Q-score Max-Clique

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Quantum at TNO

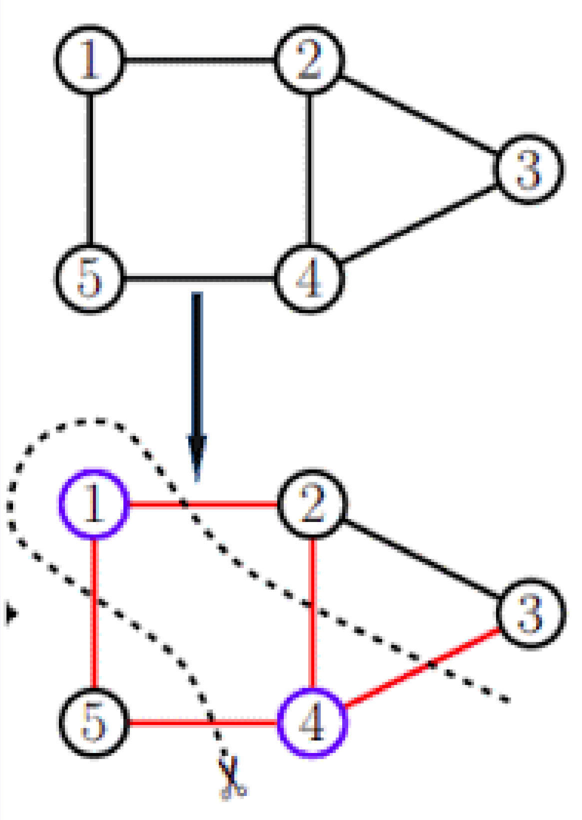
- The Netherlands Organisation for applied scientific research
- Mission:
  - To generate innovative solutions with provable impact for a safe, healthy, sustainable and digital society in the Netherlands and beyond.
- Quantum applications at TNO. Goal:
    - To enable practical implementation of relevant applications on current and/or near term quantum devices.

- Quantum benchmarking at TNO:
  - Goal:
    - Find the perfect device for a given application
  - Considered many different application-level benchmarks
  - Focus on Q-score

Original Q-score by Atos

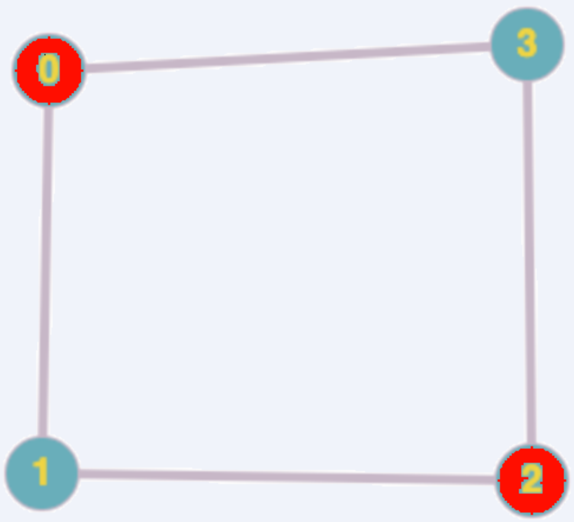
- Largest problem size N for which a device significantly outperforms a random algorithm at solving the Max-Cut problem

- Max-Cut problem:
  - Given a graph G with (possibly weighted) edges
  - Find partition P of vertices in two sets (cut) that maximizes the cost
  - Cost $C(P)$: total weight of edges between the two sets
  - Use $\left(N, \frac{1}{2}\right)$ Erdös-Renyi graphs



