Quantum speed-up for structural mechanics

AQADOC

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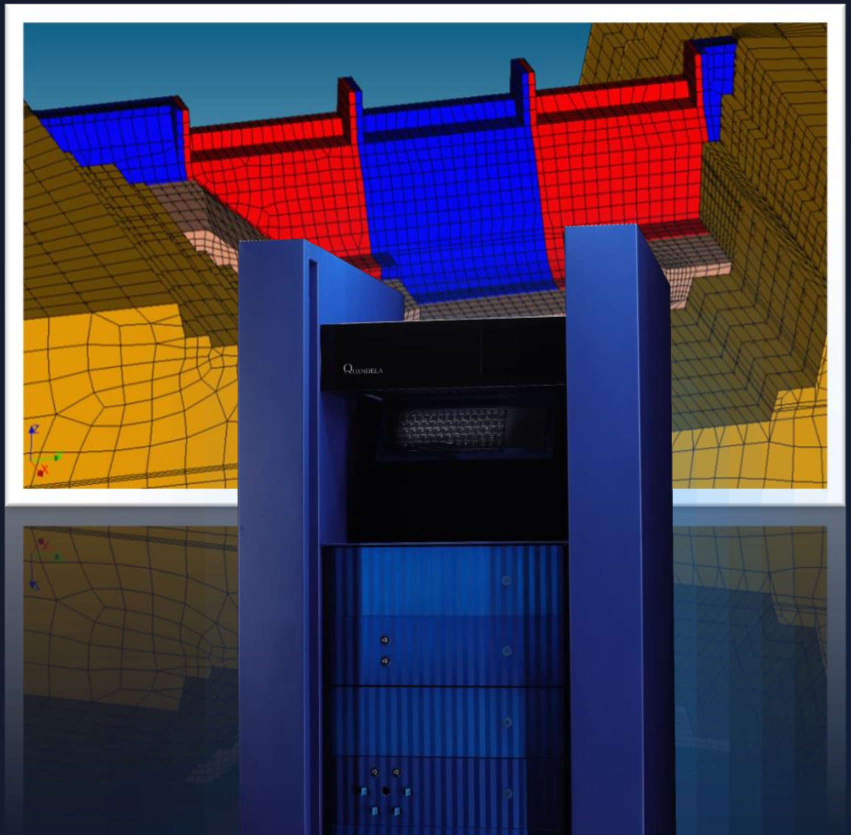

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STRUCTURAL MECHANICS

PDE Navier-Cauchy for linear elasticity

$$\nabla(\nabla \cdot \boldsymbol{u}) + (1-2\nu)\nabla^2 \boldsymbol{u} = \boldsymbol{0}$$

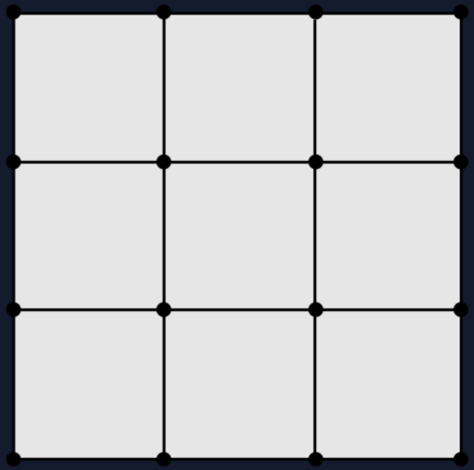
$$\boldsymbol{u} = \begin{pmatrix} u_x(x,y,z) \\ u_y(x,y,z) \\ u_z(x,y,z) \end{pmatrix}$$

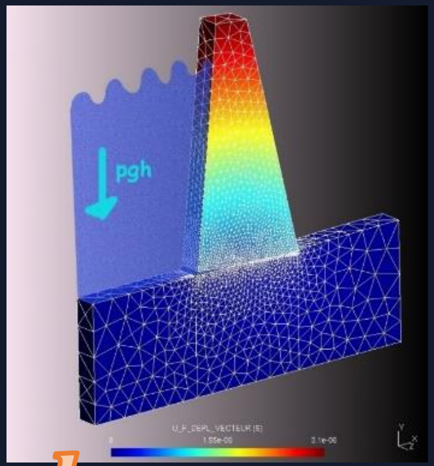
Vector-displacement field of unknowns

Finite elements discretization (mesh)

$$\mathbb{K}\vec{u} = \vec{f} \Leftrightarrow \min_{\vec{u}} (\vec{u}, \mathbb{K}\vec{u})/2 - (\vec{f}, \vec{u})$$

Linear algebra problem with large sparse mechanical rigidity matrix $\mathbb{K}$





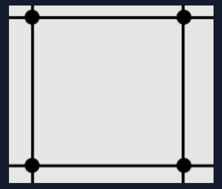
mesh

HHL ALGORITHM

All made solution : naive Qiskit HHL (2008)

