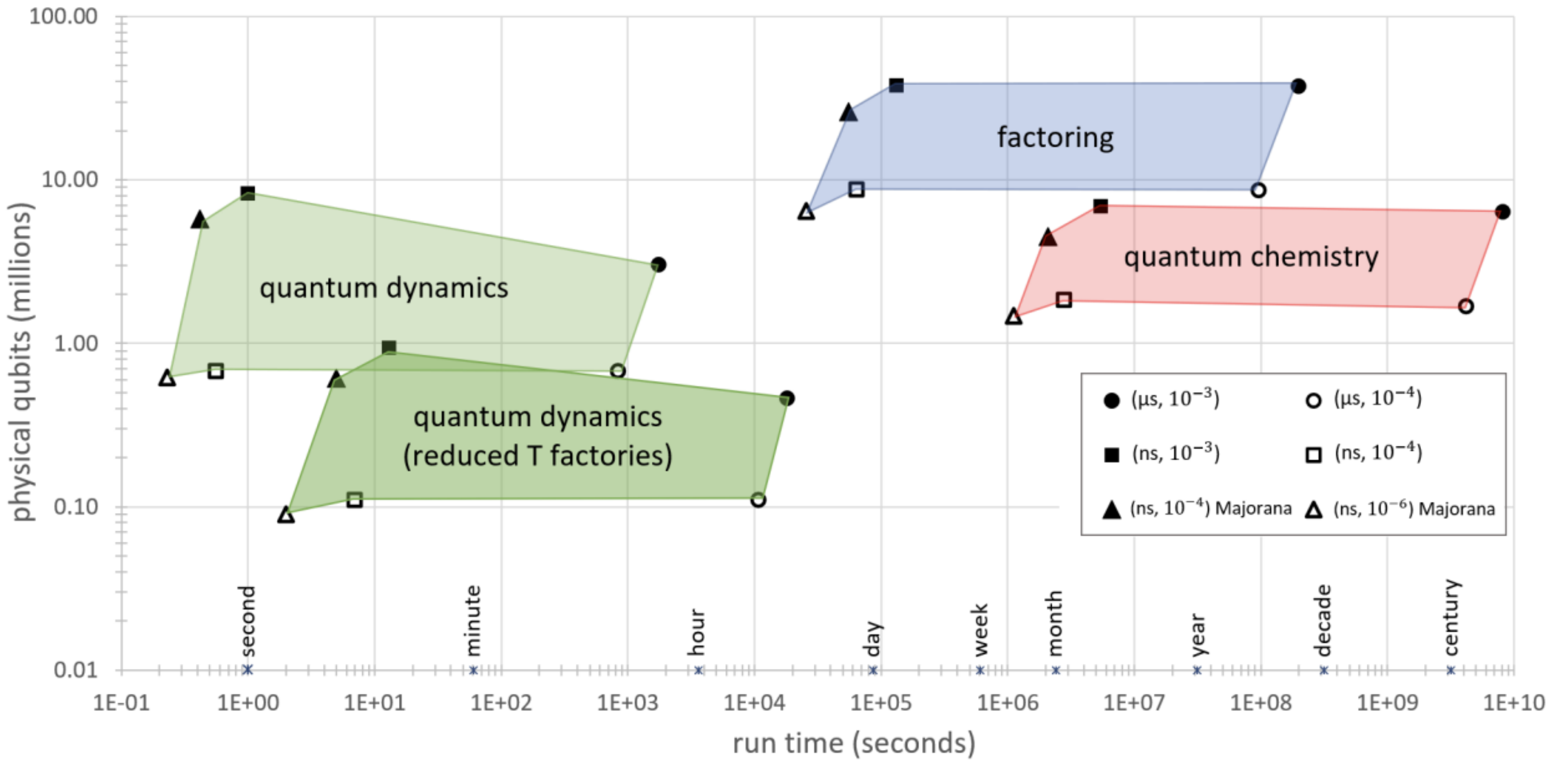
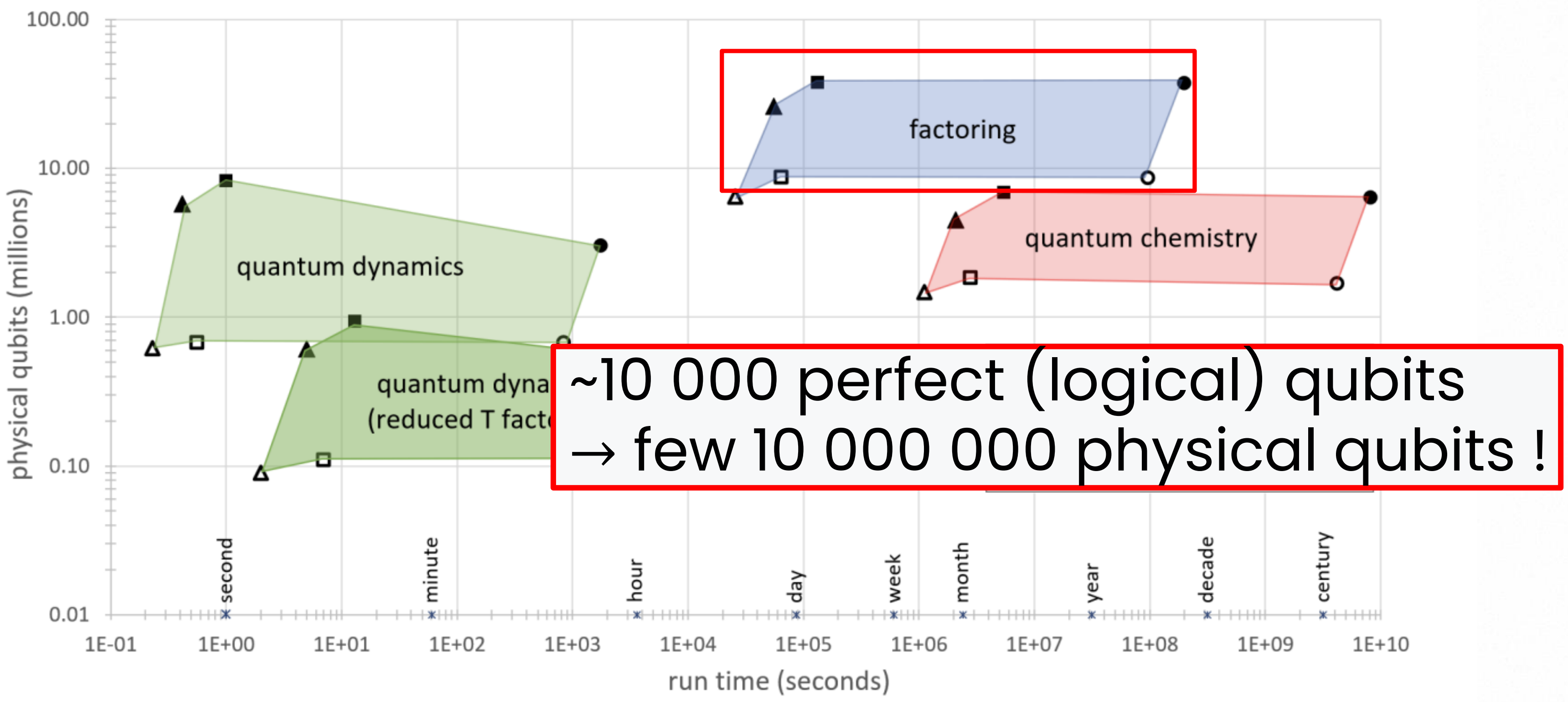


FIG. 3. Estimates of the resources required to implement three applications, assuming the qubit parameter examples specified in Table II. We explore a trade-off in the quantum dynamics application by considering two implementations: one which uses sufficient T factories to supply the needs of the shortest-depth algorithm and another which slows the algorithm down, allowing for a reduced number of T factories.

Beverland et. al, « Assessing requirements to scale to practical quantum advantage », arxiv:2211.07629

Resource estimation for full-scale FTQC



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The Quantum House Of Cards

Xavier Waintal[1]
[1] *Université Grenoble Alpes, PHELIQS, CEA, Grenoble INP, IRIG, Grenoble 38000, France*[*]

THE FUNDAMENTAL LAW OF ANALOG MACHINES

- The overall fidelity of a computation decreases exponentially with the physical fidelity
- $F \approx e^{-\epsilon_0 N_0 - \epsilon_1 N_1 - \epsilon_2 N_2 - \cdots}$

FRUITS ARE FEW AND NOT HANGING LOW

- Practical quantum advantage require super-polynomial speed-ups
- To be useful, a quantum computer must employ deep quantum circuits

THE "SALVATION IS BEYOND THE THRESHOLD" MYTH

- Error correction is required, but incurs a significant overhead (in space and in time)
- The decoding problem is hard and large (10^{15} bits/s)
- Operating below threshold is difficult

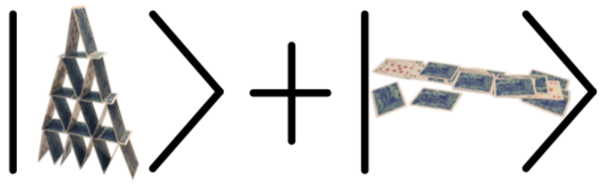

FIG. 1. A quantum computer internal state is a macroscopic quantum state described by an exponentially large set of complex numbers. Such states are subject to decoherence and very fragile.

X. Waintal, PNAS Vol.121, No.1, December 26, 2023

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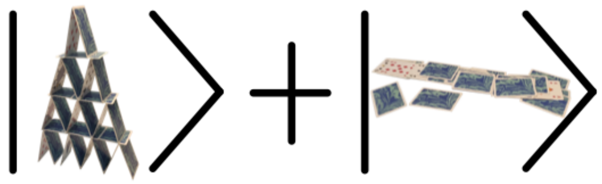


FIG. 1. A quantum computer internal state is a macroscopic quantum state described by an exponentially large set of complex numbers. Such states are subject to decoherence and very fragile.

