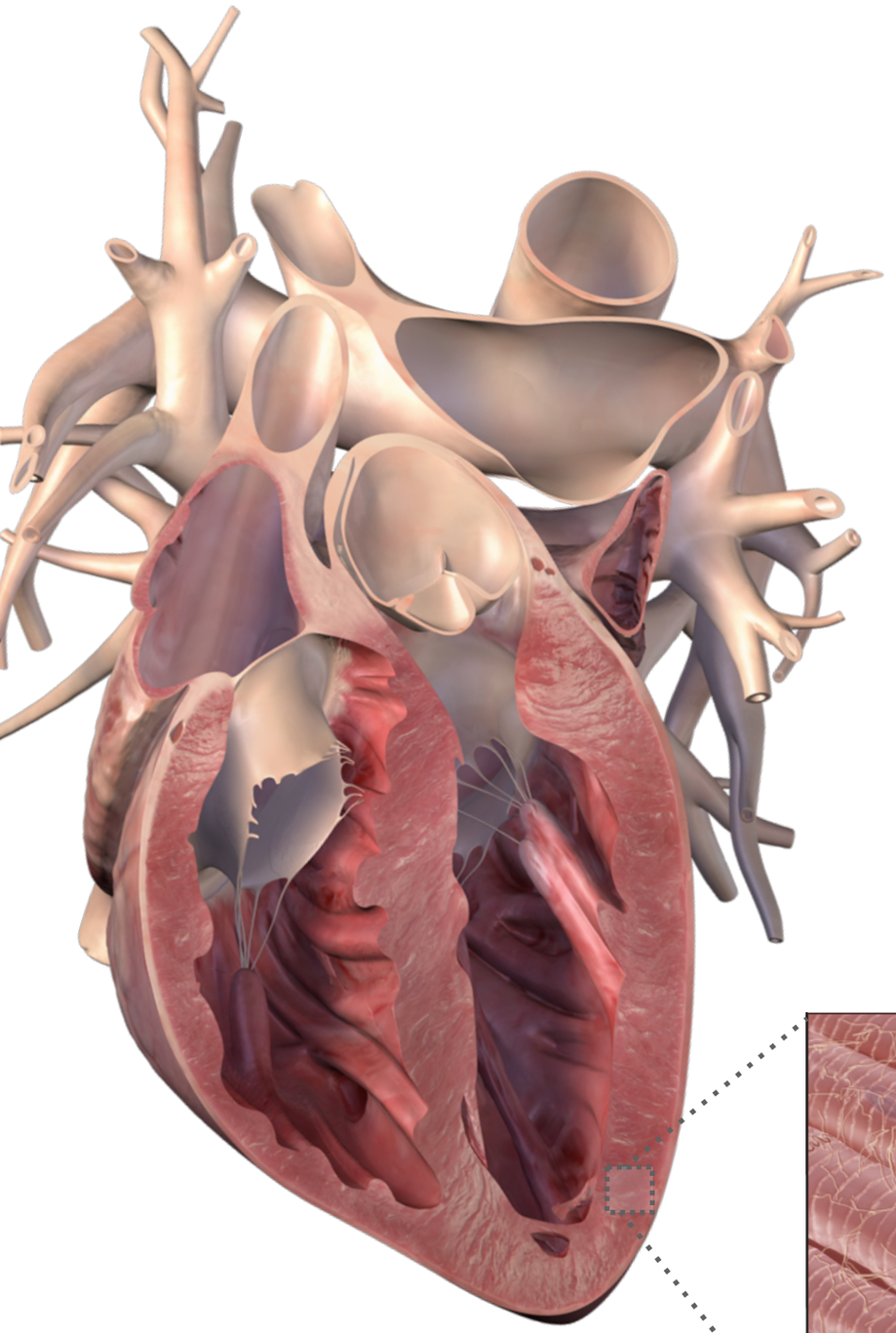
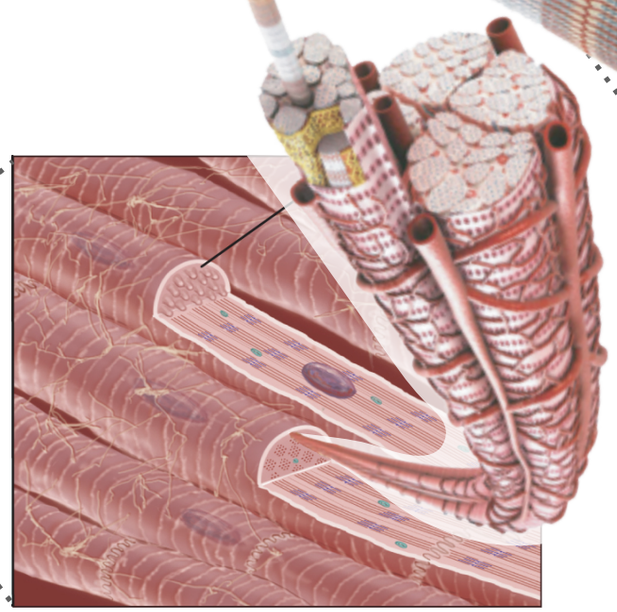
Multiscale modeling and Data interaction in cardiology

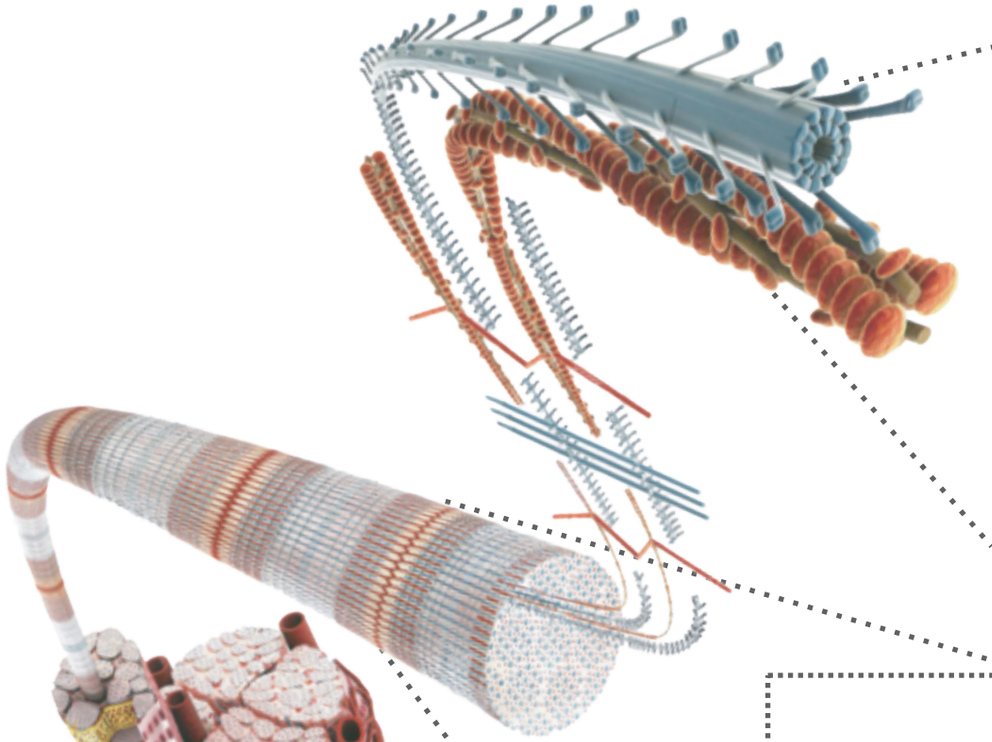
Philippe Moireau
MEDISIM team
Inria
LMS — CNRS Ecole Polytechnique —Institut Polytechnique de Paris