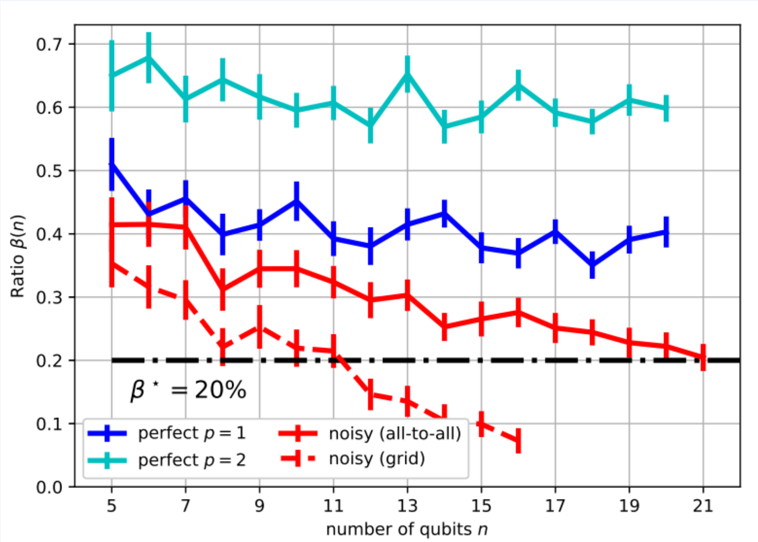
- Applicable on gate-based as well as annealing-based devices
  - Depends **not only** on the device
  - Also on the used algorithm, optimisations and resources

Q-score



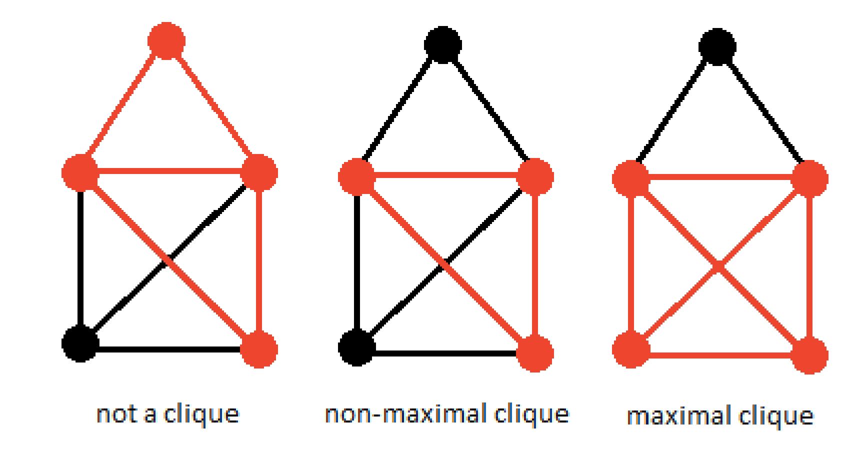
1. For increasing N, do the following steps:
2. Pick a collection of graphs of size N
3. Run a Max-Cut algorithm on each graph and compute average cost $C(N)$
4. Check whether cost is 'good': $\beta(N) = \frac{C(N) - C_{rand}}{C_{max} - C_{rand}} = \frac{C(N) - N^2/8}{0.178 N^{3/2}} > \beta^* = 0.2$
5. Q-score is highest N satisfying this

- Random cut: $C_{rand} = \frac{N^2}{8}$
- Optimal cut: $C_{max} = \frac{N^2}{8} + 0.178 N^{3/2}$



Q-score Max-Clique

- Questions about the Q-score:
  1. Why do we specifically use the Max-Cut problem?
  2. What about other devices, such as photonic quantum computers?

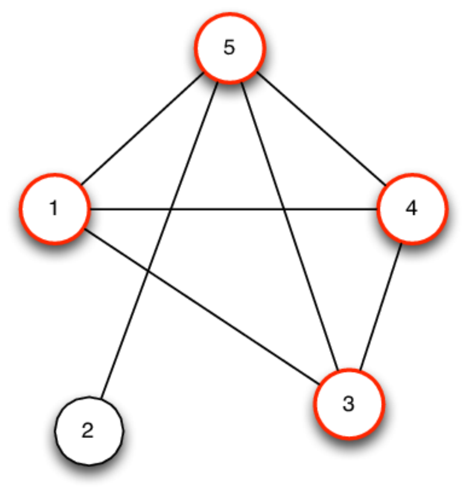


- Q-score Max-Clique
  - Q-score with the Max-Clique problem
  - Find the largest clique: complete subgraph in *G*
  - Consider $\left(N, \frac{1}{2}\right)$ Erdös-Rényi graphs

