# Towards large-scale quantum optimization solvers with few qubits

## 2nd TQCI International Seminar on Benchmarks for Quantum Computers, 05 June 2024

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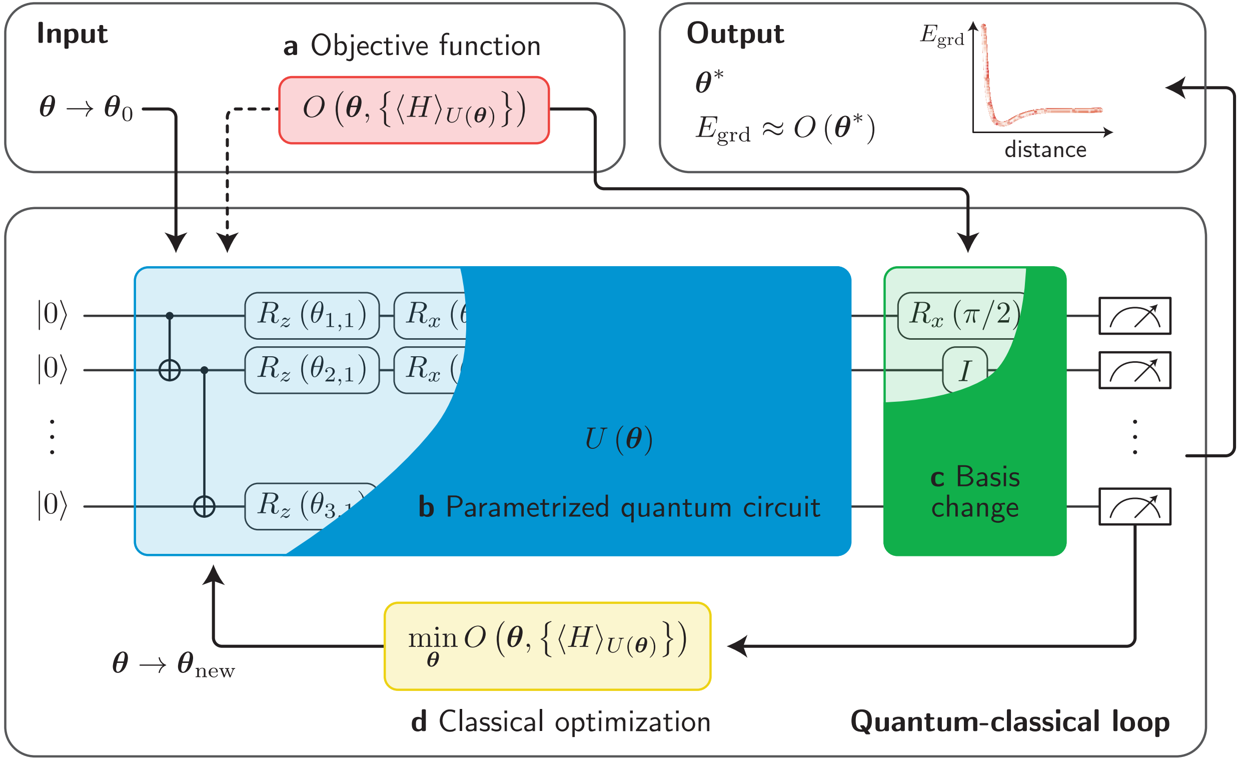
# A major promise of QCs

- Combinatorial optimizations ubiquitous in sciences & technology;
- Fault tolerant QCs: quadratic speed-ups;
- However, low clock speeds;
- Practical advantage before large-scale FT devices unclear;
- Challenge: number of qubits required to compete with classical solvers;

*Can we do something with the NISQ devices available today?*

# Variational quantum algorithms

- Parametrized quantum circuit on a NISQ computer
- Optimize parameters via feedback loop between classical optimizer and quantum circuit




K. Bharti et al., **Noisy intermediate-scale quantum algorithms** (2022)

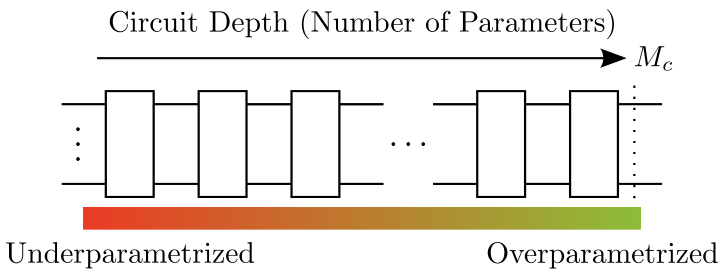
Examples:

- VQE: quantum chemistry
  Peruzzo et al. (2013)
- QAOA: combinatorial optimizations
  Farhi, Goldstone, & Gutmann (2014)

# Challenges of variational quantum algorithms

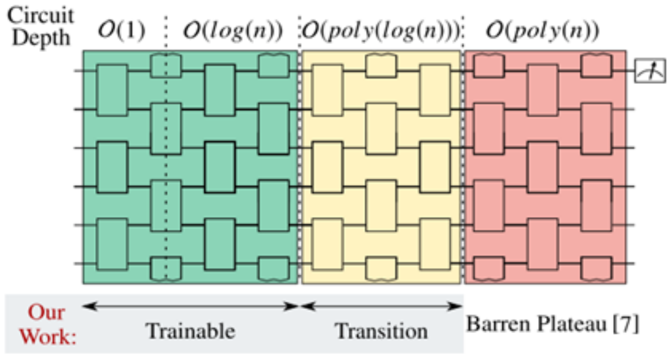
- Deep quantum circuits needed for smooth optimization landscapes.

E. Anschuetz, **Critical Points in Quantum Generative Models** (2021);
M. Larroca et al., **Theory of over-parametrization in quantum neural networks** (2021).




- Barren plateaus: gradient variances decay exponentially with # qubits.

J. R. McClean et al, **Barren plateaus in quantum neural network training landscapes** (2018);
S. Wang et al., **Noise-induced barren plateaus in variational quantum algorithms** (2021).




- Noise: state becomes useless at depths ~ $O(1 / \text{noise strength})$.

D. Stilck-França & R. García-Patron, **Limitations of optimization algorithms on noisy quantum devices** (2021); G. De Palma (2022); Y. Kweck et al (2023).

*Near-term q optimization solvers restricted to small quantum circuits*

🥺

## Can we still do something useful/interesting?

# Outline of the talk

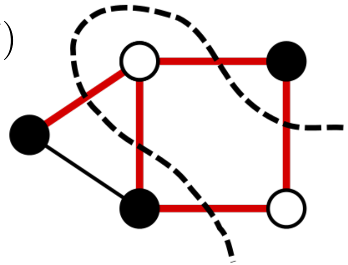
- Quantum QUBO solvers with polynomial qubit-number compressions
- Circuit complexity and performance (numerics)
- Barren plateaus mitigation as a built-in feature
- Experimental results from IonQ and Quantinuum deployments

M. Sciorilli, L. Borges, T. Patti, Diego García-Martín,  G. Camillo, A. Anandkumar, and LA,
**Towards large-scale quantum optimization solvers with few qubits**, arXiv:2401.09421.

# MaxCut and weighted MaxCut

- Paradigmatic NP-hard problem
- APX-hard too: no Poly(time) approx. algorithm for arbitrary approx. ratio $r$
- Best efficient classical solver with performance guarantees: Goemmans-Williamson (SDP), $r \approx 0.878$

$G := (E, V)$



*Find the bipartition cutting the maximum number of (weighted) edges*

$$\underset{\boldsymbol{x} \in \{-1,1\}^m}{\text{maximize}} \quad \sum_{i,j \in [m]} W_{ij}(1 - x_i x_j)$$

*Ising model formulation: find the ground state of*

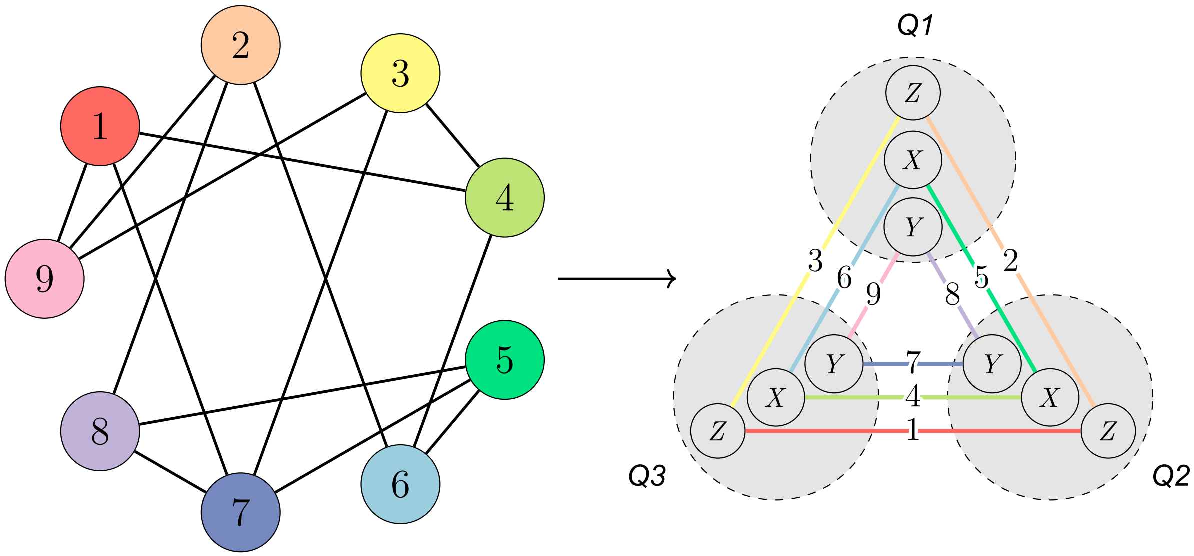
$$H := \sum_{i,j \in [m]} W_{ij} Z_i \otimes Z_j$$