A multiscale structure

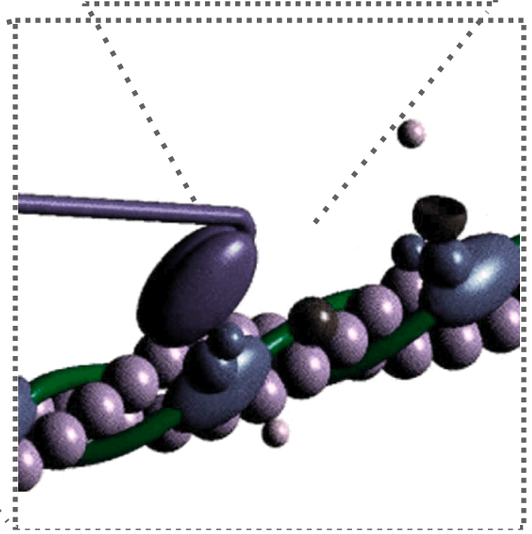


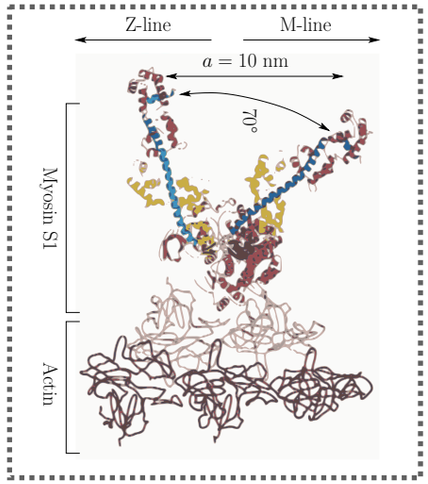
cm scale



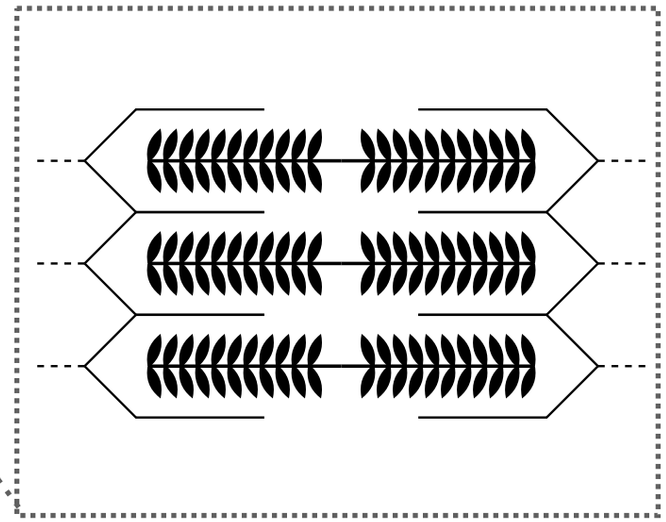


µm scale





nm scale



© 2007 Dorling Kindersley Limited

© 2012 Encyclopædia Britannica, Inc.

A multiscale structure

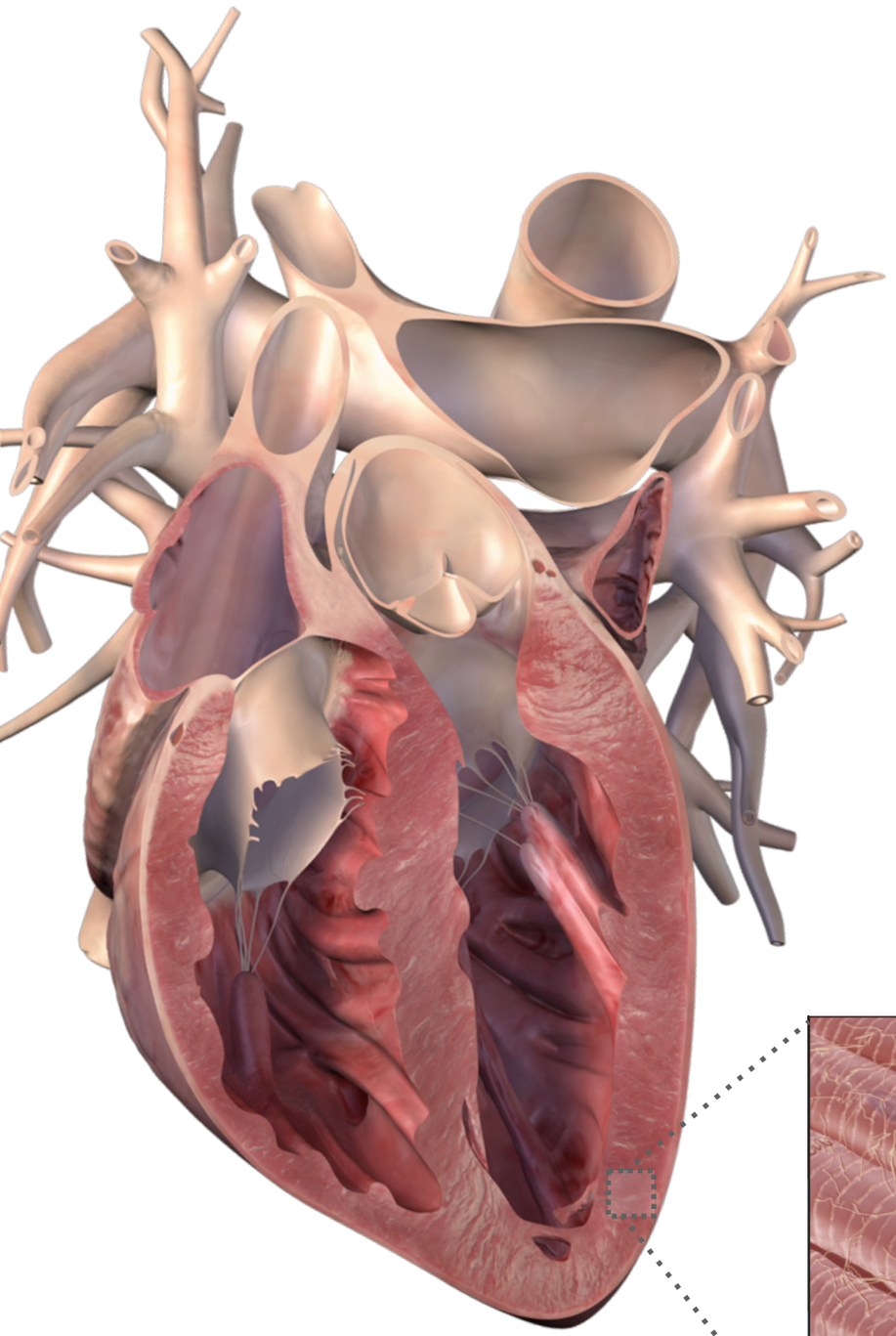
$$dX^t = \left(\alpha^t \dot{e}_c - (1-\alpha^t)\frac{\partial}{\partial_x} w_\alpha(X^t, Y^t)\right) dt + \delta(t_s)(s(t) - X^t) + (1-\alpha^t)\sqrt{2\beta^{-1}}dB_x^t$$