XAVIER'S CONCLUSION

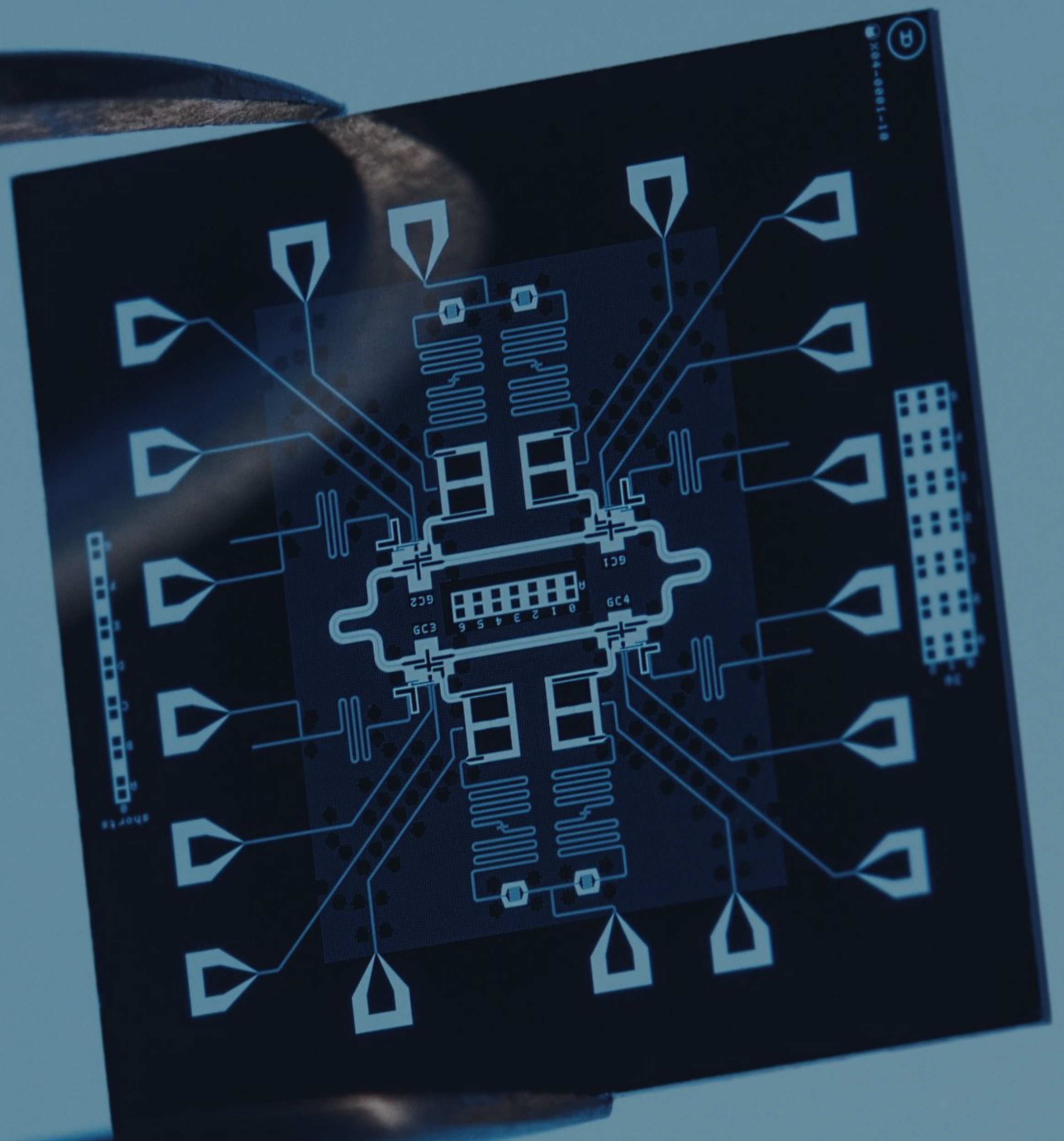
- "I have tried to convey the idea that, perhaps, quantum computing as it has been envisioned so far is simply too difficult to happen."

ALICE & BOB'S CONCLUSION

- We need new types of qubits that implement hardware-efficient error correction

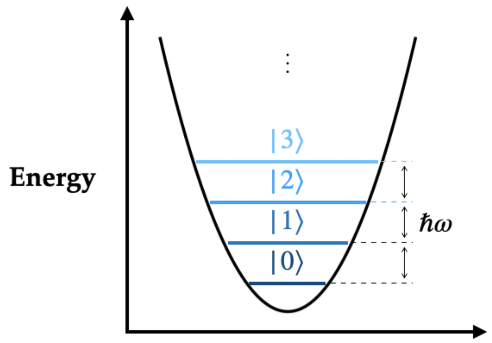
X. Waintal, PNAS Vol.121, No.1, December 26, 2023

Cat qubits
Stable by design



Hardware-efficient error correction with bosonic qubits

A. Joshi, K. Noh, Y. Gao,
Quantum Sci. Technol. 6 033001 (2021)



$$\mathcal{H} = \text{span}\,\{|n\rangle, n \in \mathbb{N}\}$$

Hardware-efficient error correction with bosonic qubits

A. Joshi, K. Noh, Y. Gao,
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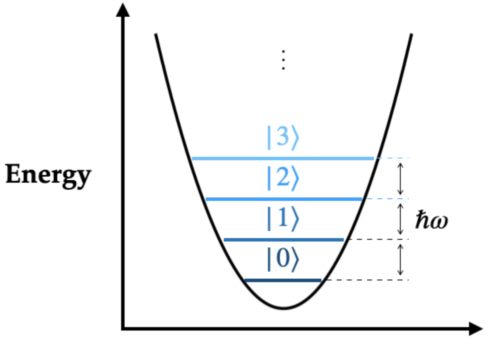

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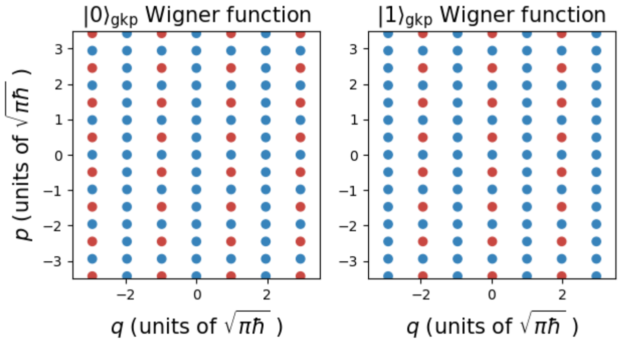
GKP qubit



D. Gottesman, A. Kitaev, J. Preskill
Phys. Rev. A 64, 2001.

Nord Quantique

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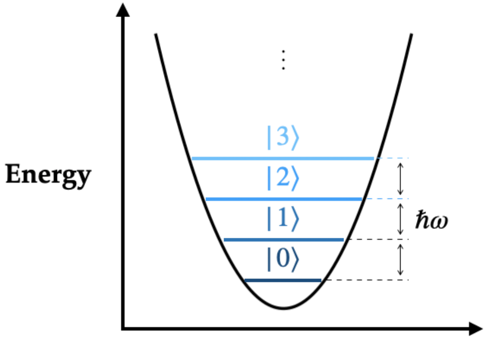

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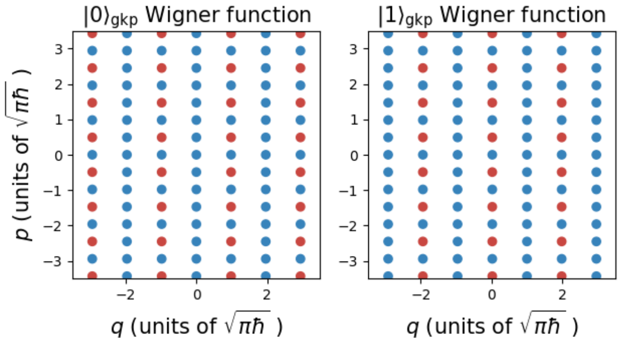

D. Gottesman, A. Kitaev, J. Preskill
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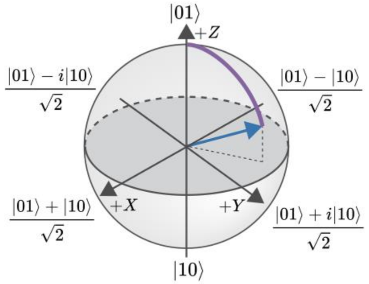
Nord Quantique

Dual-rail qubit



A. Kubica et al, arxiv:2208.05461
JD Teoh et al, arxiv:2212.12077

qci aws

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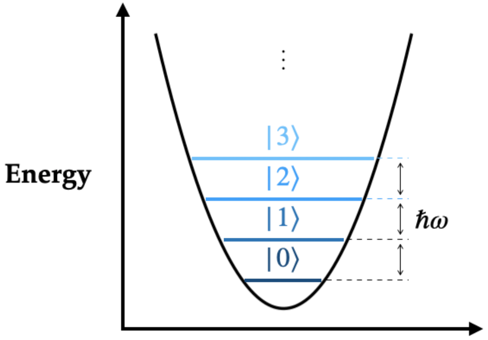

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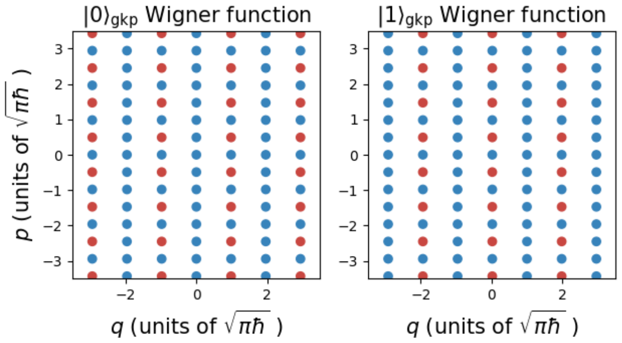

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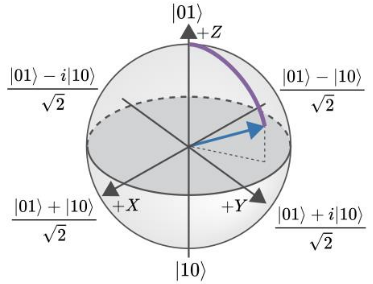
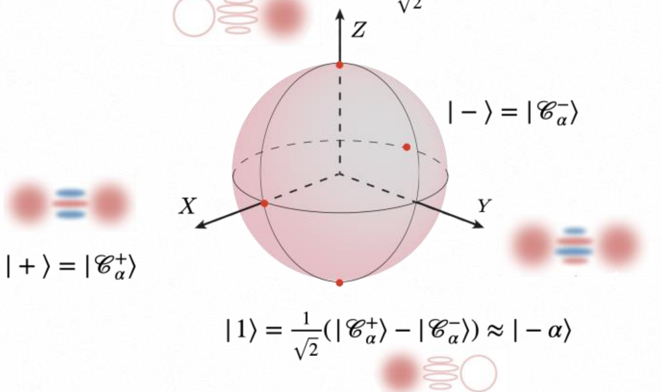