*Native quantum encoding:*

$x_i \leftarrow$ *Eigenstate of Pauli matrix* $Z_i$

# A quantum solver with polynomial space compression

Pauli correlation encoding: $x_i := \mathrm{sgn}(\langle \Pi_i \rangle)$ ← *2-qubit Pauli string*




- Graph-instance agnostic
- Quadratic compression in qubit number: $m = \mathcal{O}(n^2)$

- Generalizes single-qubit QRACs

T. Patti et al. (2022); B. Fuller et al. (2022)

- Polynomial compression: sweeter spot than exponential compressions

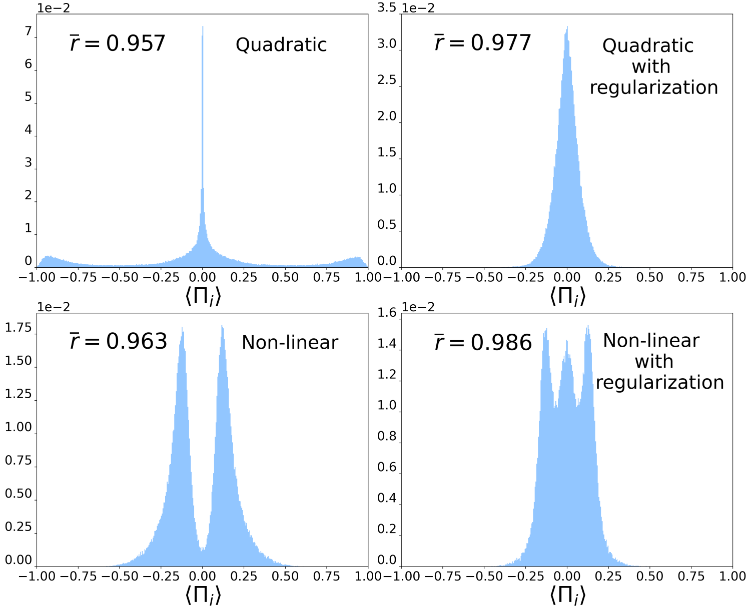
B. Tan et al. (2021); E. X. Huber et al. (2023); I. D. Leonidas et al. (2023); Y. Tene-Cohen et al. (2023)

# A quantum solver with polynomial space compression

Loss function to optimize: $$\mathcal{L} = \sum_{(i,j)\in E} W_{ij} \tanh\left(\alpha \langle \Pi_i \rangle\right) \tanh\left(\alpha \langle \Pi_j \rangle\right) + \mathcal{L}^{(\mathrm{reg})}$$

*(forces small Pauli correlators)*

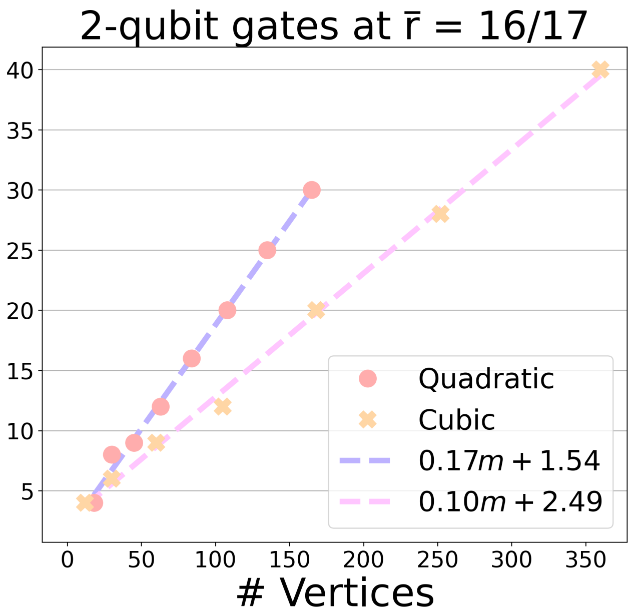
- Encodes graph instance
- Circuit-Ansatz agnostic
- Highly non-linear relaxation of the binary problem
- Ameanable to standard q error mitigation