$$dY^t = \left(-\frac{\partial}{\partial y} w_\alpha(X^t, Y^t)\right) dt + \sqrt{2\beta^{-1}}dB_y^t$$

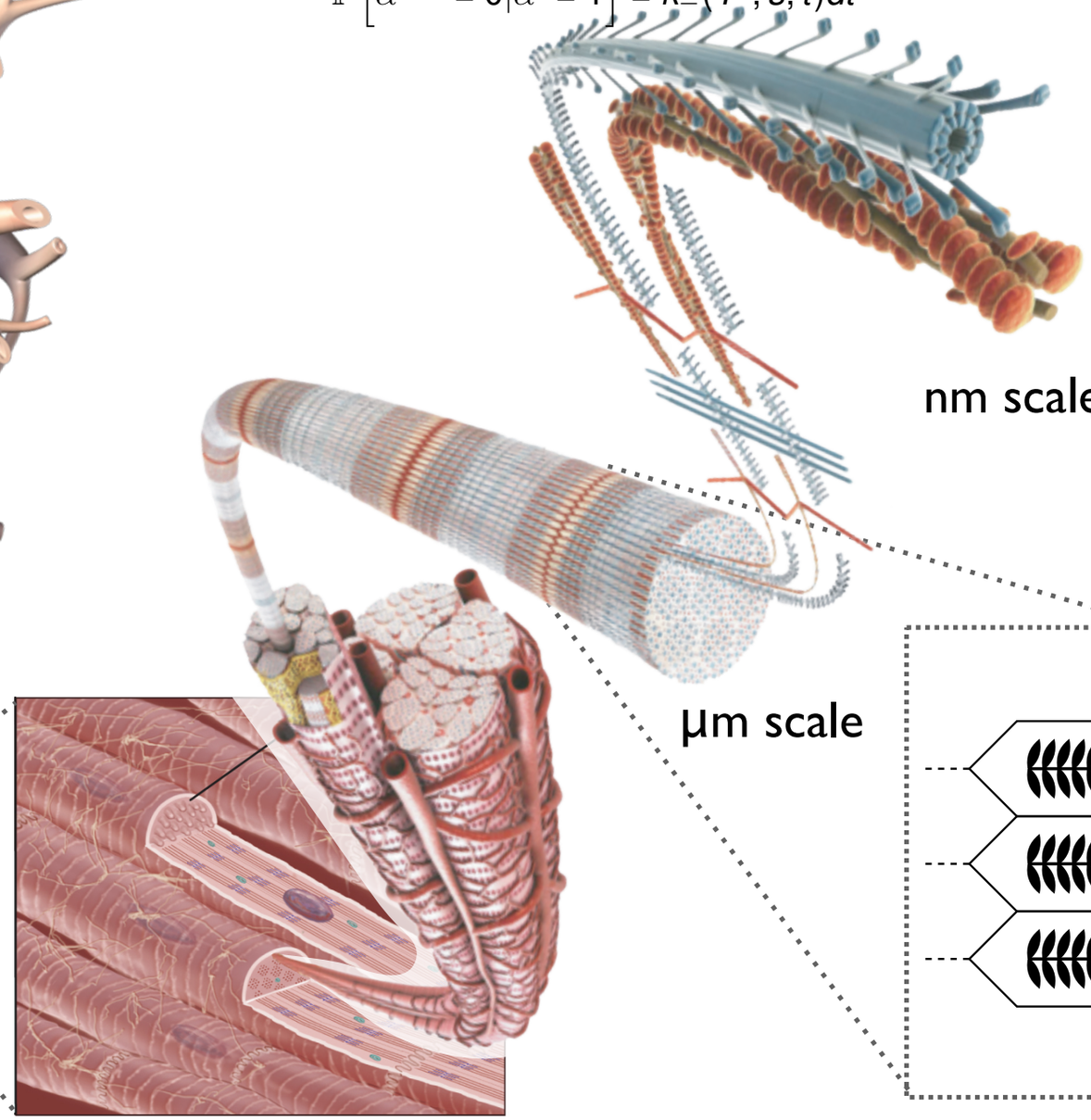
$$\mathbb{P}\left[\alpha^{t+dt} = 1 | \alpha^t = 0\right] = k_+(X^t, Y^t, s, t)dt$$