A. Kubica et al, arxiv:2208.05461
JD Teoh et al, arxiv:2212.12077

Cat qubit



$|0\rangle = \frac{1}{\sqrt{2}}(|\mathcal{C}_\alpha^+\rangle + |\mathcal{C}_\alpha^-\rangle) \approx |\alpha\rangle$

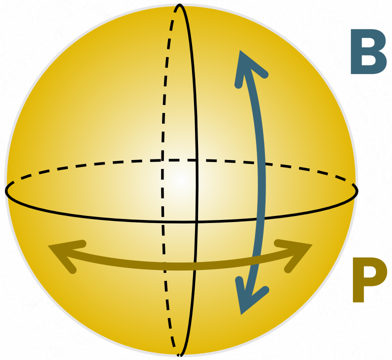
$|-\rangle = |\mathcal{C}_\alpha^-\rangle$

$|+\rangle = |\mathcal{C}_\alpha^+\rangle$

$|1\rangle = \frac{1}{\sqrt{2}}(|\mathcal{C}_\alpha^+\rangle - |\mathcal{C}_\alpha^-\rangle) \approx |-\alpha\rangle$

M. Mirrahimi et al, New J. Phys. 16 045014

The cat qubit: a « biased noise » qubit



Bit-flip $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow |\psi\rangle = \alpha|1\rangle + \beta|0\rangle$

Phase-flip $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow |\psi\rangle = \alpha|0\rangle - \beta|1\rangle$

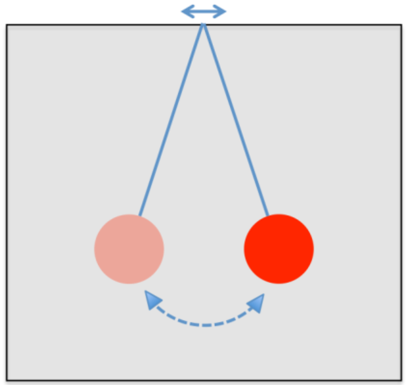
TRANSMON QUBITS	CAT QUBITS
$T_X = T_Z = 10 - 100\ \mu s$ $T_Z/T_X \sim 1$	$T_X = 10\ s \quad T_Z = 1\ \mu s$ $T_Z/T_X \sim 10^7$

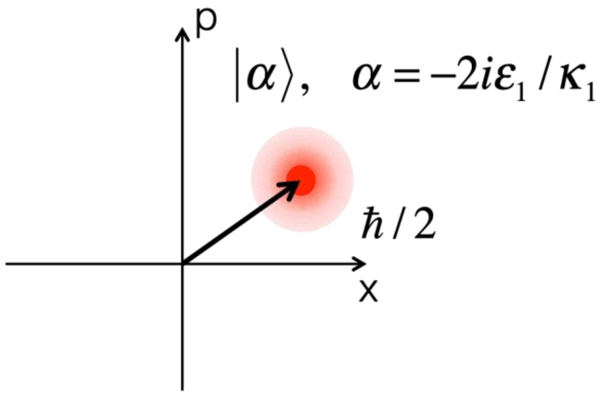
The driven-dissipative cat qubit

« Single-photon » driven-damped harmonic oscillator

$$H = \epsilon_1^* a + \epsilon_1 a^\dagger \quad + \quad L = \sqrt{\kappa_1} a$$

$$\equiv \quad L = \sqrt{\kappa_1}(a - \alpha)$$





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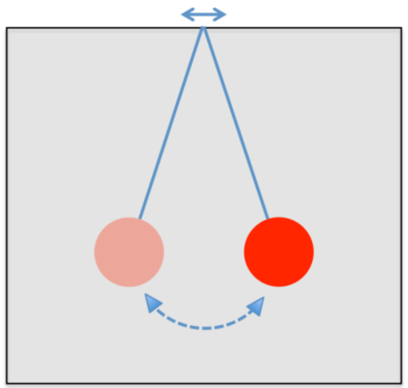
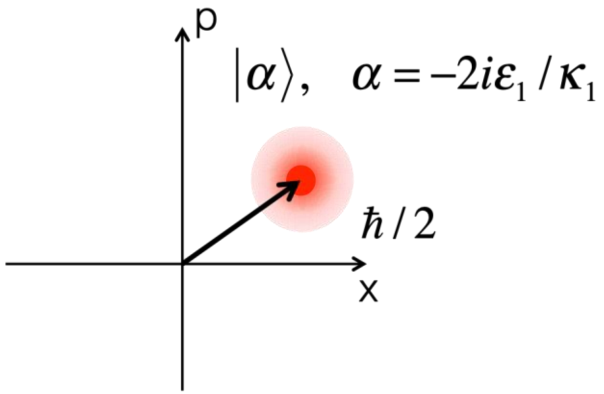
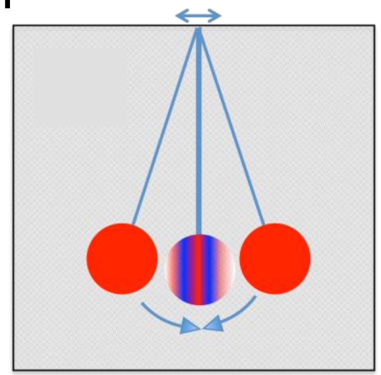




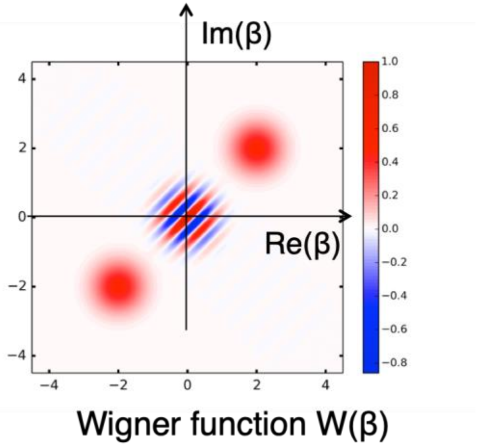
« Two-photon » driven-damped harmonic oscillator

$$H = \epsilon_2^* a^2 + \epsilon_2 a^{\dagger 2} \quad + \quad L = \sqrt{\kappa_2}\, a^2$$

$$\equiv \quad L = \sqrt{\kappa_2}(a^2 - \alpha^2)$$



$$\{|\alpha\rangle, |-\alpha\rangle\}$$

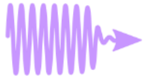


$$\alpha = \pm\sqrt{-2i\varepsilon_2 / \kappa_2}$$

Stabilizing the cat manifold: the two-photon exchange

Parametric pumping for 2-photon coupling

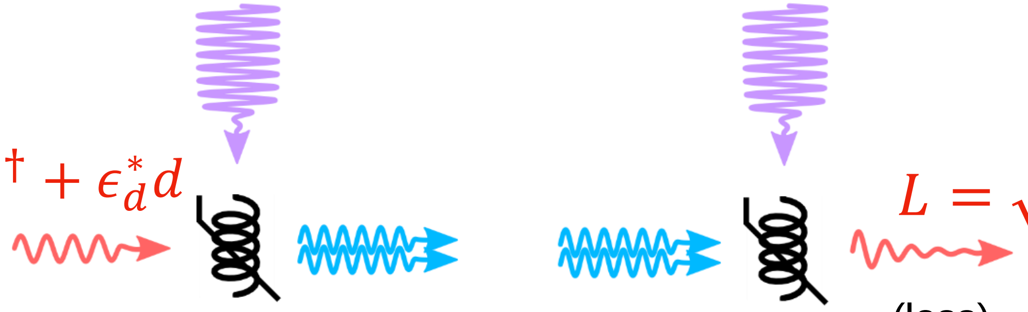
$$H = g_2 a^{\dagger 2} d + g_2 a^2 d^\dagger$$



$$\omega_p = 2\omega_a - \omega_d$$

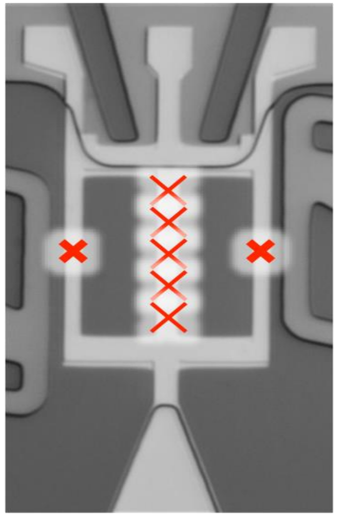
$H = \epsilon_d d^\dagger + \epsilon_d^* d$

(drive)



$L = \sqrt{\kappa_d}\, d$

(loss)