Size	2x2	4x4	8x8
CPU time, sec	0.1	1.3	184
# gates	335	5331	314 753
# qubits	5	8	11

$$\mathbb{K}\vec{u} = \vec{f} \Leftrightarrow \min_{\vec{u}}(\vec{u}, \mathbb{K}\vec{u})/2 - (\vec{f}, \vec{u})$$



Linear algebra problem with

~~large sparse~~ mechanical rigidity matrix $\mathbb{K} \in mat(8x8)$

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ENERGETIC VARIATIONAL ALGORITHM

VQA algorithm for structural mechanics

$$\min(\vec{u}, \mathbb{K}\vec{u})/2 - (\vec{f}, \vec{u})$$ *classical*

$$\min\langle u|\mathbb{K}|u\rangle/2 - \langle f|u\rangle$$ *quantum*

$$\vec{u} = \begin{pmatrix} u_\alpha^0 \\ u_\alpha^1 \\ u_\alpha^2 \\ \cdots \\ u_\alpha^N \end{pmatrix} \Longrightarrow |u\rangle \qquad 2^n = N$$

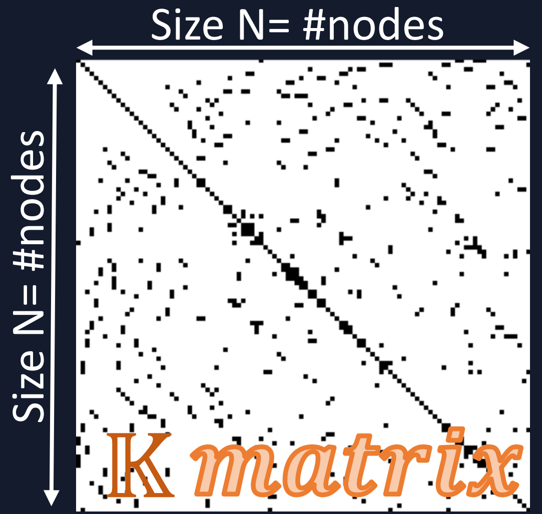
$$\vec{u} \Longrightarrow |u\rangle = u_\alpha^0|\cdots 00\rangle + u_\alpha^1|\cdots 01\rangle + u_\alpha^2|\cdots 10\rangle + \cdots + u_\alpha^N|\cdots 11\rangle$$

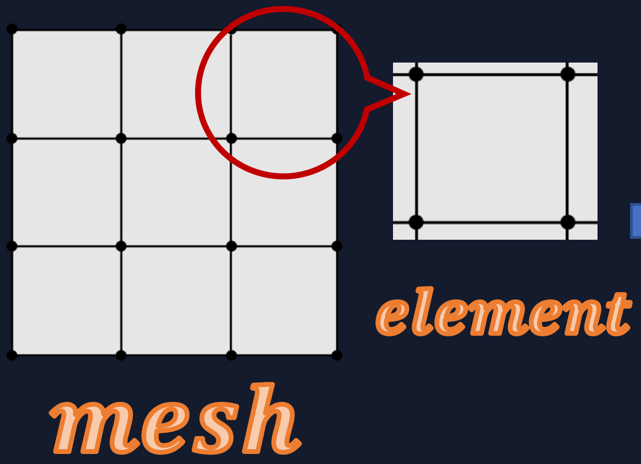
Searching of the ground state of a quantum system with Hamiltonian $\mathbb{K}$