$$\mathbb{P}\left[\alpha^{t+dt} = 0 | \alpha^t = 1\right] = k_-(Y^t, s, t)dt$$

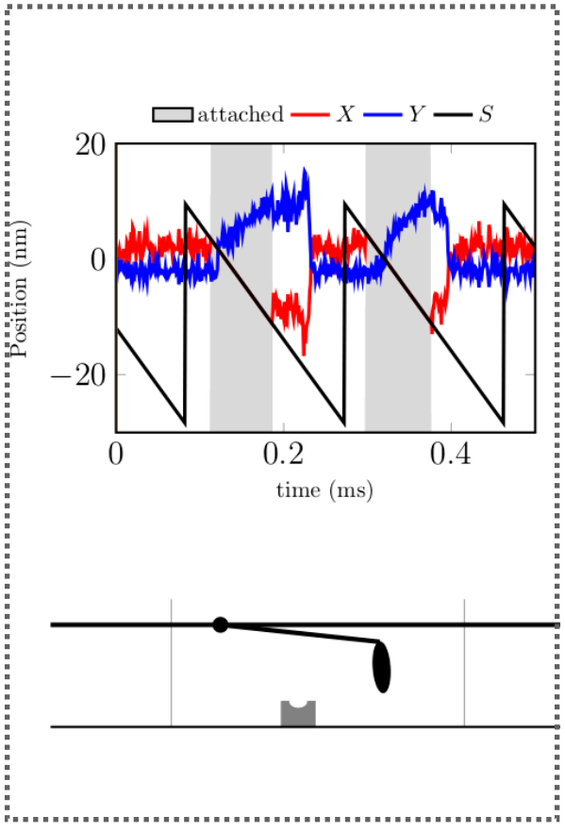
Langevin equations



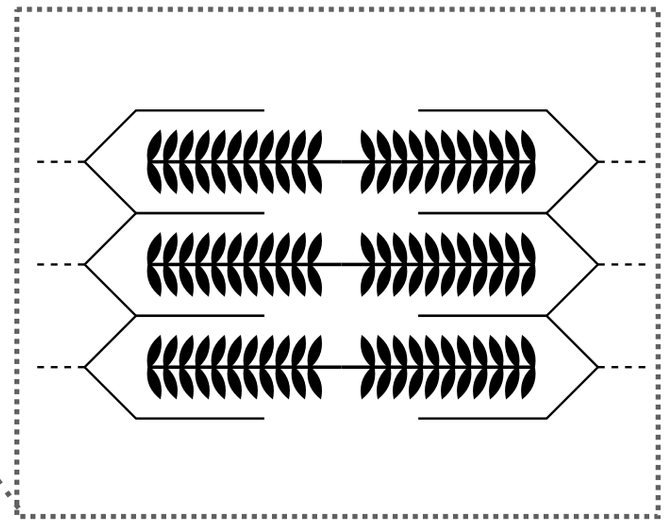
cm scale



µm scale



nm scale

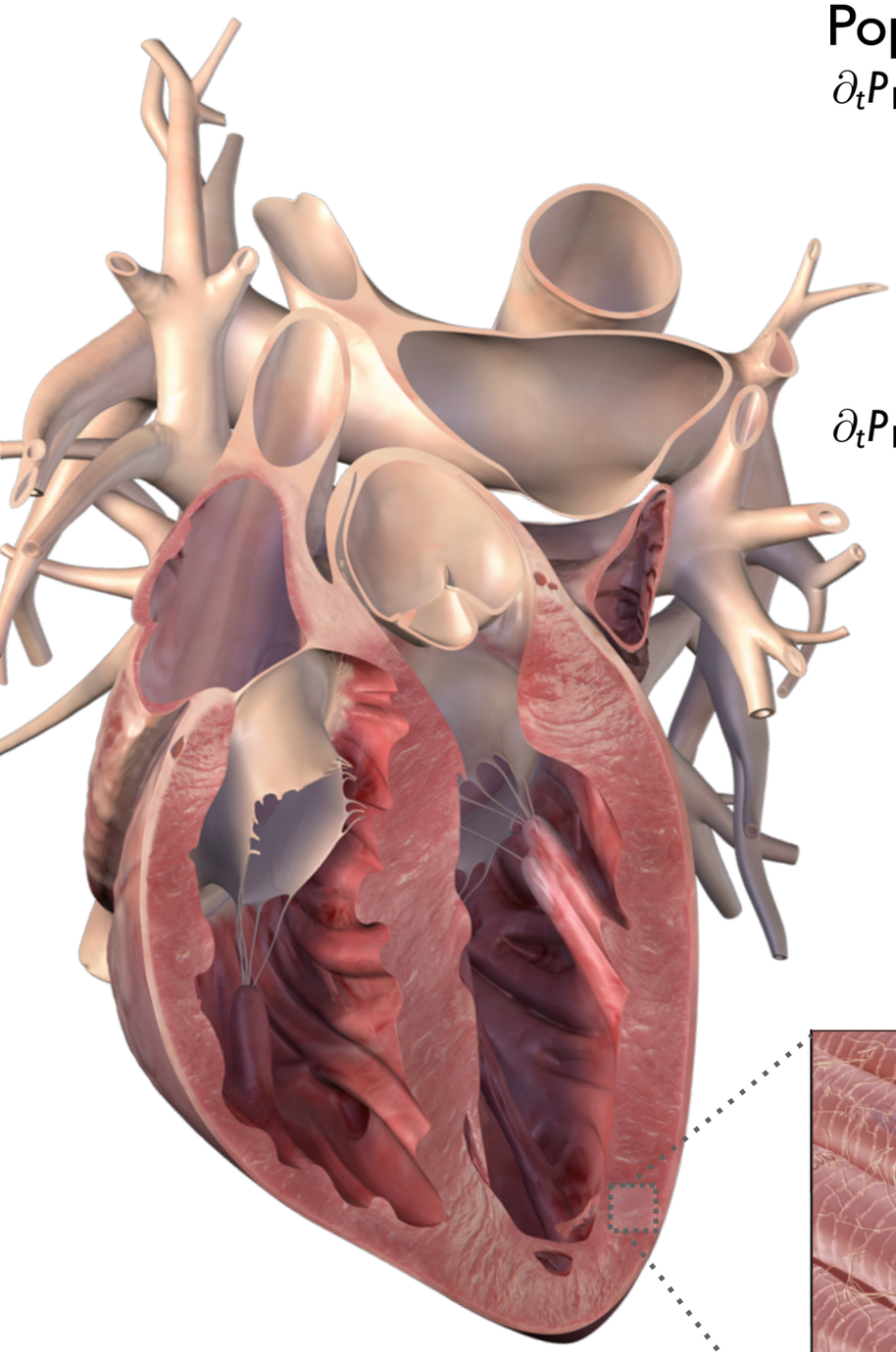


© 2012 Encyclopædia Britannica, Inc.