ATS (parametric mixing device)

$$H_{eff} = \epsilon_2^* a^2 + \epsilon_2 a^{\dagger 2}$$

(two-photon drive)

$$L_{eff} = \sqrt{\kappa_2}\, a^2$$

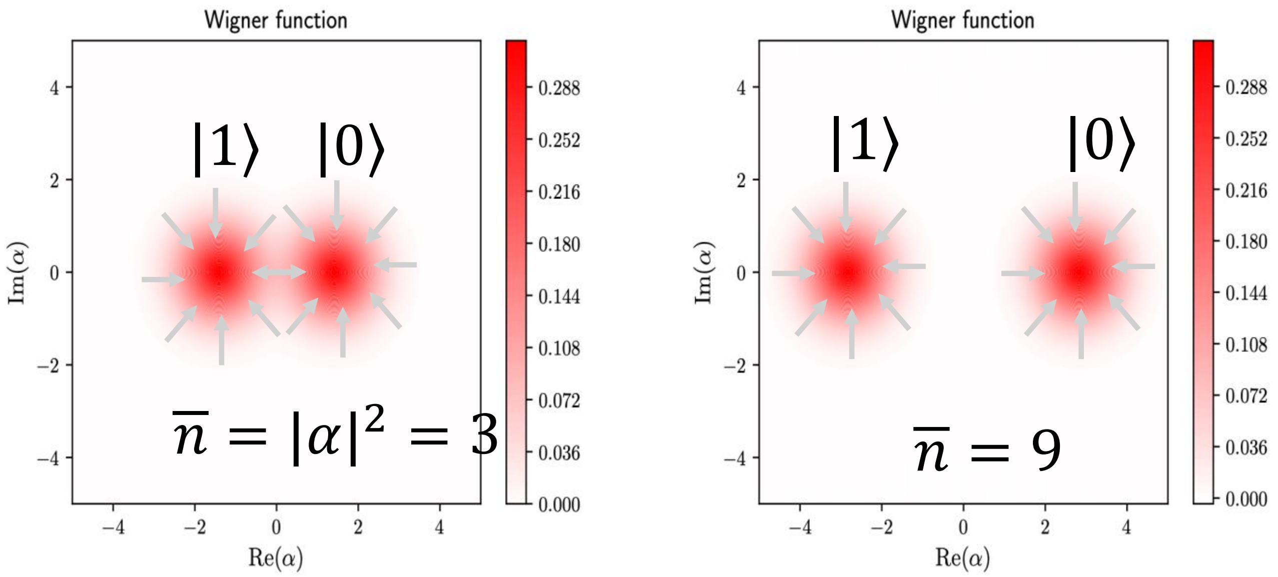
(two-photon loss)

Z. Leghtas et al, Science 2015

R. Lescanne et al., Nature Physics 2020

A biased noise qubit with « tunable bias »

Exponential reduction of bit-flips



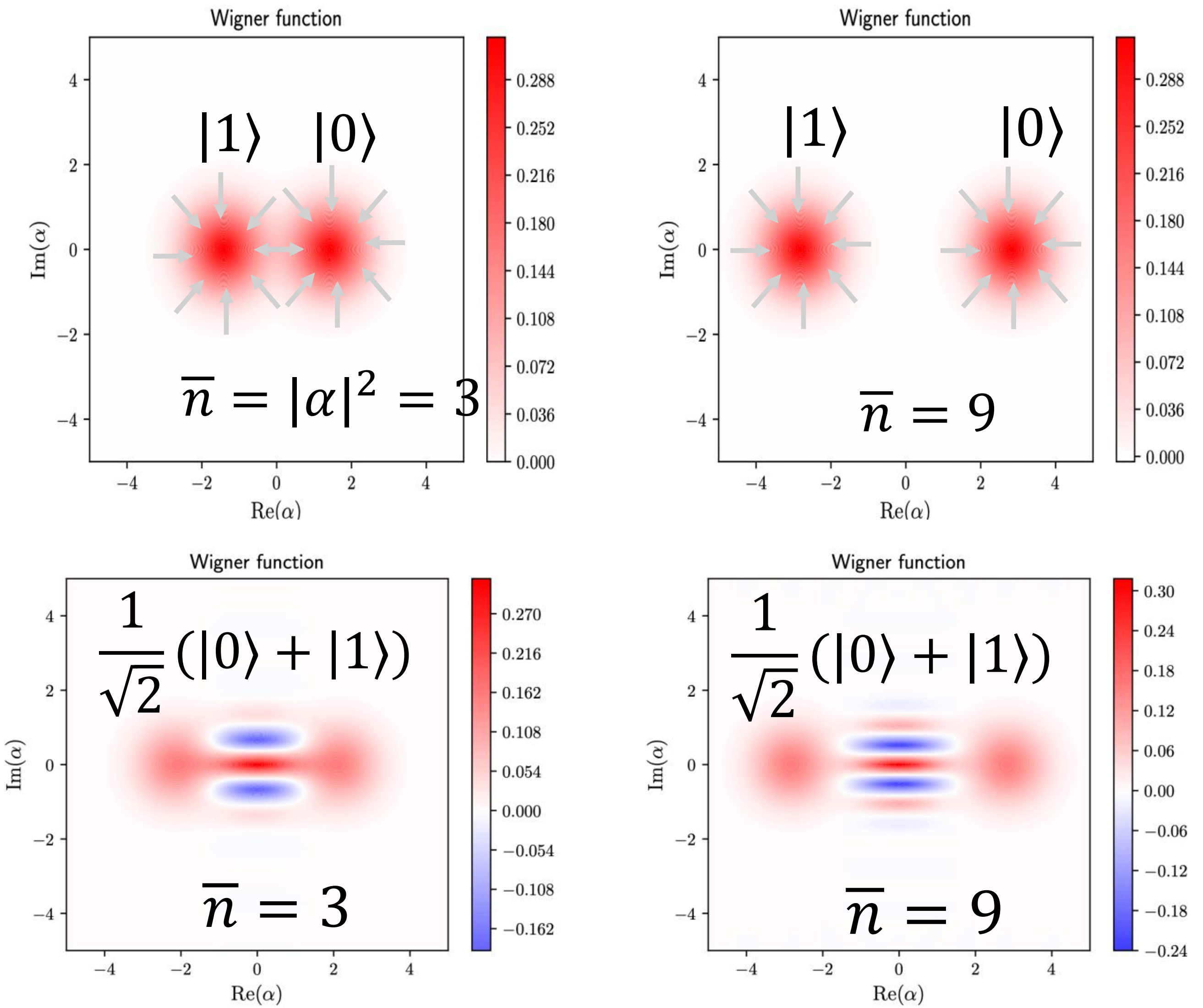
$+Z$

see J. Cohen, PhD thesis, 2017

A biased noise qubit with « tunable bias »

Exponential reduction of bit-flips

Linear increase of phase-flips

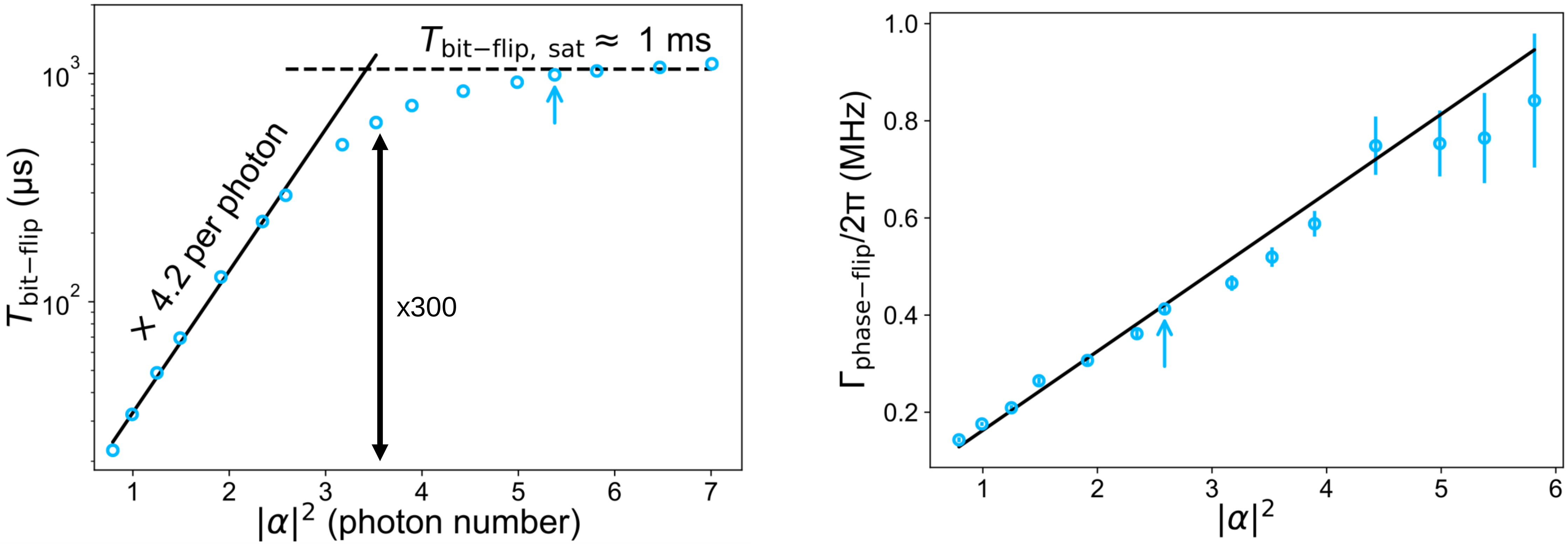


$+Z$

$+X$

see J. Cohen, PhD thesis, 2017

Experimental suppression of bit-flips (1/3)