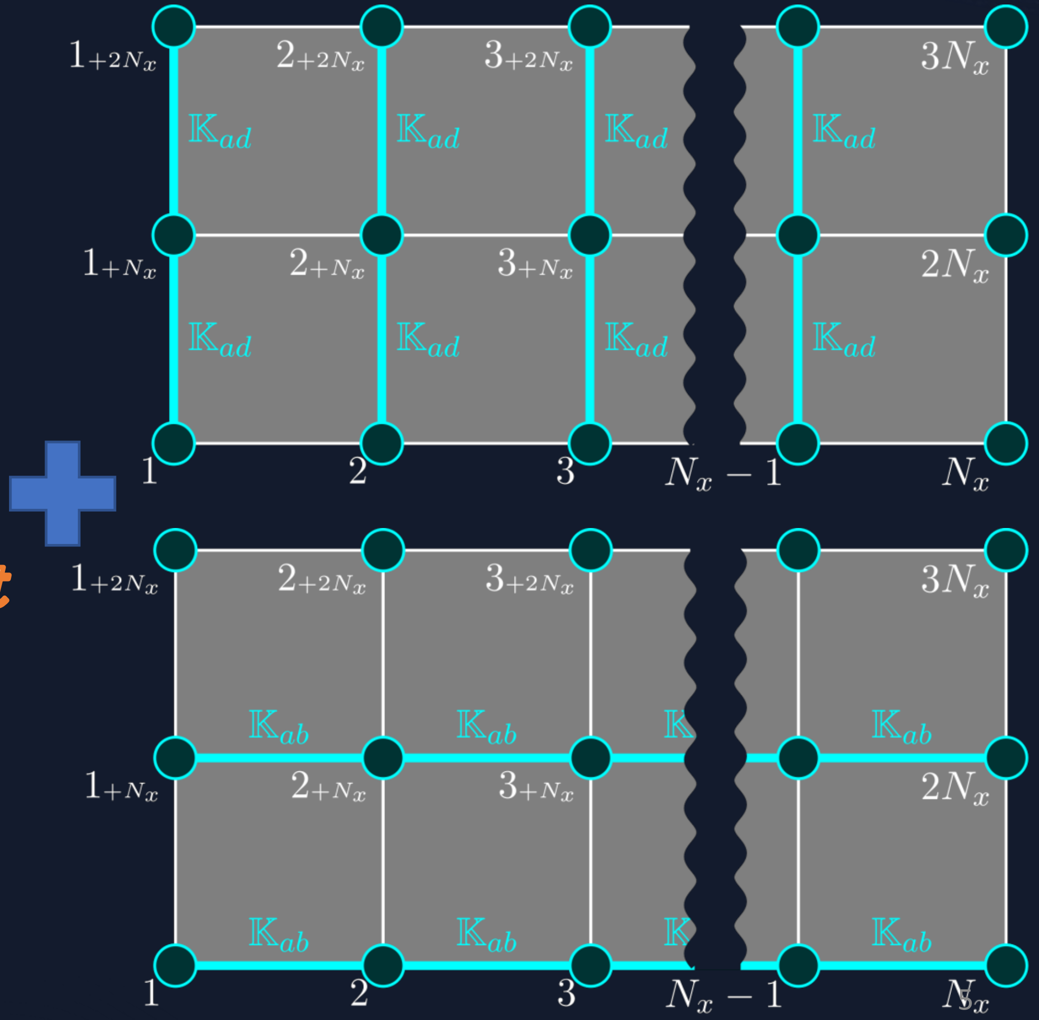
ENCODING THE PROBLEM

VQA algorithm for structural mechanics

$$\min\langle u|\mathbb{K}|u\rangle/2 - \langle f|u\rangle$$





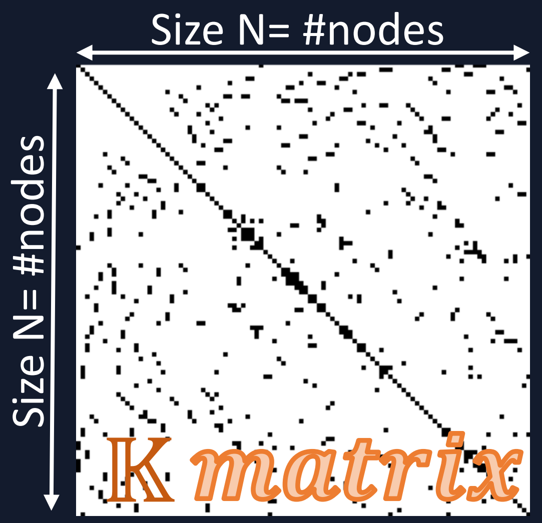


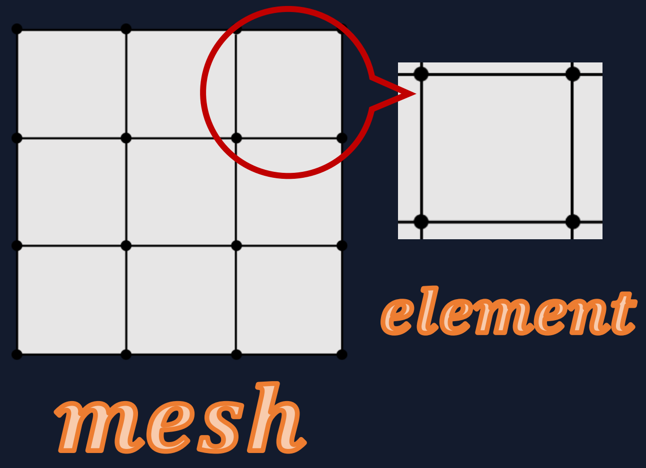
Encoding of the Hamiltonian $\mathbb{K}$