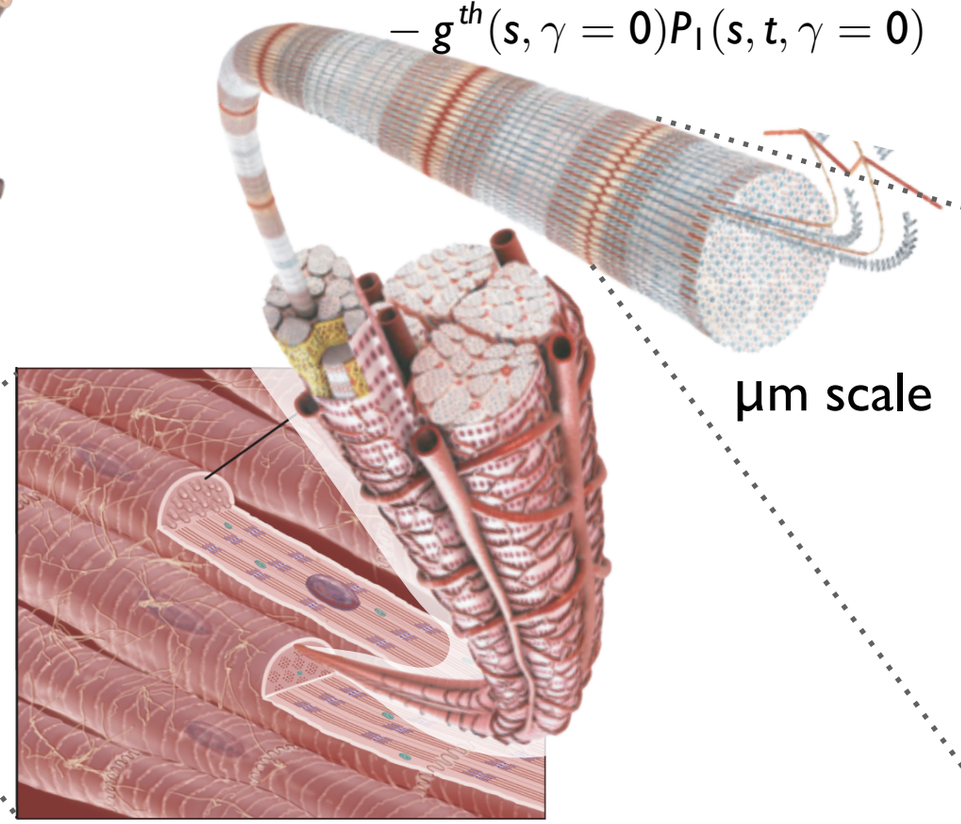
A multiscale structure

Population dynamics

$$
\begin{aligned}
\partial_t P_1(s,t,\gamma=1) + \dot{e}_c \partial_s P_1(s,t,\gamma=1) \\
&+ \frac{|\dot{n}_0|_+}{n_0}\left(P_1(s,t,\gamma=1) - P_1(s,t,\gamma=0)\right) \\
&= f^{th}(s,\gamma=1)(1-P_1(s,t,\gamma=1)) \\
&\quad - g^{th}(s,\gamma=1)P_1(s,t,\gamma=1) \\
\partial_t P_1(s,t,\gamma=0) + \dot{e}_c \partial_s P_1(s,t,\gamma=0) \\
&+ \frac{|\dot{n}_0|_-}{1-n_0}\left(P_1(s,t,\gamma=0) - P_1(s,t,\gamma=1)\right) \\
&= f^{th}(s,\gamma=0)(1-P_1(s,t,\gamma=0)) \\
&\quad - g^{th}(s,\gamma=0)P_1(s,t,\gamma=0)
\end{aligned}
$$