R. Lescanne, Z. Leghtas et al., Nature Physics, 2020

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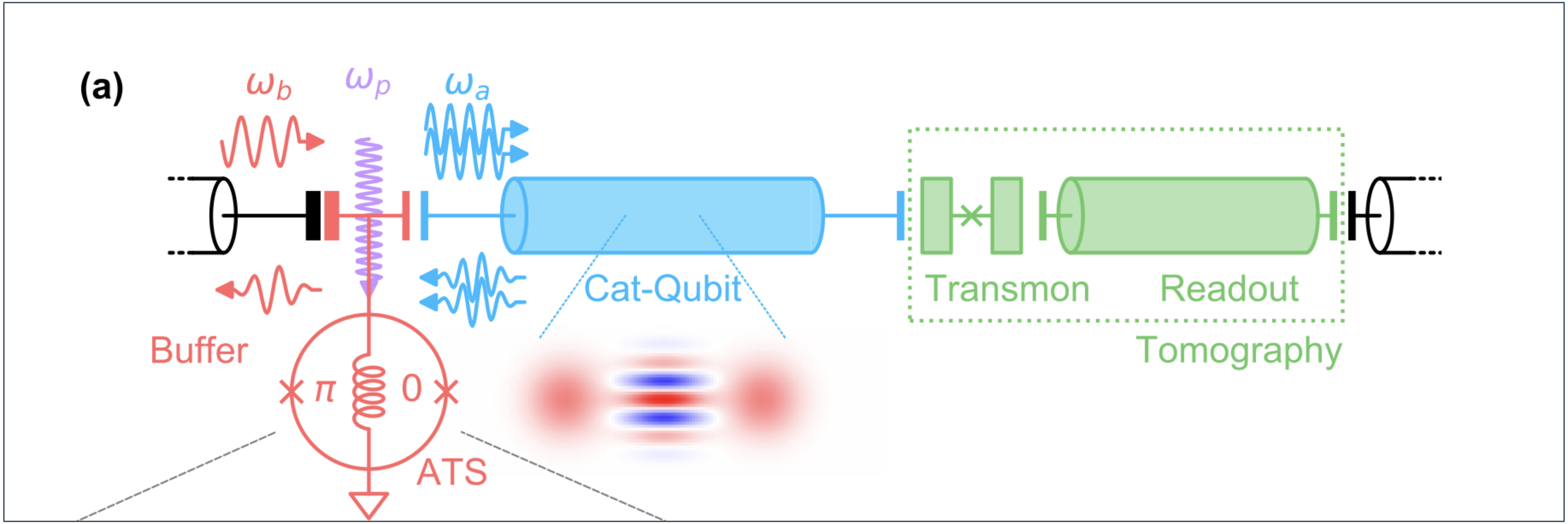
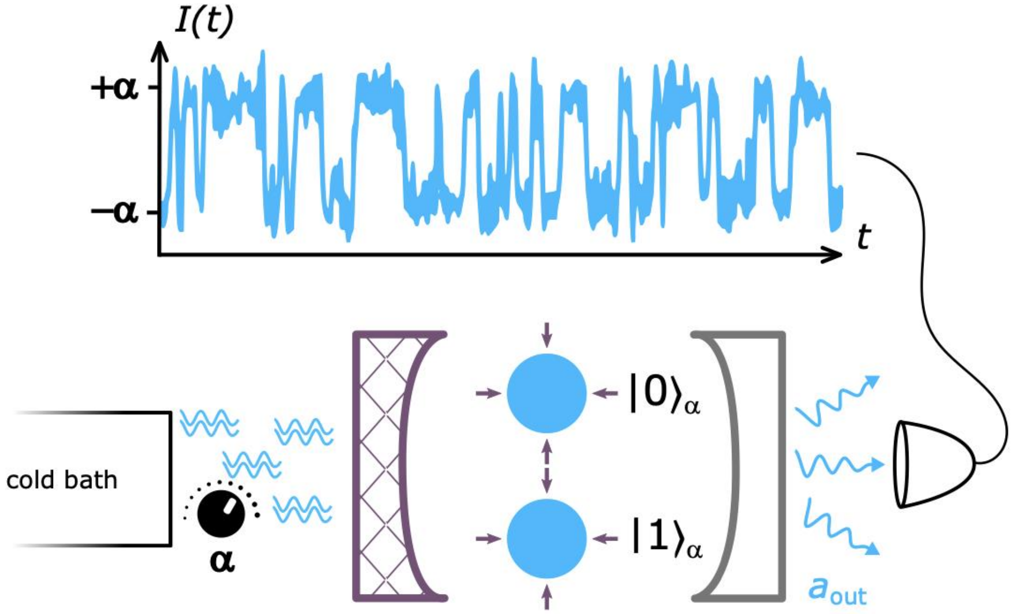
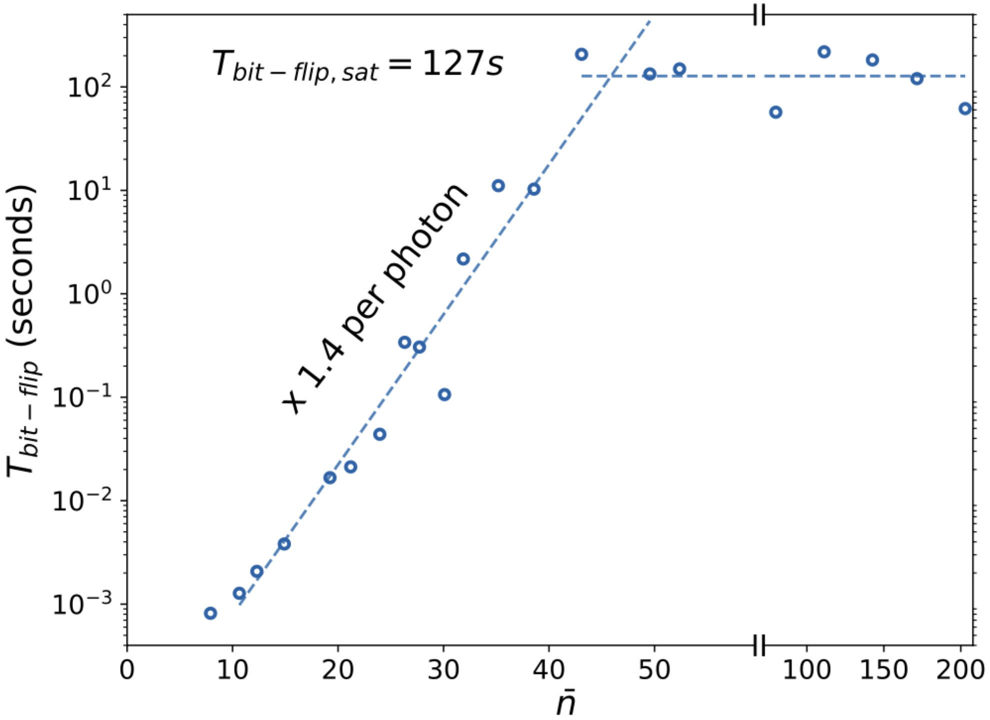


R. Lescanne, Z. Leghtas et al., Nature Physics, 2020

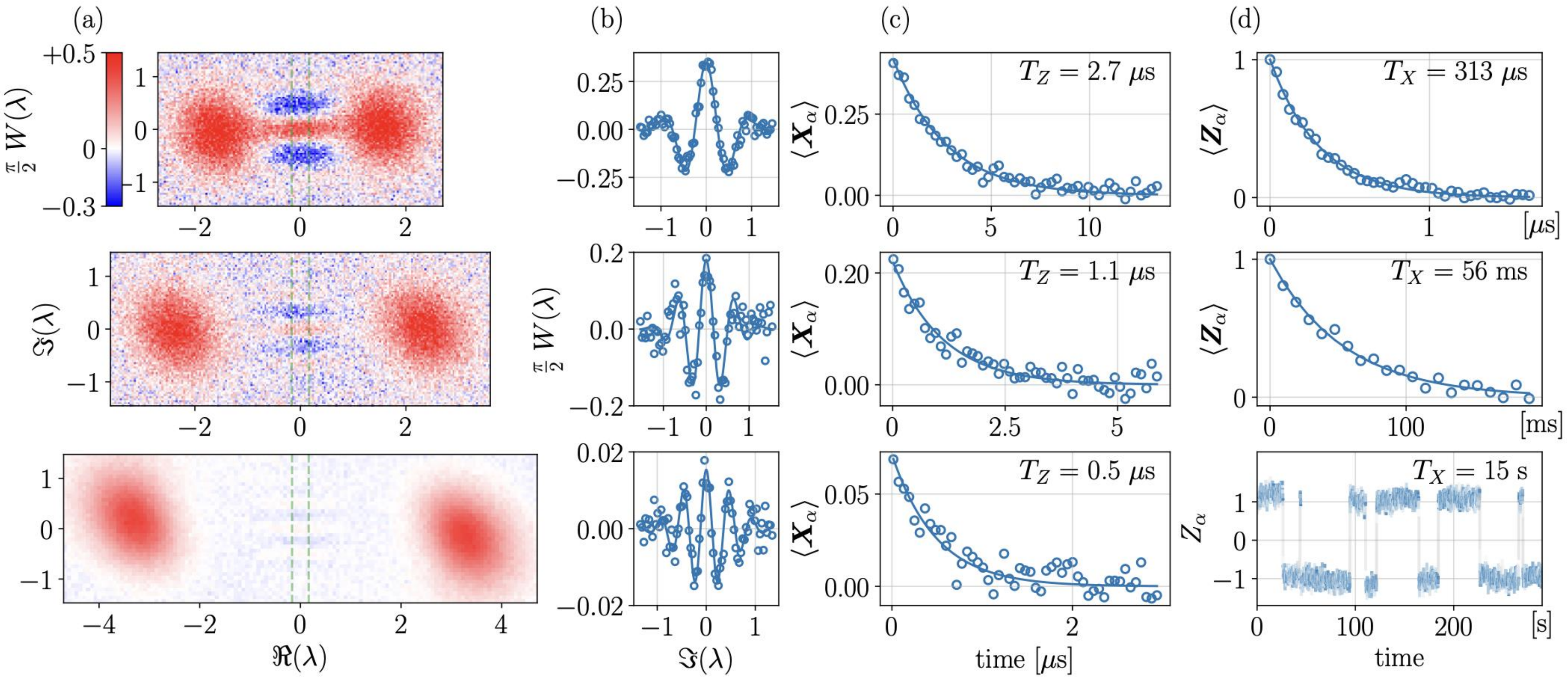
Experimental suppression of bit-flips (2/3)





C. Berdou, Z. Leghtas et al., PRX Quantum, 2023

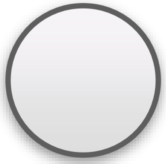
Experimental suppression of bit-flips (3/3)



U.Réglade, Z. Leghtas et al., arXiv2307.06617, July 2023

« Hardware-efficient » protection against bit-flips

$p_X = p_Z$



$$|0\rangle_L = |0\ldots0\rangle = |0\rangle^{\otimes d}$$

$$|1\rangle_L = |1\ldots1\rangle = |1\rangle^{\otimes d}$$

$$|\pm\rangle_L = \frac{1}{\sqrt{2}}(|0\rangle_L \pm |1\rangle_L)$$

$\mathbb{P}[X_L] = \mathbb{P}[$majority of qubits bit-flipped $] \propto \binom{d}{\frac{d+1}{2}} p_X^{\frac{d+1}{2}}$ $\rightarrow 0$

$\mathbb{P}[Z_L] = \mathbb{P}[$any of the qubits phase-flipped $] \propto d \times p_Z$

« Hardware-efficient » protection against bit-flips

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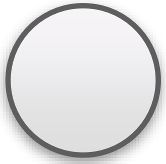

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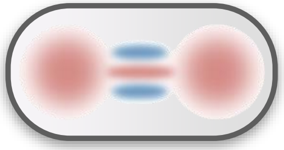
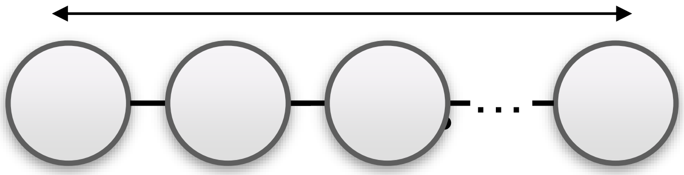
α^2