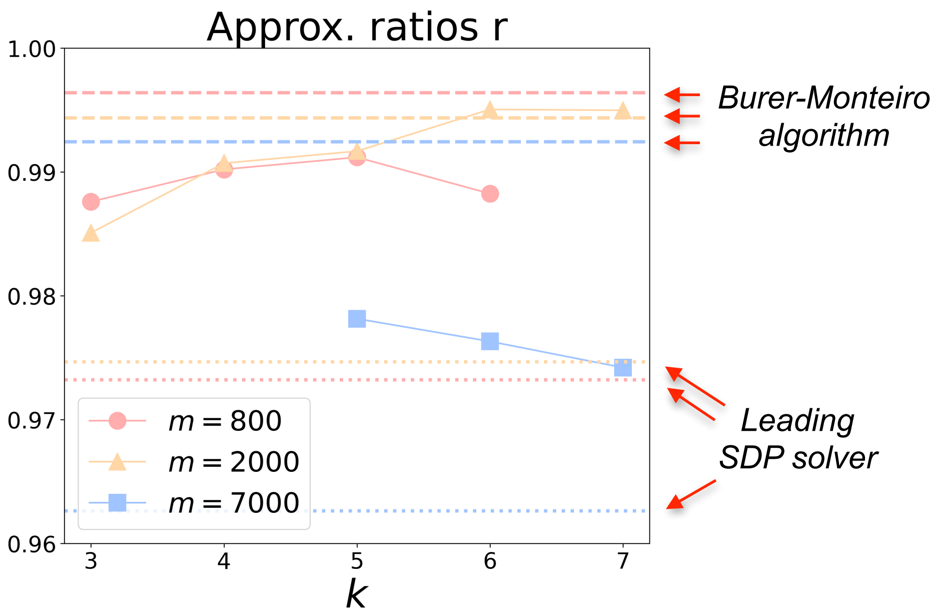


# Numerical performance

# Circuit complexity and performance

- Non-trivial random MaxCut instances
- 1D brickwork (hardware-efficient) Ansatz




- **Depth sub-linear in *m*!**
- Trains well (even in under-parametrization!)

***Competitive with state of the art solvers!***

# A convenient by-product of the encoding: intrinsic barren plateau mitigation

# Barren plateaus mitigation as a built-in feature

**BPs (concentration of measure):**

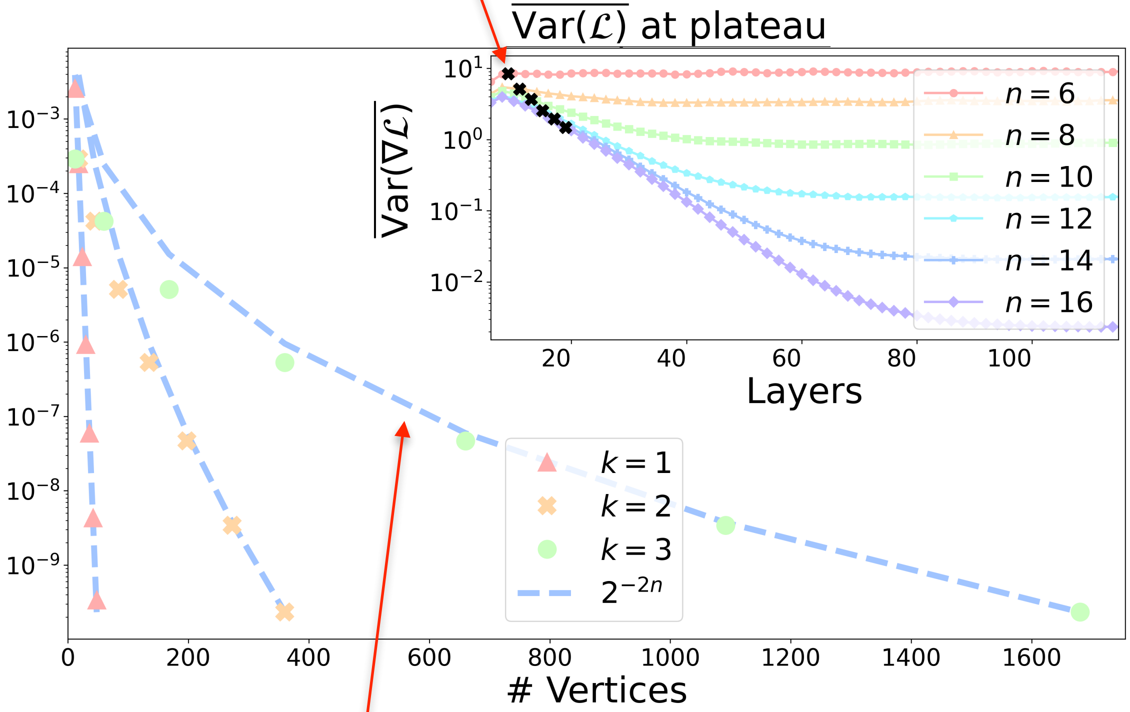
- Exponential decay in *n* of gradient variance over random initializations.