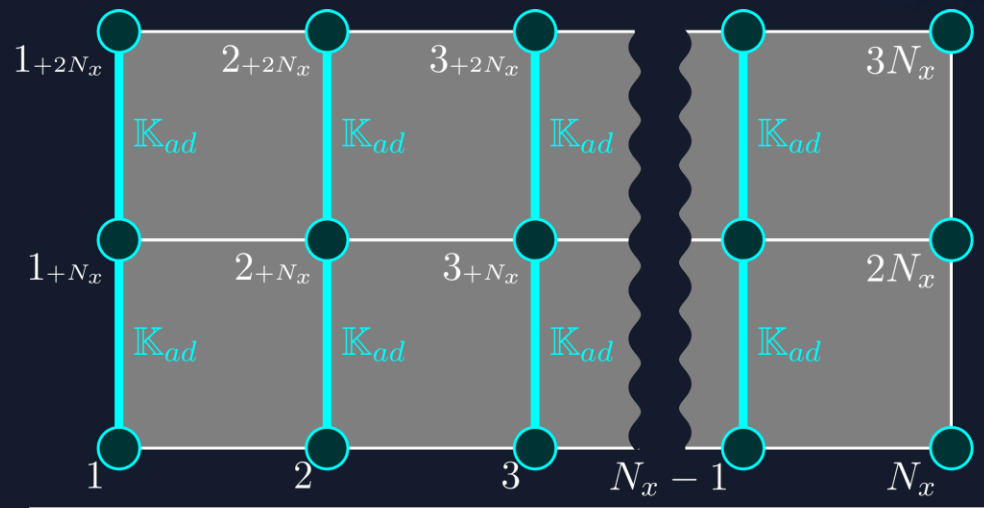
ENCODING THE PROBLEM

VQA algorithm for structural mechanics

$$\min\langle u|\mathbb{K}|u\rangle/2 - \langle f|u\rangle$$







Tensor product decomposition

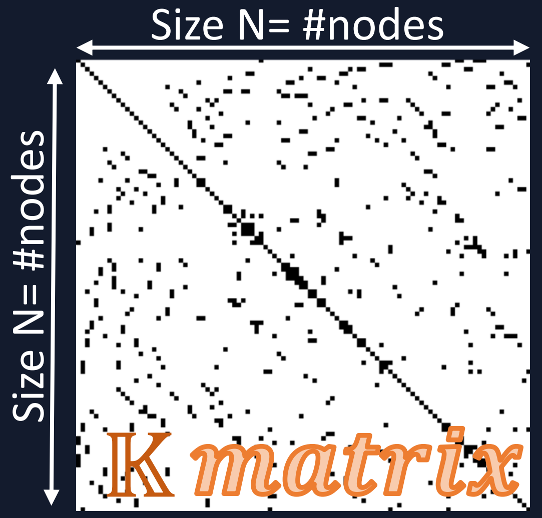
for vertical contributions $\mathbb{K}_{ad}$

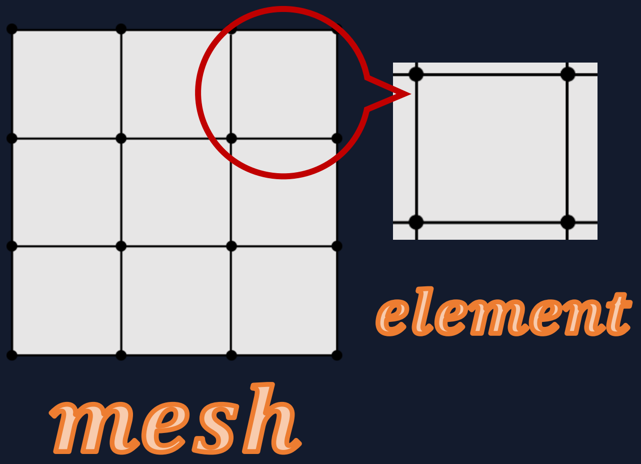
Encoding of the Hamiltonian $\mathbb{K}$

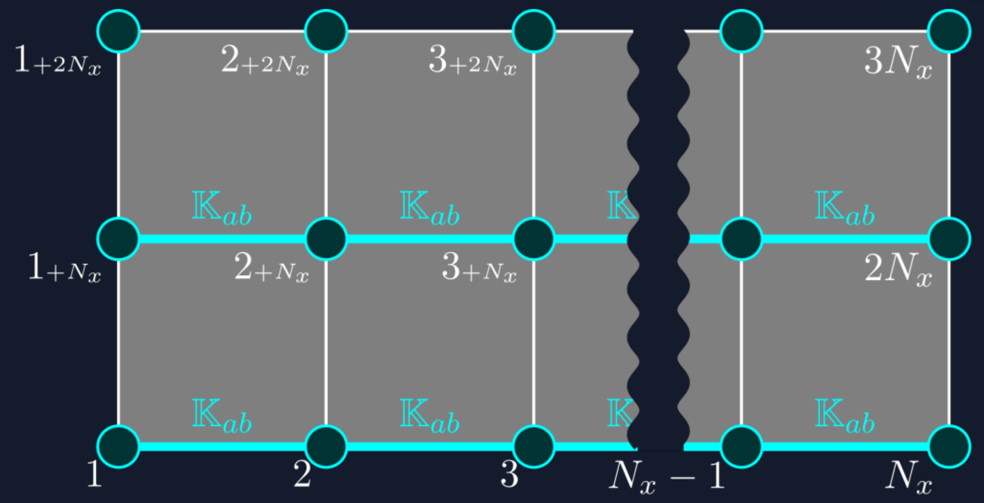
$$\mathbb{T}_{N_y} \otimes \mathbb{D}_{N_x} \otimes \mathbb{K}_{ad}$$