« $\Leftrightarrow$ »

d

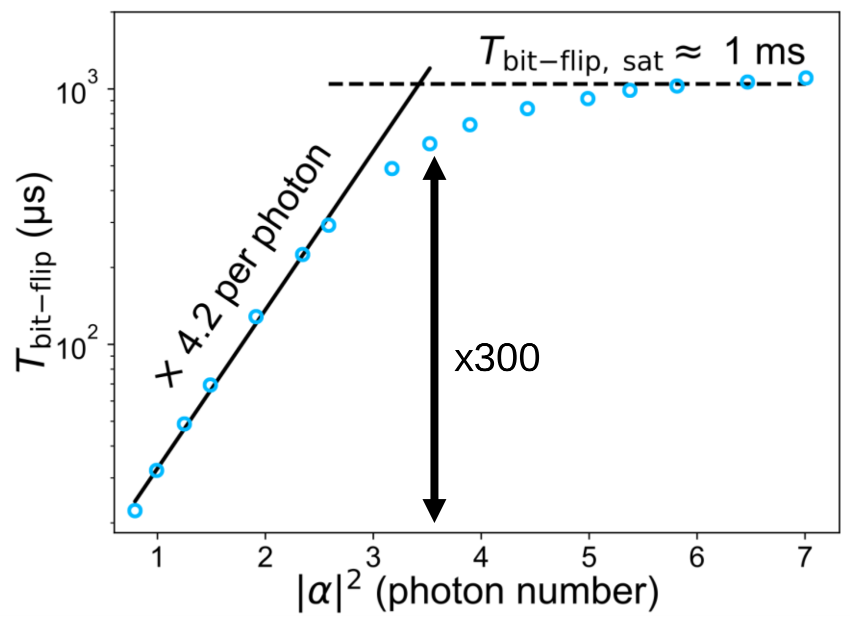


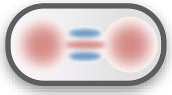
Google Quantum AI, Nature (2021)

« Hardware-efficient » quantum error correction

Exponential suppression of bit-flips in a qubit encoded in an oscillator

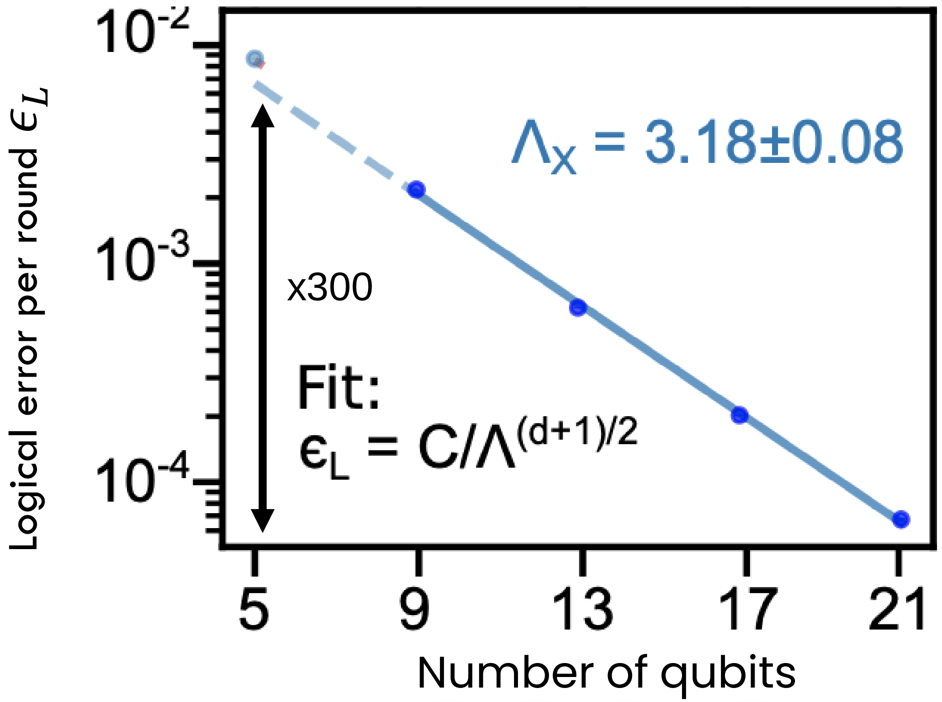
R. Lescanne, Z. Leghtas et al., Nature Physics, 2020

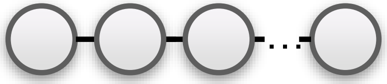




Exponential suppression of bit or phase flip errors with repetitive error correction

Google Quantum AI, Nature (2021)

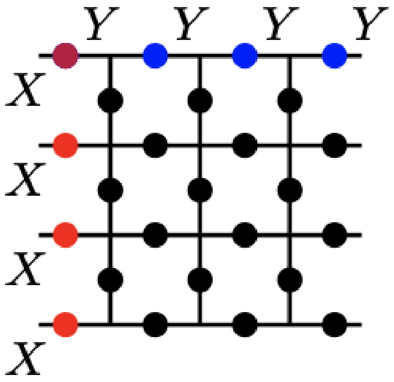




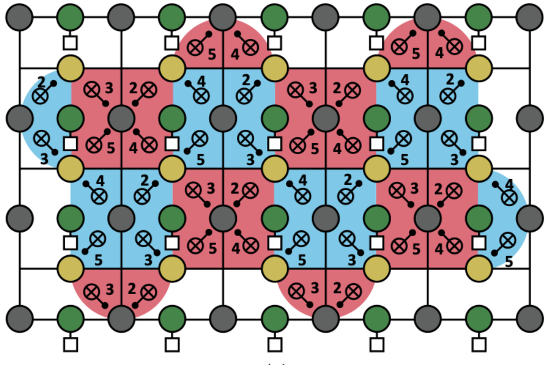
Fully protected logical qubit?

Include some bit-flip error correction capability ?

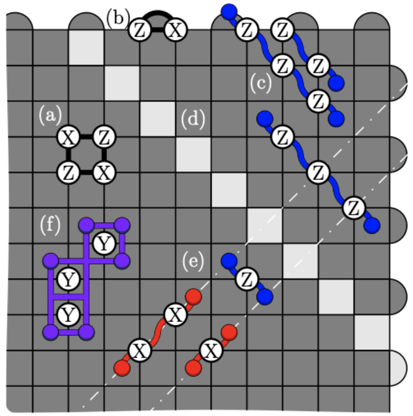
D. Tuckett *et al*, PRL 120,0505 5 (2018)



C. Chamberland et al,
PRX Quantum 3, 010329 (2022)



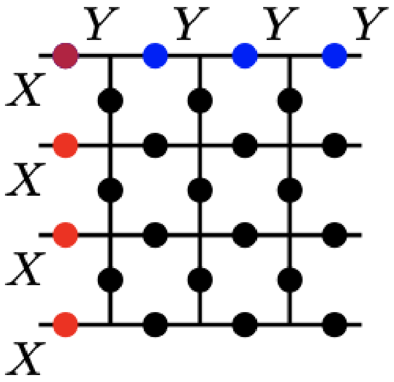
J. Pablo Bonilla Ataides *et al*,
Nature Com. 12, 2171 (2021)



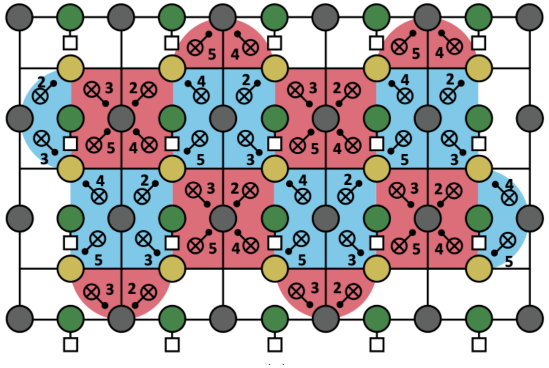
Fully protected logical qubit?

Include some bit-flip error correction capability ?

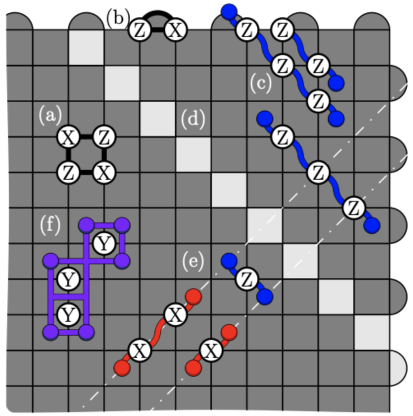
D. Tuckett *et al*, PRL 120,0505 5 (2018)