J. R. McLean et al. (2021)

**BPs for Pauli-correlation encoding:**

- Proven supra-polynomial suppression in variance decay in *m*!
- Plateaus at depth ~ 8.5 *n*.

Aver. approx. ratio 0.941 at depth ~ *n*




Analytical scaling

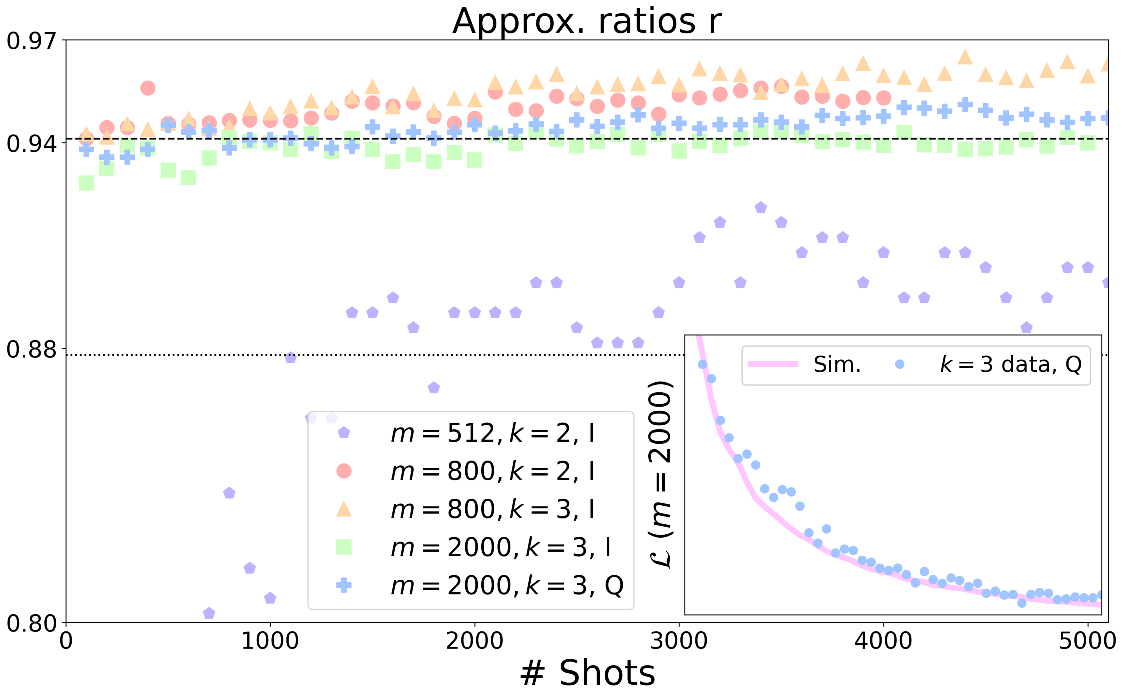
# Experimental deployment on trapped-ion quantum hardware

# Weighted MaxCut on IonQ's Aria-1 and Quantinuum's H1-1 devices

1D brickwork Ansatz: native IonQ and Quantinuum gates.

*Worst-case hardness threshold (no U.G.C)* →

*Goemans-Williamson threshold* →




- First-ever experiment with such high-quality solutions for these sizes!
- Previous experiments with QAOA: $m$= 414 and aver. (max.) $r$ = 0.57 (0.69).

A. Abbas et al. (2023)

# Conclusions on part I

- Pauli-correlation encoding: QUBOs with polynomially fewer qubits
- Non-linear cost function trains even in under-parametrization
- Barren plateau mitigation as a provable built-in feature
- Unprecedented performance both numerically and experimentally
- Amenable to standard error-mitigation
- Experimental training: (pre-)train classically?

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# The quantum algorithms team

## Positions open!

★ Paper co-authors


Thank you for your attention!