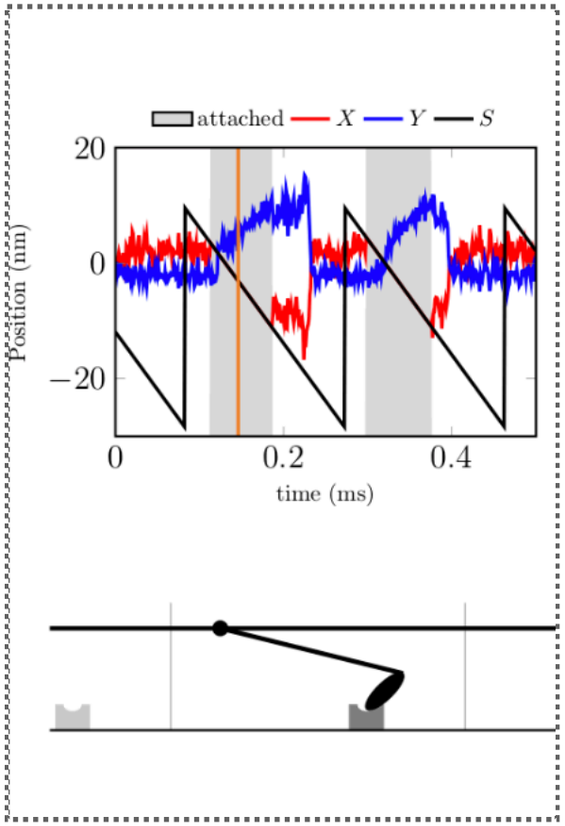


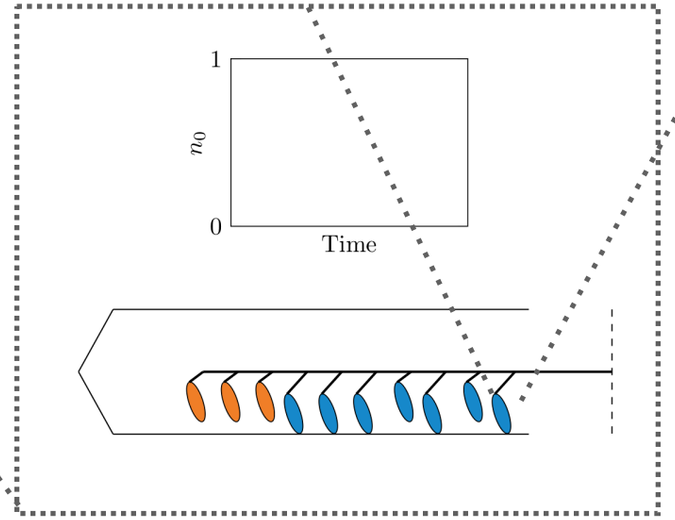
cm scale



µm scale

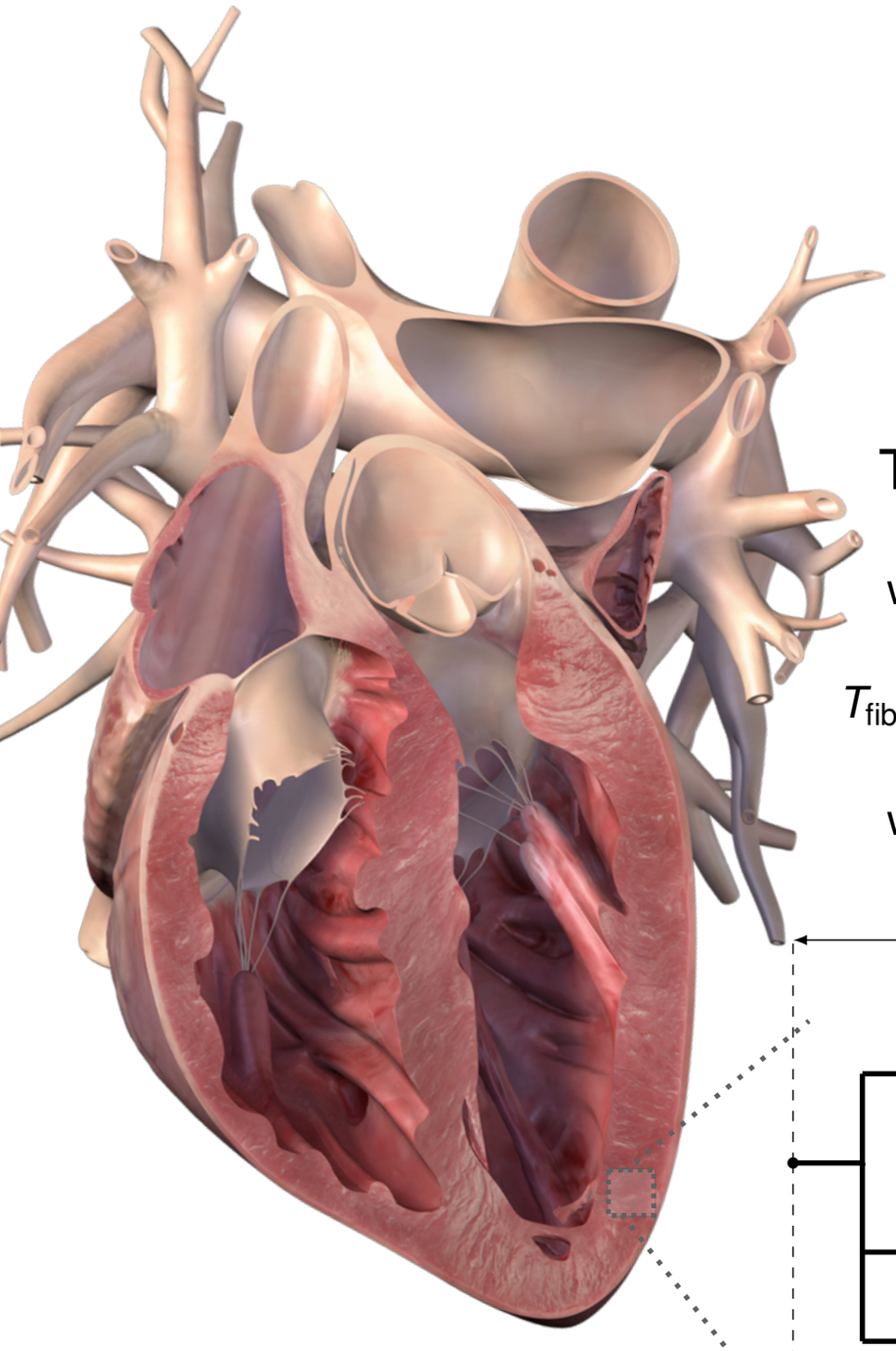
nm scale





© 2012 Encyclopædia Britannica, Inc.

A multiscale structure



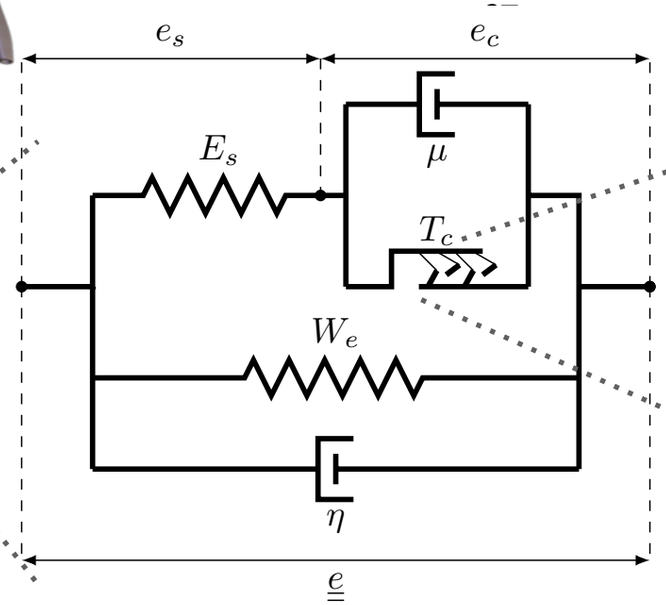
cm scale

Tissue rheology

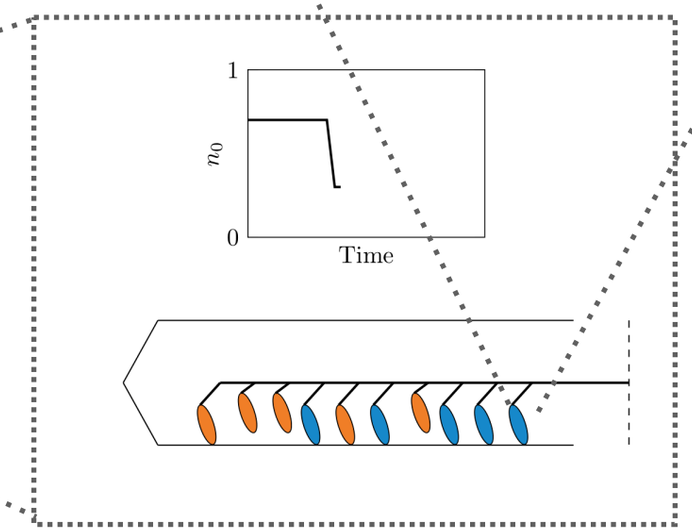
$$\text{with } \underline{\underline{\Sigma}} = \frac{\partial \Psi}{\partial \underline{\underline{e}}} + \eta \underline{\underline{\dot{e}}} + \frac{T_{\text{fib}}}{(1 + 2\underline{\tau} \cdot \underline{\underline{e}} \cdot \underline{\tau})^{\frac{1}{2}}} \underline{\tau} \otimes \underline{\tau}$$