- (Natively) solvable on photonic, annealing and gate-based quantum hardware[1]
  - First quantum metric suitable for all three quantum paradigms, as well as classical devices

[1] *Molecular Docking with Gaussian Boson Sampling (2019) arXiv:1902.00462*

Q-score Max-Clique



1. For increasing N, do the following steps:
2. Pick a collection of graphs of size N
3. Run a Max-Clique algorithm on each graph and compute average clique size $C(N)$
4. Check whether clique size is 'good': $\beta(N) = \frac{C(N)-C_{rand}}{C_{max}-C_{rand}} = \frac{C(N)-1.6416325}{2\ln(N)-1.6416325} > \beta^* = 0.2$
5. Q-score Max-Clique is highest N satisfying this

- How do we estimate the max and random clique size C_{max} and C_{rand}?
- Random clique size:
  Adding nodes randomly until no clique yields $C_{rand} = \sum_{i=0}^{N} i(1-p^i)p^{0.5i(i-1)} \approx 1.6416325$
- Max clique size:
  Literature study plus quantum and classical brute force suggest $C_{max} = 2\ln(N)$

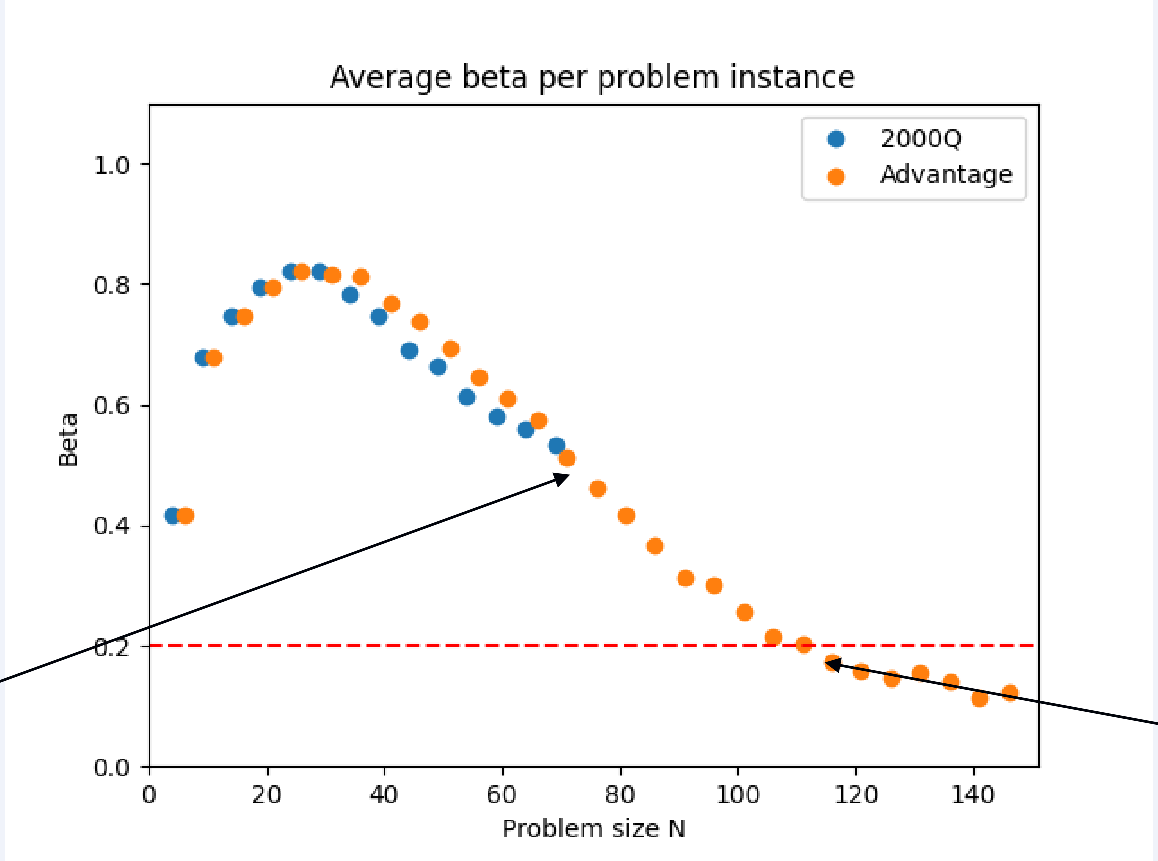
Experiments

- Calculate Q-score Max-Clique for following solvers
  - Quantum gate-based hardware — IBM Guadalupe, Quantum Inspire Starmon-5
  - Quantum annealing hardware — D-Wave Advantage, D-wave 2000Q
  - Quantum photonic hardware — TODO (Quix Quantum, Quandela?)
  - Quantum photonic simulator — Xanadu
  - Classical algorithms — Simulated annealing, Tabu search
  - Hybrid quantum-classical solvers — D-wave hybrid solver

- Extra constraints for a fair comparison
  - Maximum calculation time: 60 seconds
  - Use out-of-the-box solvers:
    - Standard parameters
    - No extra optimisation allowed

Q-score Max-Clique Annealing Hardware

D-wave 2000Q
2048 qubits
6 qubit couplings



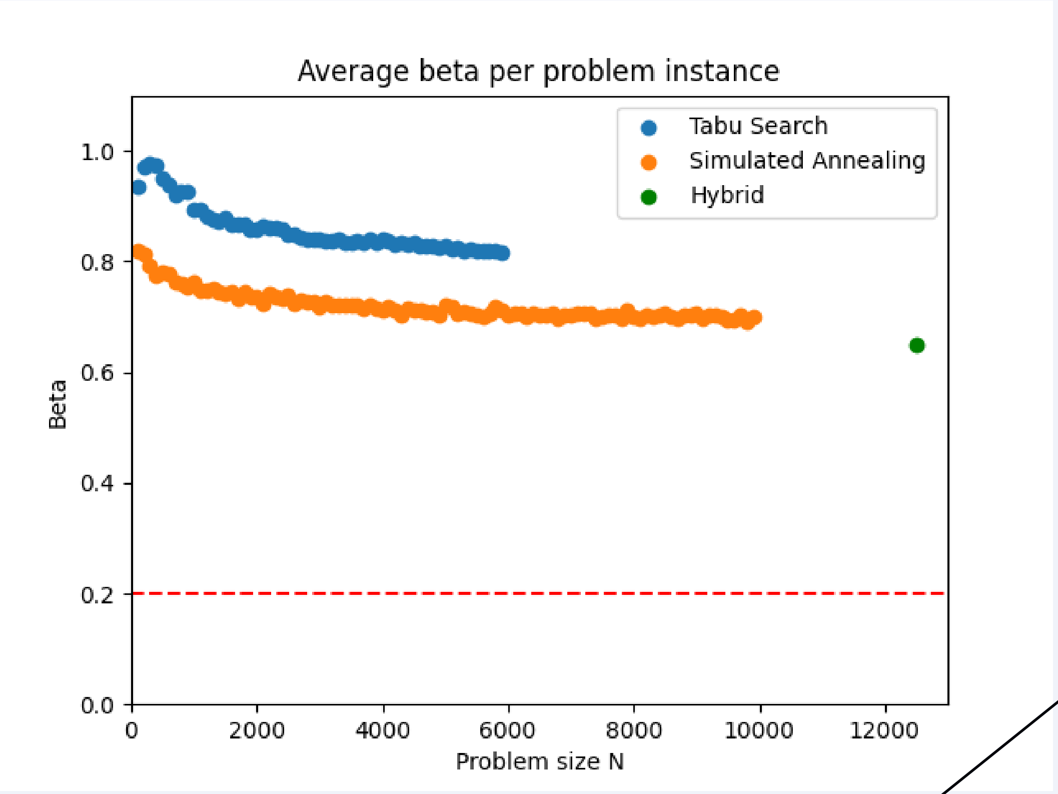
D-Wave Advantage
5640 qubits
15 qubit couplings