C. Chamberland et al,
PRX Quantum 3, 010329 (2022)

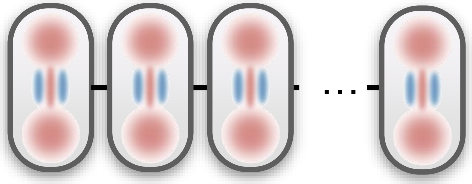


J. Pablo Bonilla Ataides *et al*,
Nature Com. 12, 2171 (2021)



$\bar{n} = |\alpha|^2$ **large enough to handle bit-flips alone ?**

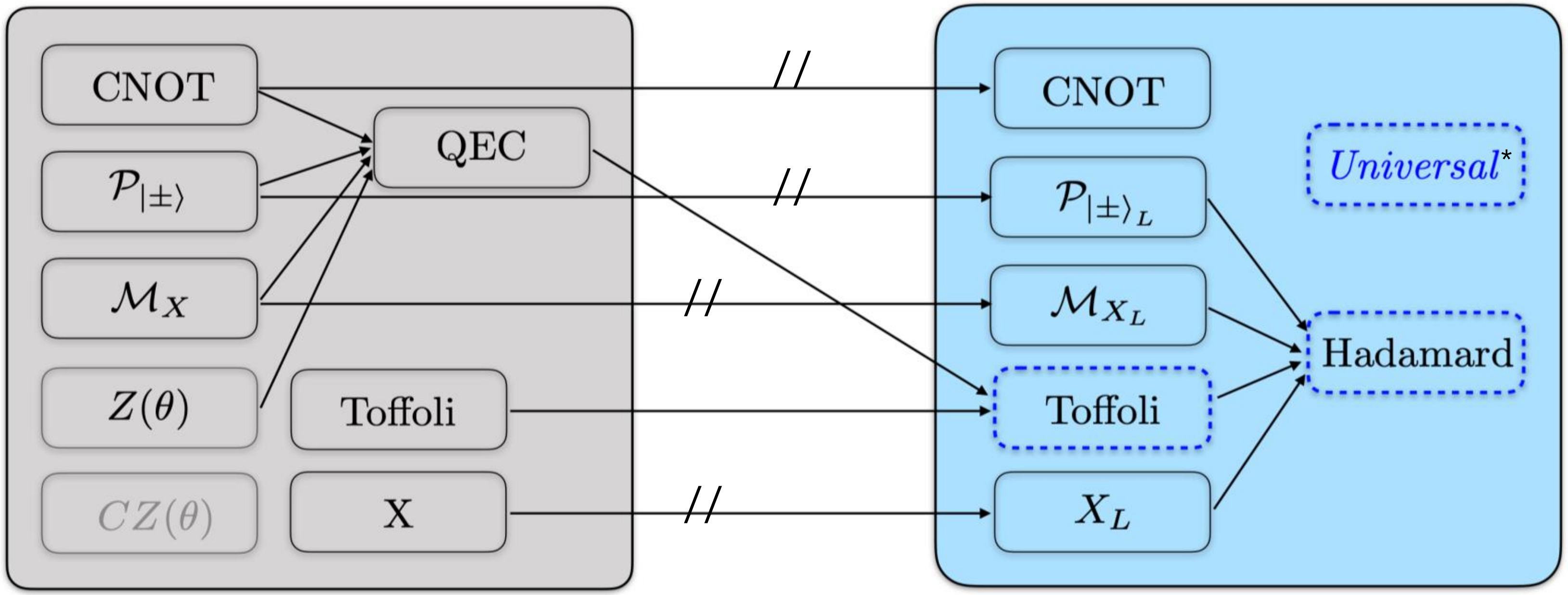
JG and MM, Phys. Rev. X 9, 041053



Scheme for universal quantum computation

« Bias-preserving » operations

Fault-tolerant logical operations

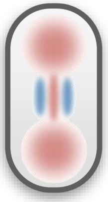


// Transversal

Single cat-qubit level

$$|+\rangle = |\mathcal{C}_\alpha^+\rangle$$

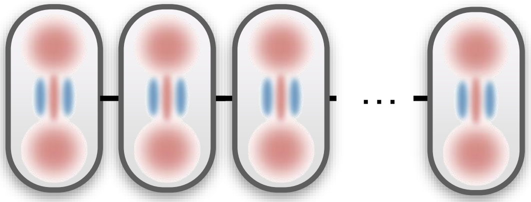
$$|-\rangle = |\mathcal{C}_\alpha^-\rangle$$



Repetition cat-qubit level

$$|+\rangle_L = |\mathcal{C}_\alpha^+\rangle^{\otimes n}$$

$$|-\rangle_L = |\mathcal{C}_\alpha^-\rangle^{\otimes n}$$



JG and MM, Phys. Rev. X 9, 041053

Hardware-efficient QEC

How does this translate to practical applications?

Elliptic Curve Cryptography

Diffie-Hellman key exchange

Shared knowledge (public)

$$y^2 = x^3 + ax + b$$

$$G = (x_0, y_0)$$

k_a

$k_a G$ →

← $k_b G$

Public channel

k_b

$k_a(k_b G)$

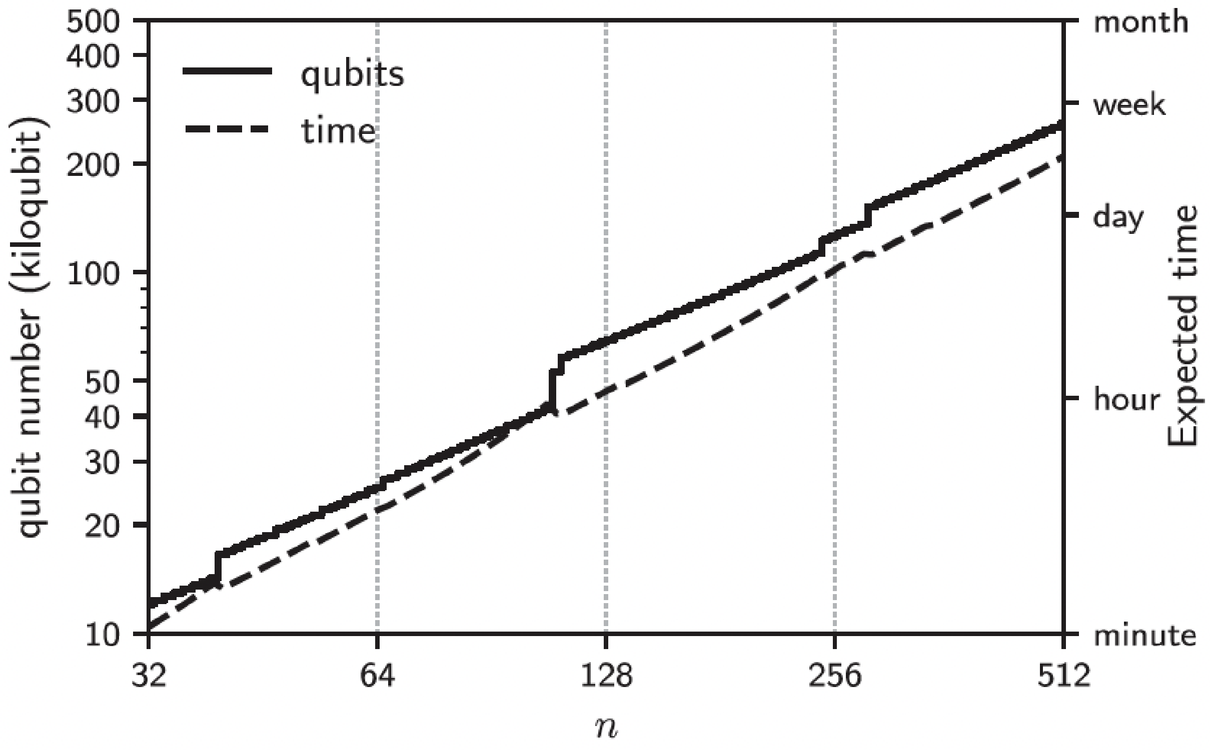
$k_b(k_a G)$

Elliptic Curve Cryptography: resource analysis

PHYSICAL REVIEW LETTERS **131,** 040602 (2023)

Performance Analysis of a Repetition Cat Code Architecture: Computing 256-bit Elliptic Curve Logarithm in 9 Hours with 126 133 Cat Qubits

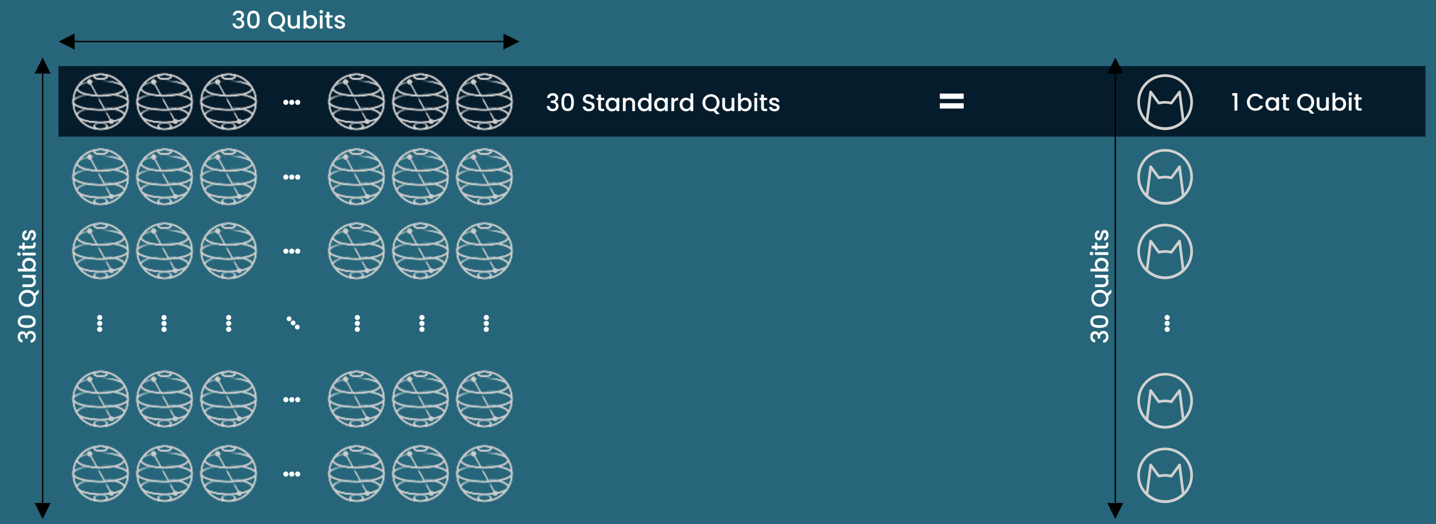
Élie Gouzien,[1,*] Diego Ruiz,[2,3] Francois-Marie Le Régent,[2,3] Jérémie Guillaud,[2] and Nicolas Sangouard[1,†]