$$T_{\text{fib}} = \nu \dot{e}_c + \check{T}_c = E_s e_s$$

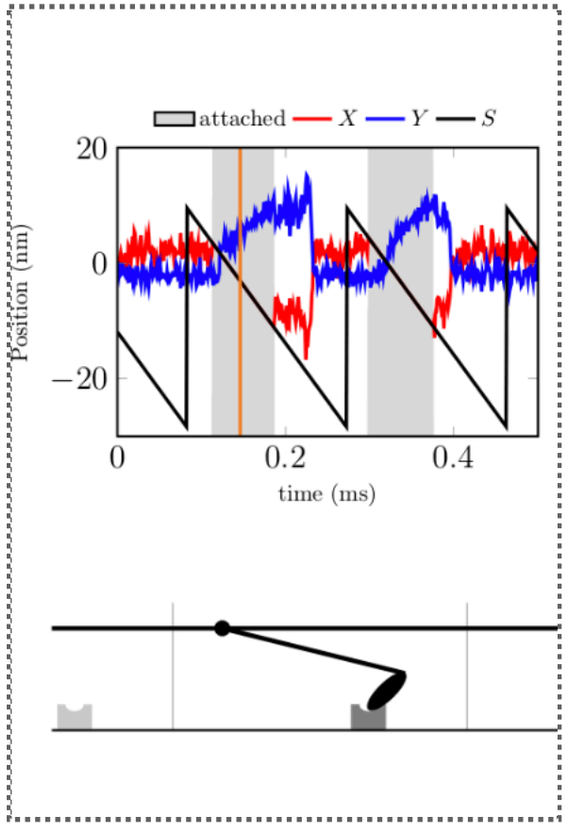
$$\text{with } \check{T}_c(\underline{x}, t) = \frac{\rho_{\text{surf}}}{d_a} \int^{s^+} a(\underline{x}, s, t) \frac{\partial w_1}{\partial s}(s, t) ds$$