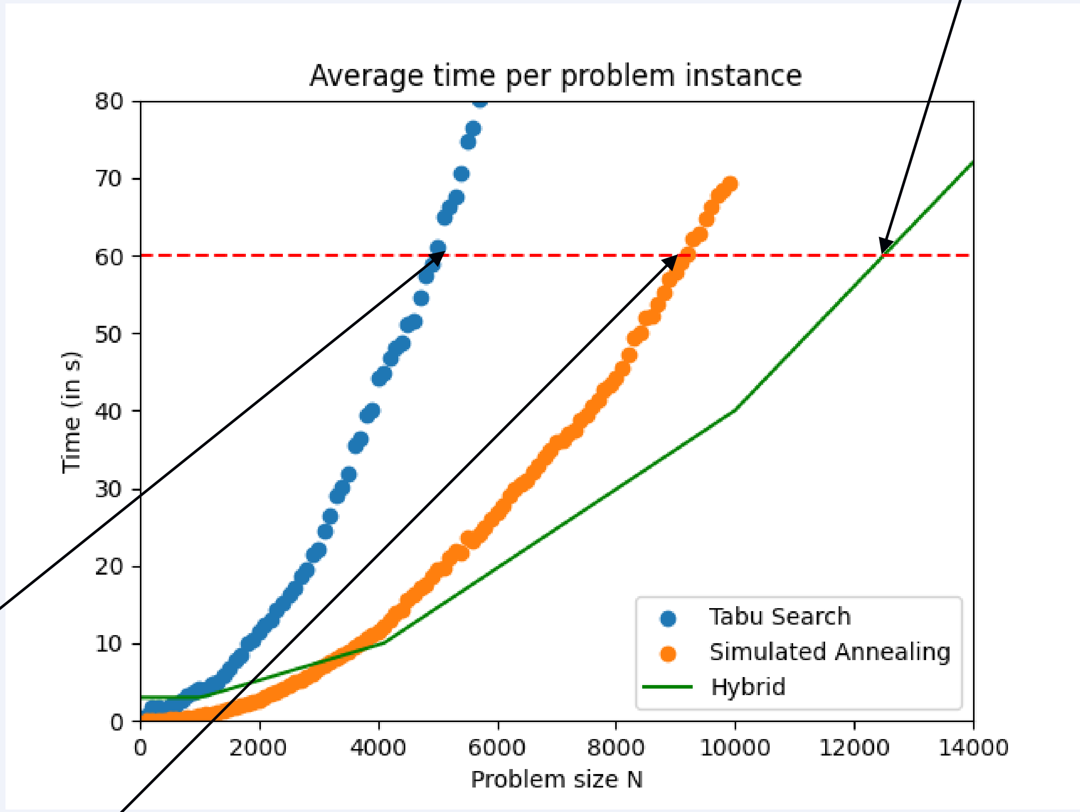
Q-score = 70

Q-score = 110

Q-score Max-Clique Classical & Hybrid Solvers

Q-score = 12,500





Q-score = 4,900

Q-score = 9,100

Results

Approach	Q-score
1. Tabu search	4,900
2. Simulated annealing	9,100
3. D-Wave Advantage	110
4. D-Wave 2000Q	70
5. Hybrid annealing solver	12,500
6. Quantum Inspire Starmon-5	5*
7. IBM Guadalupe	≥5*
8. Simulated gate-based	13*
9. Simulated photonic-based	20*

- * = more than 60 seconds used

Conclusion Q-score Max-Clique

- A comparison between all different devices is indeed possible
  - Track development of single device or technology
  - Compare fully developed devices in the future

- Flexibility of Q-score:
  - Any problem for which a scalable expression to the random and maximal solution exists can be used in this sense
  - Time constraint
  - Degree of optimisation

- Goal:
  - Suite of metrics with plug-and-play degrees of freedom
  - Matching between application(s) and device(s)

Thank you

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