Cat qubits for low-overhead FTQC

"Quantitative" Approach
Standard Qubits + Surface Code

"Qualitative" Approach
Cat Qubits + Repetition Code



Shor's algorithm to break RSA encryption

22M physical qubits
C. Gidney et al. 2019

350k cat qubits
E. Gouzien et al. 2022

ALICE & BOB

1
A&B cat qubit

≈

Google

49
Google physical qubits

ALICE & BOB

100%
of the scientific articles cited are from A&B

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mentions of A&B technology

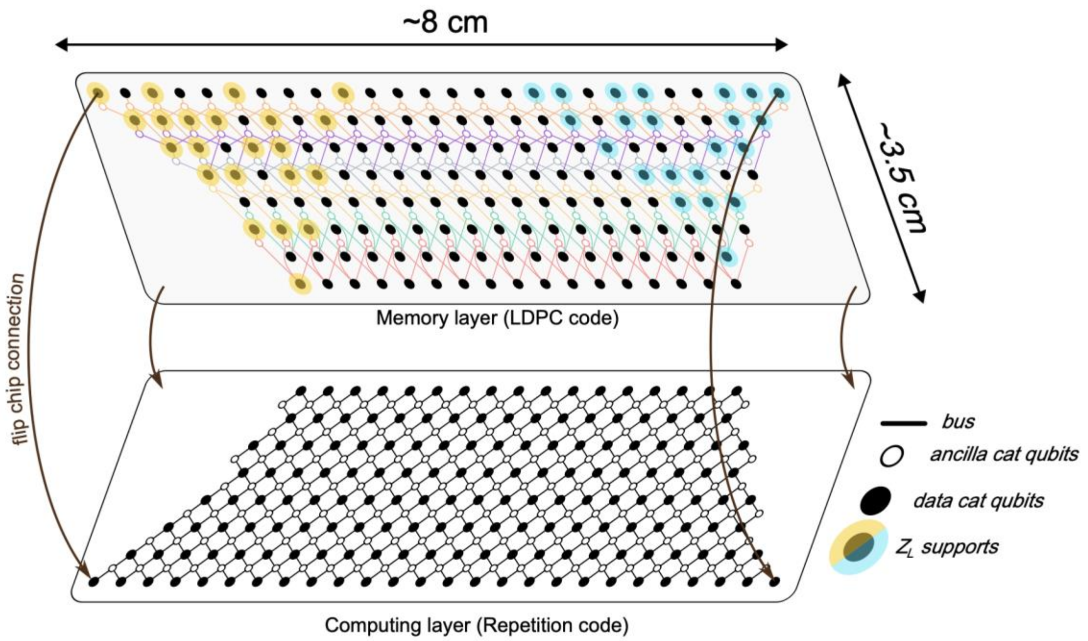
LDPC code + cat qubits for extremely dense QEC

	Surface code + sc qubits [1, 2]	High-rate qLDPC codes + sc qubits [3]	Repetition code + cat qubits [4]	**High-rate LDPC code + cat qubits**
Short-range interactions	yes (2D)	no (2D)	yes (1D)	yes (2D)
Tanner graph degree	3-4	6	2	4
$N_L = 100$ footprints; $\left.\begin{matrix}\epsilon = 10^{-3} \\ \kappa_1/\kappa_2 = 10^{-4}\end{matrix}\right\} \to \epsilon_L \leq 10^{-8}$	N = 33,700 -	2,400 (N/14) -	- 2,100 (N/16)	- 758 (N/44)

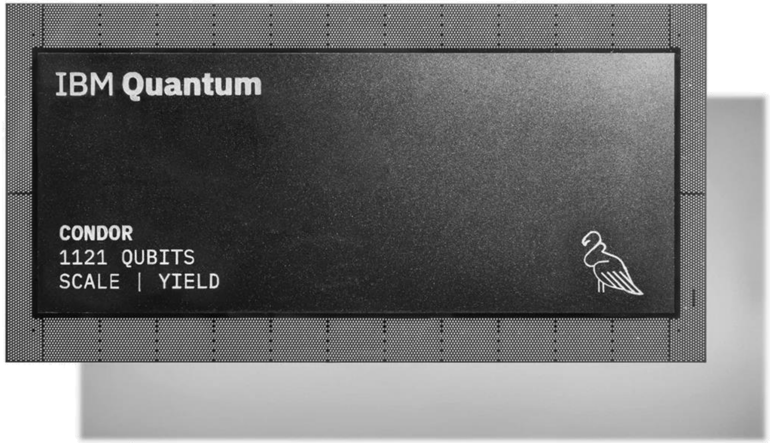
D. Ruiz et al, arxiv:2401:09541

Summary

FTQC
100 logical qubits, 10^{-8} logical error
10 000 logical qubits, 10^{-15} logical error



Hardware-efficient QEC needed
- Cat qubits: 10s bit-flip lifetime (10^{-7})
- LDPC codes: high-encoding rates
- 100 logical qubits, 10^{-8} logical error
  → 758 cat qubits



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LDPC code + cat qubits for extremely dense QEC

LDPC code
34 logical qubits
Distance 22

Equivalent repetition code
17 logical qubits
Distance 8

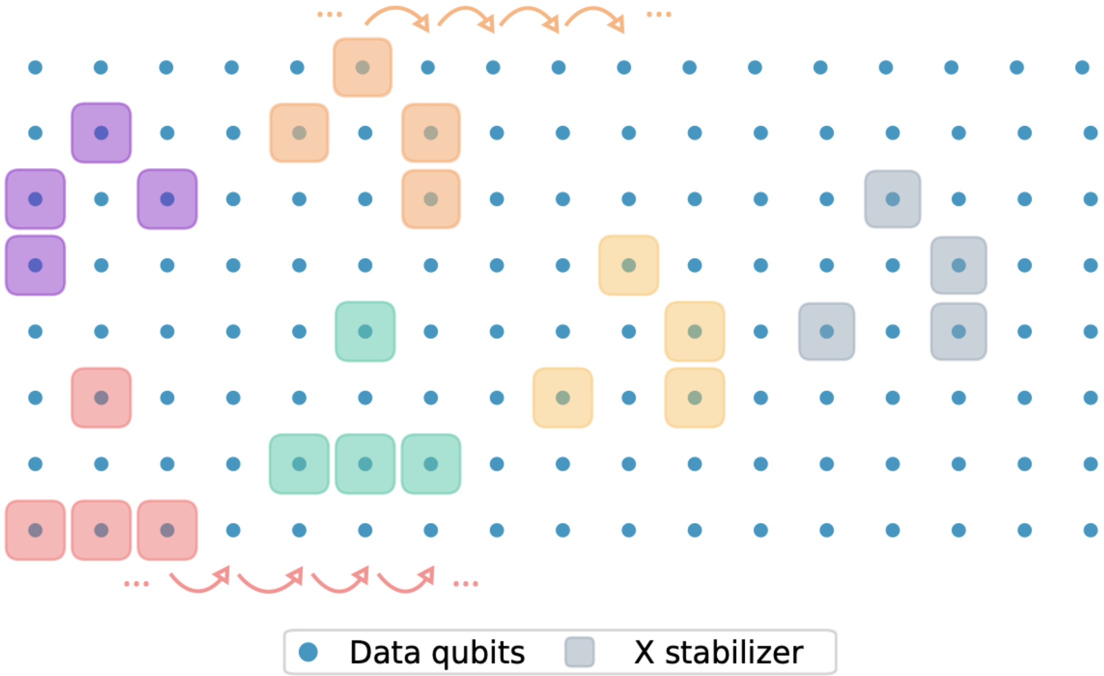
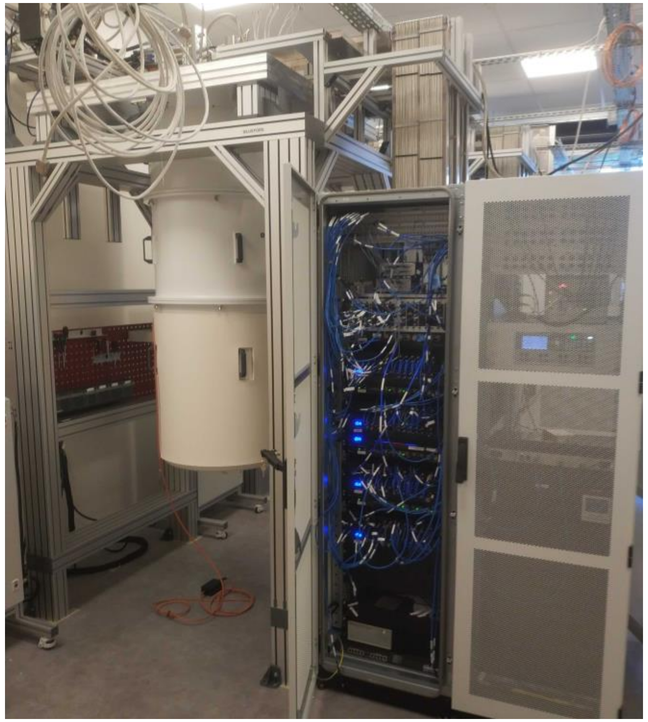
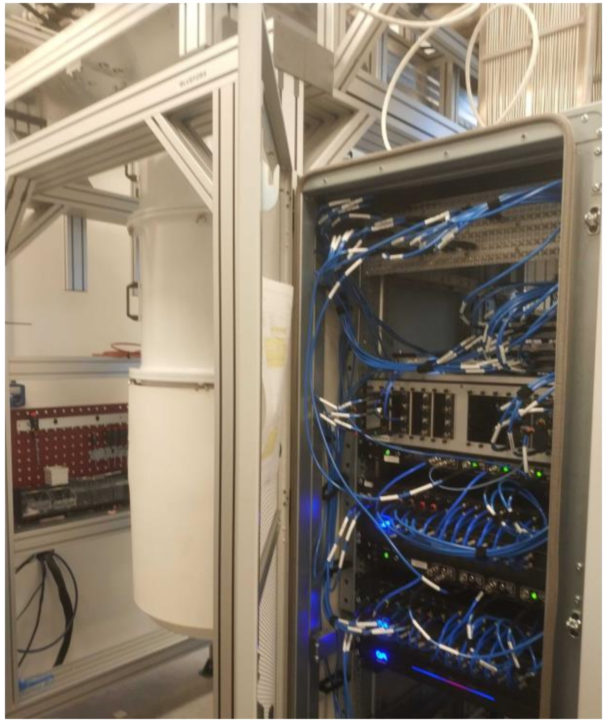


FIG. 4. Layout of the $[136, 34, 22]^{\star}$ phase-flip code. The data qubits are represented as blue dots and the 6 patterns of X-type stabilizers as colored squares. The code belongs to the family of quasi-cyclic codes [74], the weight-4 stabilizer on each row is repeated $L = 17$ times in the horizontal direction (for a total of 85 stabilizers). Here, the code is represented with periodic boundary conditions on the lateral sides, but this constraint can be safely removed for an experimental realization (see Section V).

Lab – zoom



Idéfix



Obélix

Cétautomatix

Cléopâtre