VQA algorithm for structural mechanics

$$\min\langle u|\mathbb{K}|u\rangle/2-\langle f|u\rangle$$







Tensor product decomposition for vertical contributions $\mathbb{K}_{ad}$

Encoding of the Hamiltonian $\mathbb{K}$

$$\mathbb{D}_{N_y}\otimes\mathbb{T}_{N_x}\otimes\mathbb{K}_{ab}$$

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ENCODING THE PROBLEM LOGARITHMICALLY

VQA algorithm for structural mechanics

$$\min\langle u|\mathbb{K}|u\rangle/2 - \langle f|u\rangle$$

$$\mathbb{K} = polynomial(\mathbb{G}_{2x2}) \otimes^{\log N_x \log N_y}$$

$$\mathbb{G}_{2x2} \equiv \{p_{\pm}; \sigma_{\pm}; I_2\} \qquad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$p_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad p_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Optimized tensor product decomposition rigidity $\mathbb{K}$

See Liu et al. 2020

Sato et al. 2021

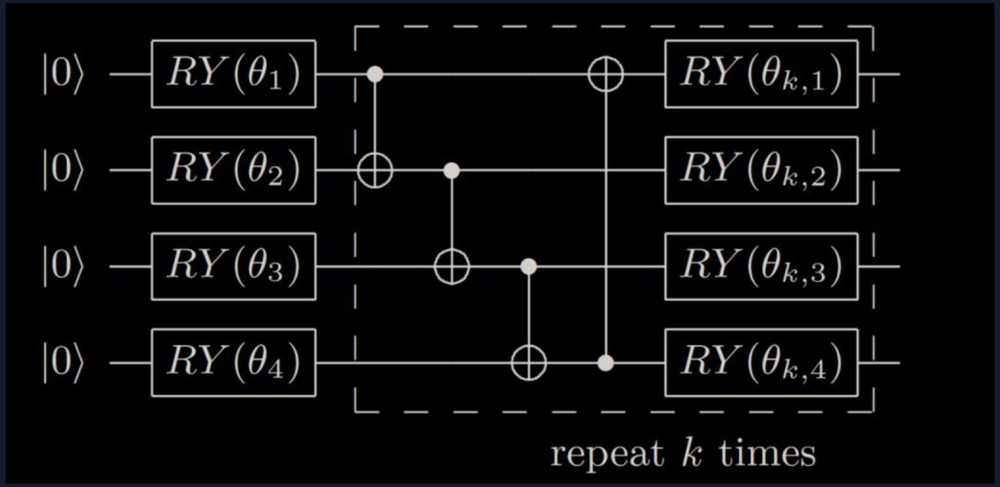
$$\mathbb{T}_{N_y} \otimes \mathbb{D}_{N_x} \otimes \mathbb{K}_{ad}$$

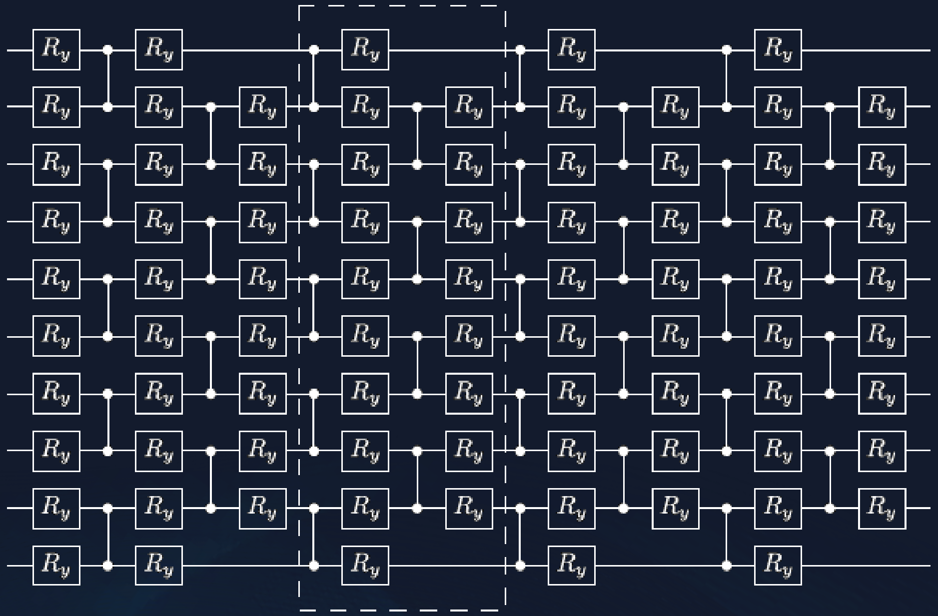
$$\mathbb{D}_{N_y} \otimes \mathbb{T}_{N_x} \otimes \mathbb{K}_{ab}$$

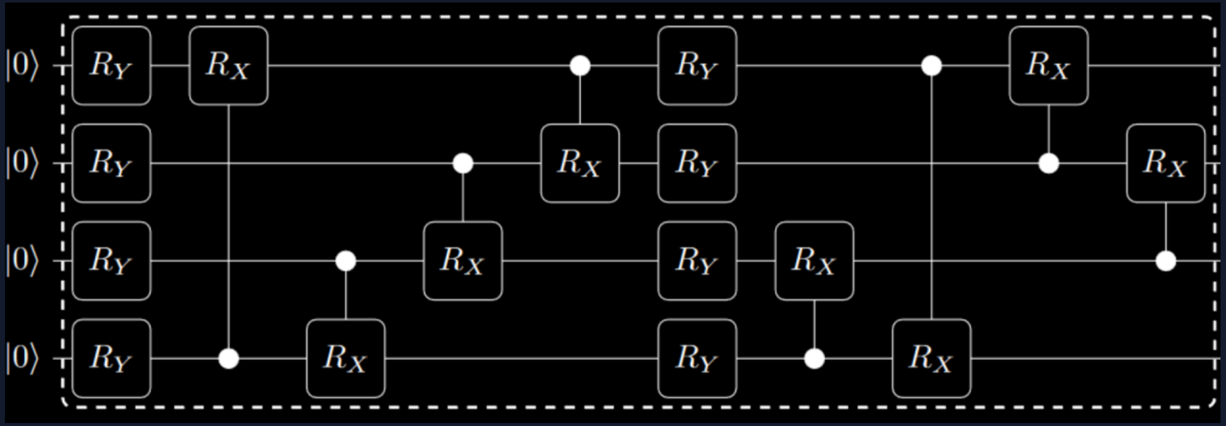
ANSATZ PREPARATION

Ansätze: quantum parametrization

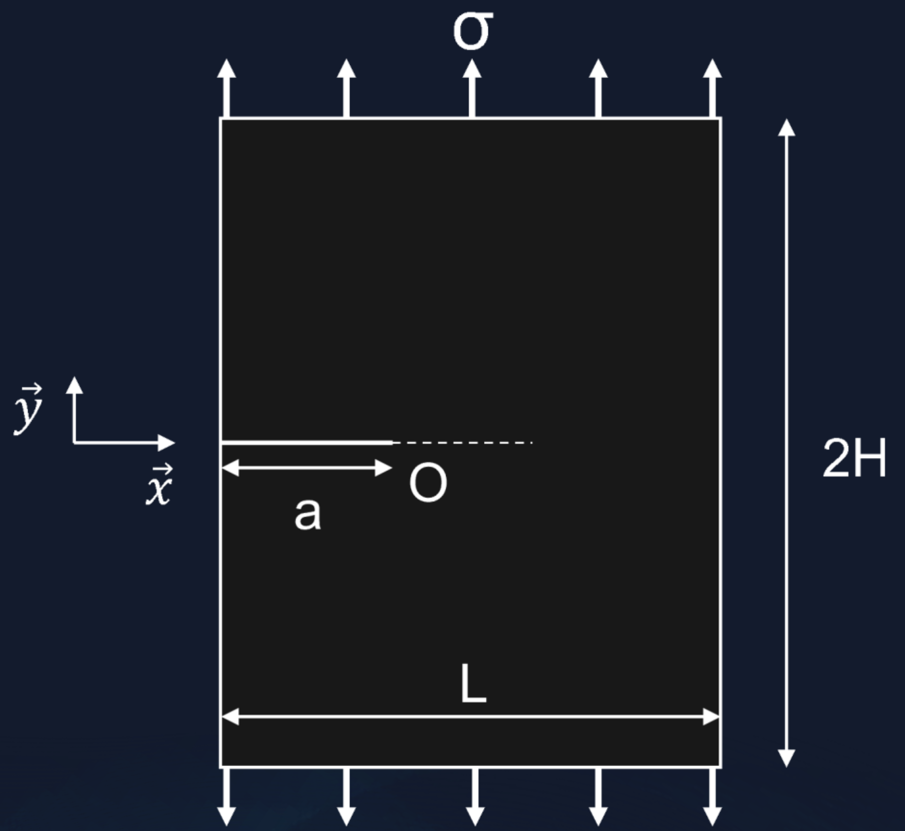
$$|u\rangle = U^{\otimes layer}\,|0\cdots 00\rangle$$

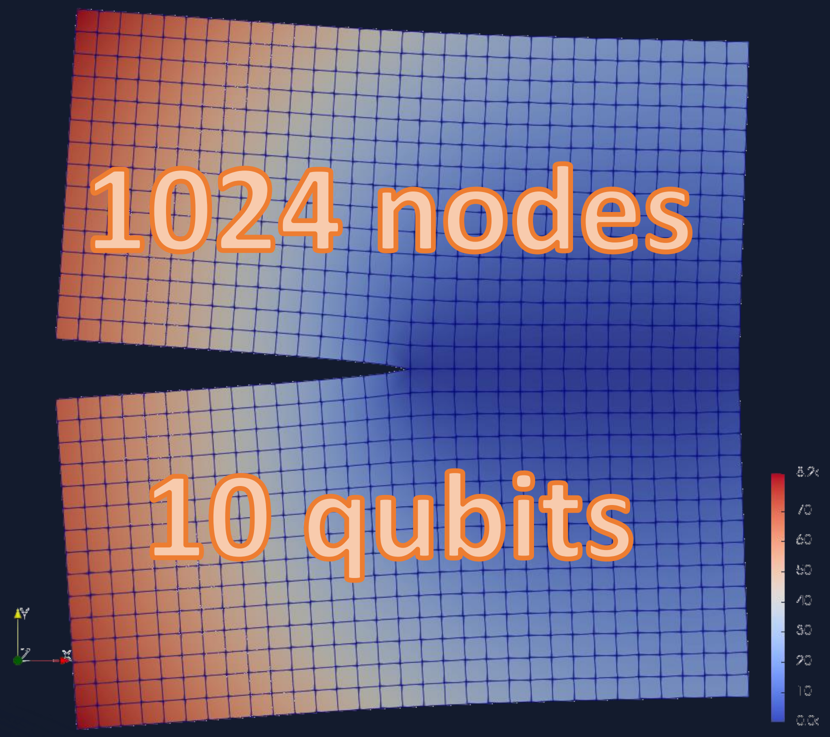


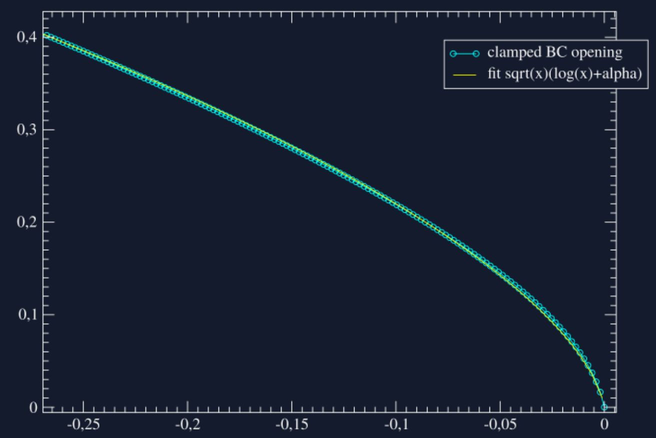




Application for edge-cracked plate: classical FEM simulations



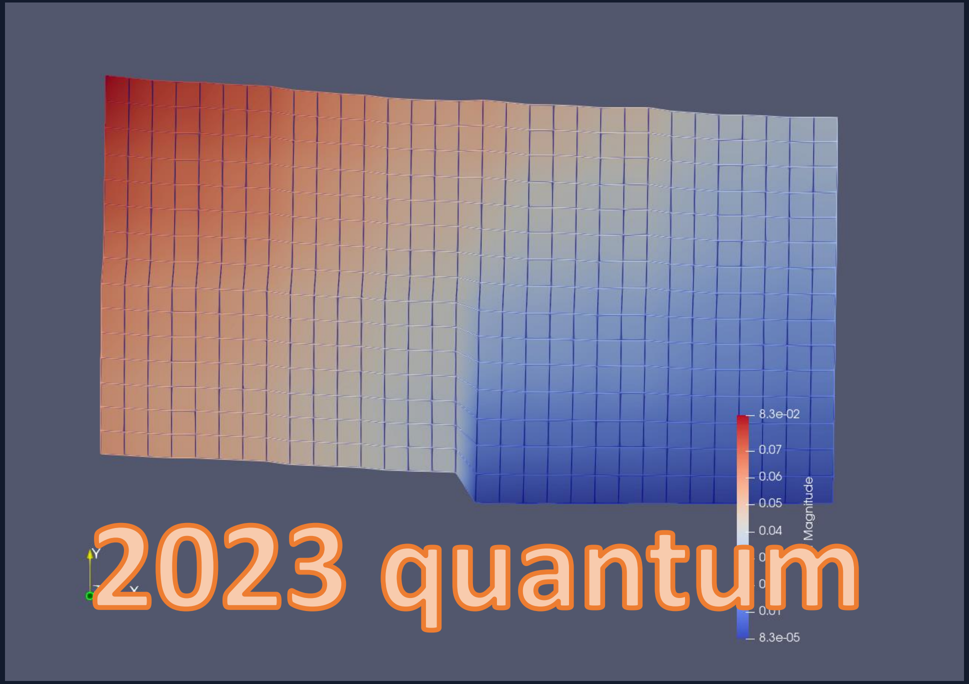


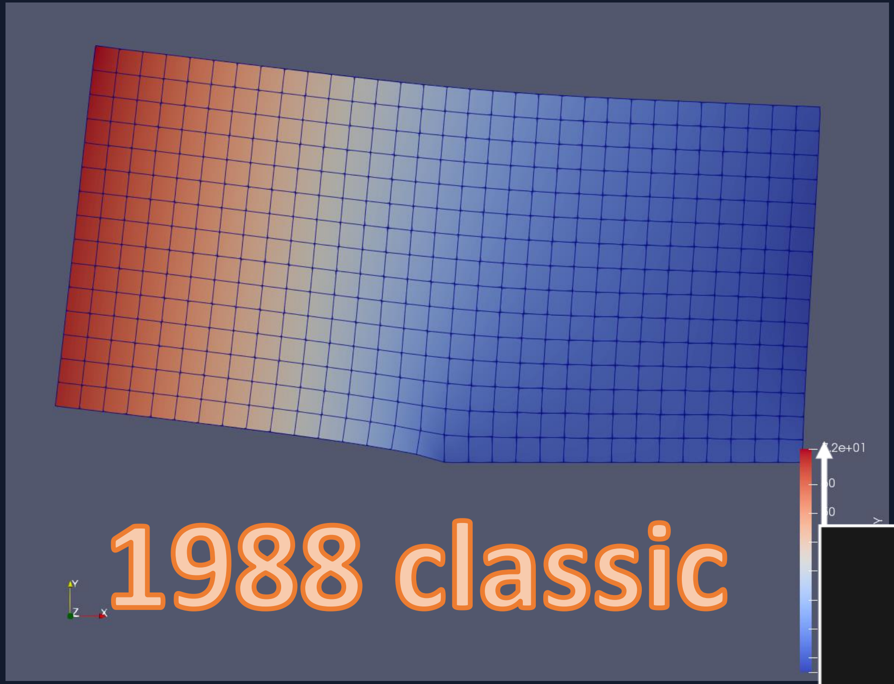


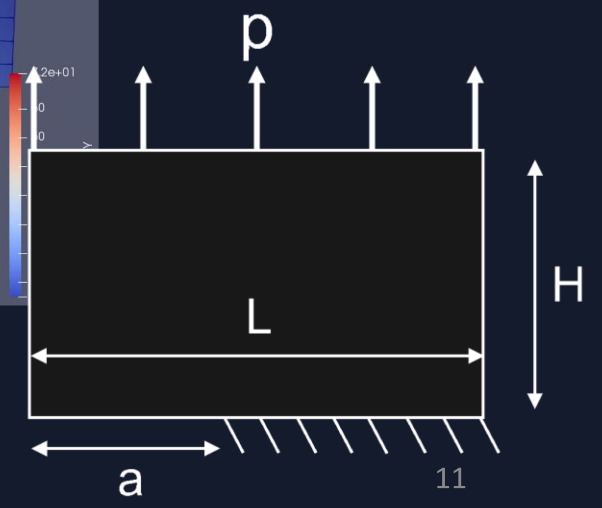
Crack opening observable

APPLICATION TO CRACKS

Application for edge-cracked plate: classical versus quantum



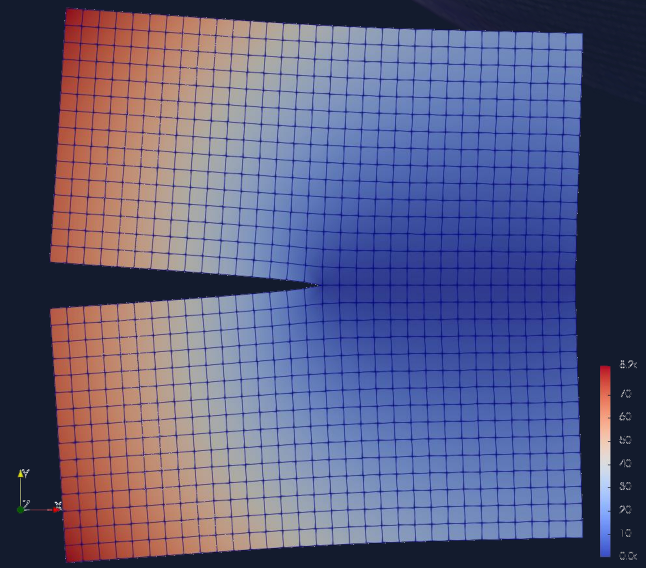




Conditions for an **efficient** VQA algorithm :

- **Unitary transformation** U_f such that $|f\rangle = U_f|0\rangle^{\otimes n}$ ✅
- A state $|\psi(\boldsymbol{\theta})\rangle$ that is prepared thanks to the ansatz $U(\boldsymbol{\theta})$: $|\psi(\boldsymbol{\theta})\rangle = U(\boldsymbol{\theta})|0\rangle^{\otimes n}$ ✅
- Decompose the rigidity matrix in a **simple tensorial product** of (polynomially many at most) operators. ✅
- **Few measurements** (around the crack only) to retrieve the solution ✅

- Structural mechanics



- Structural mechanics
- Energetic formulation $\min\langle u|\mathbb{K}|u\rangle/2-\langle f|u\rangle$

OVERVIEW

- Structural mechanics
- Energetic formulation
- VQA algorithm that is:
  - Efficient for the encoding ✅
  - Efficient for the decomposition of the rigidity matrix ✅
  - Requires a few measurements to get the value of interest ✅

- Structural mechanics
- Energetic formulation
- VQA algorithm that is:
  - Efficient for the encoding ✅
  - Efficient for the decomposition of the rigidity matrix ✅
  - Requires a few measurements to get the value of interest ✅
- Precision depends on the number of qubits: scaling up!