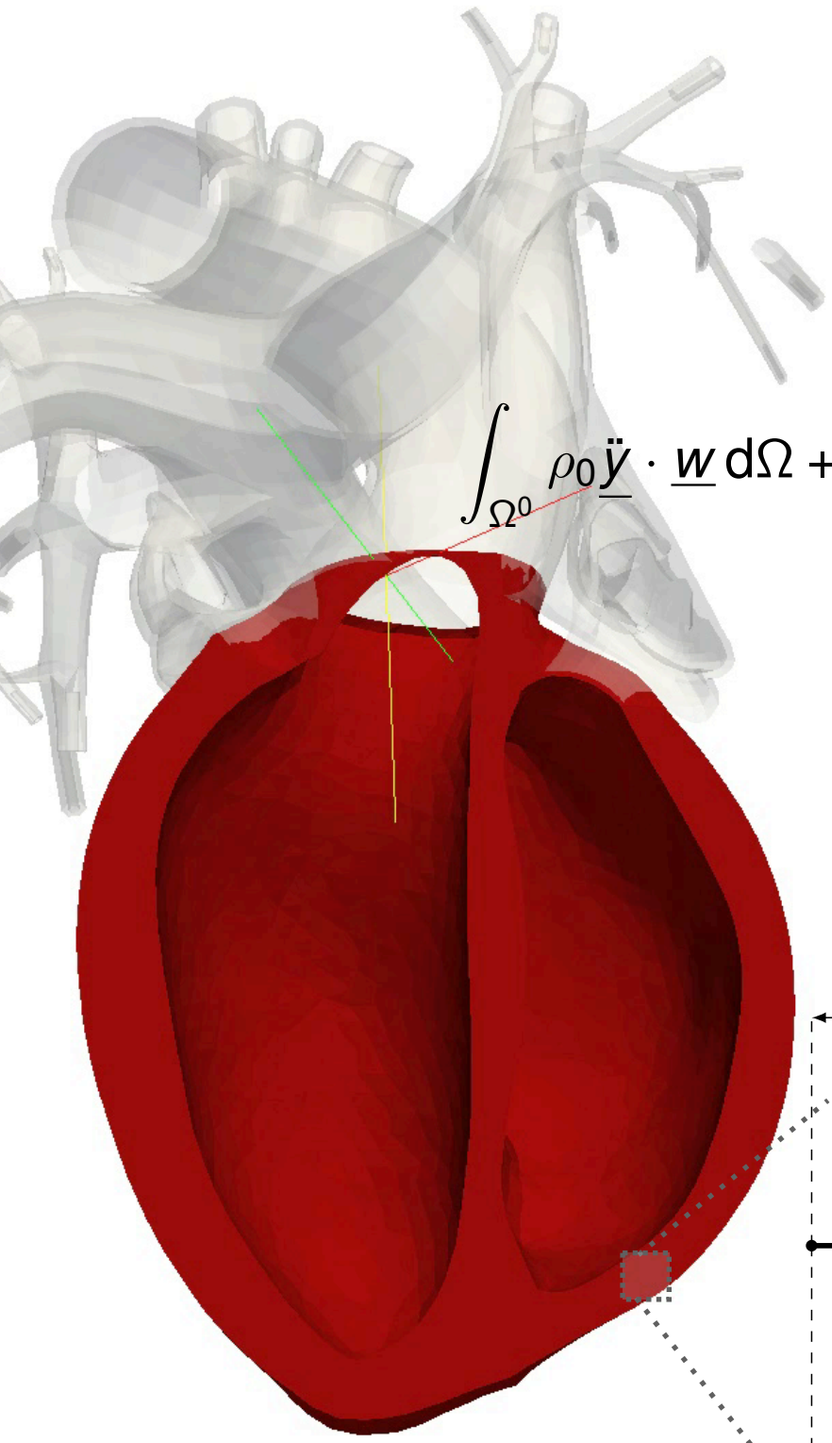
µm scale



nm scale



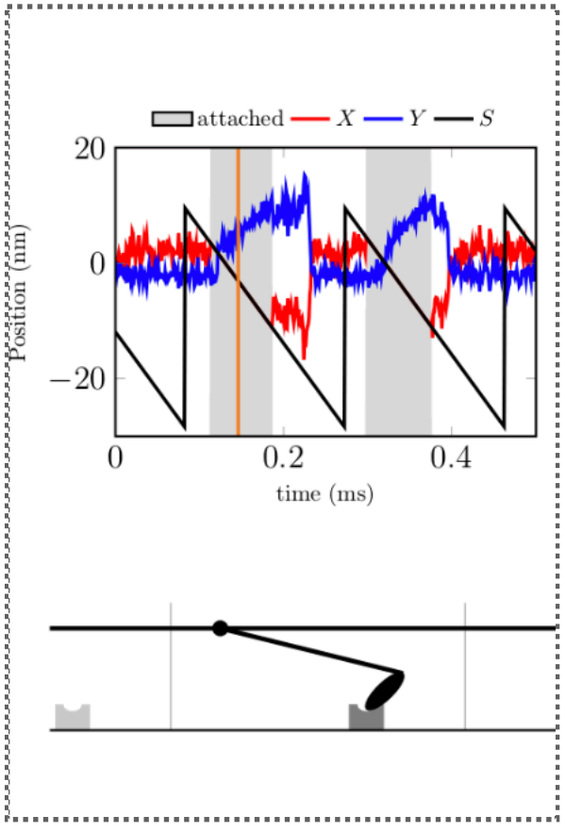
A multiscale structure

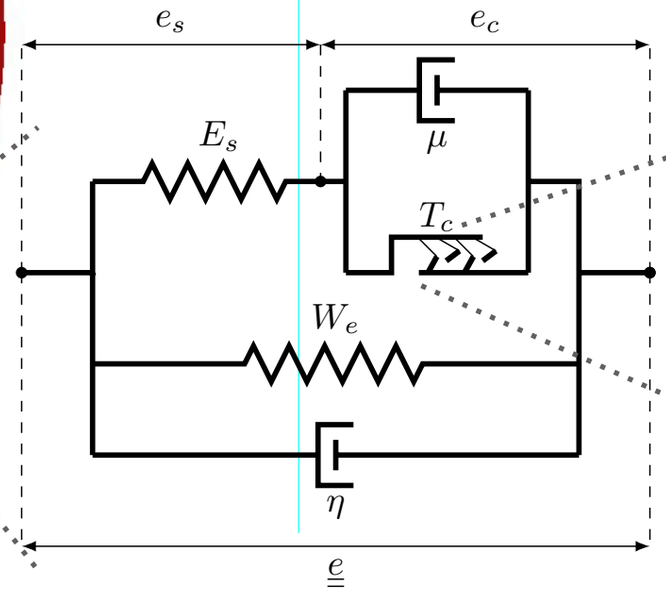


$$\int_{\Omega^0} \rho_0 \ddot{\underline{y}} \cdot \underline{w} \, \mathrm{d}\Omega + \int_{\Omega^0} \underline{\underline{\Sigma}} : D\underline{\underline{e}} \cdot \underline{w} \, \mathrm{d}\Omega = \int_{\Gamma_N^0} \underline{t}_N \cdot \underline{w} \, \mathrm{d}\Gamma, \quad \forall \underline{w} \in \mathscr{V}_{\mathrm{ad}}$$

Fundamental Principle of Dynamics

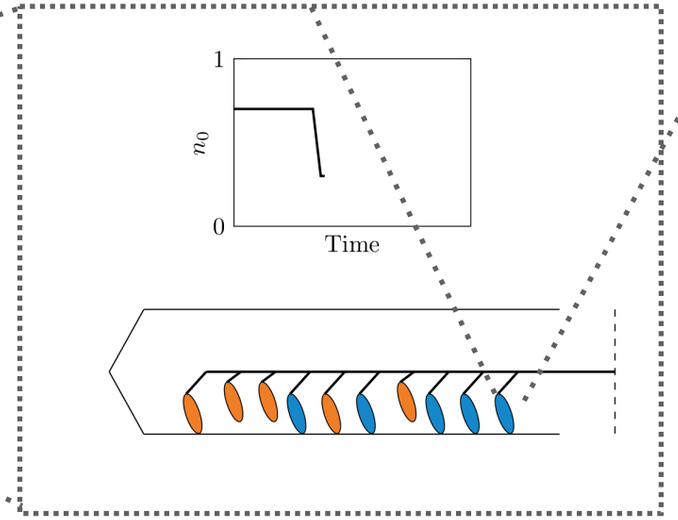
nm scale





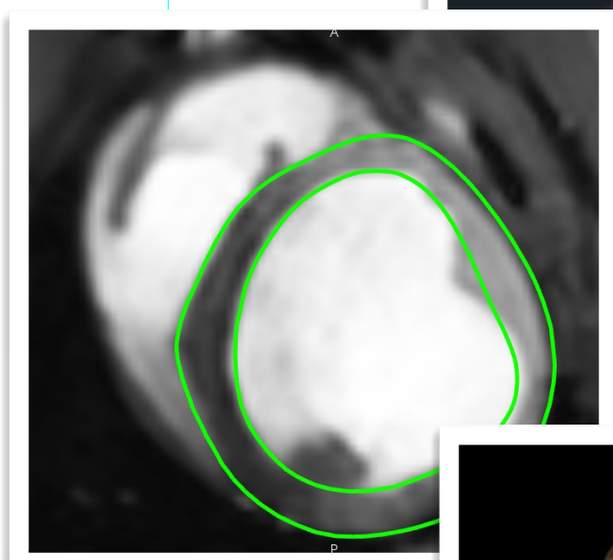
cm scale

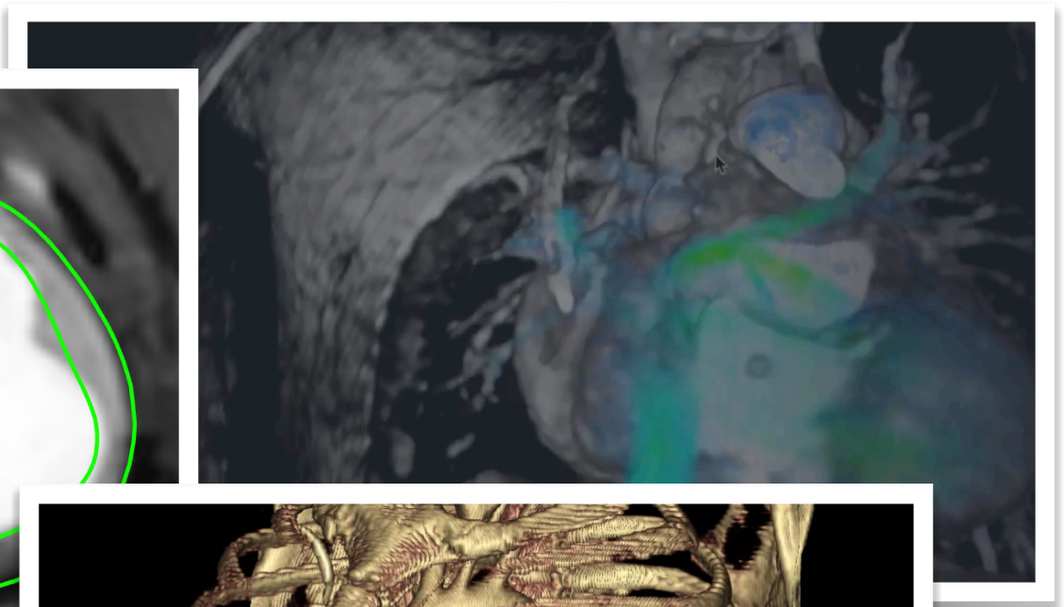
µm scale

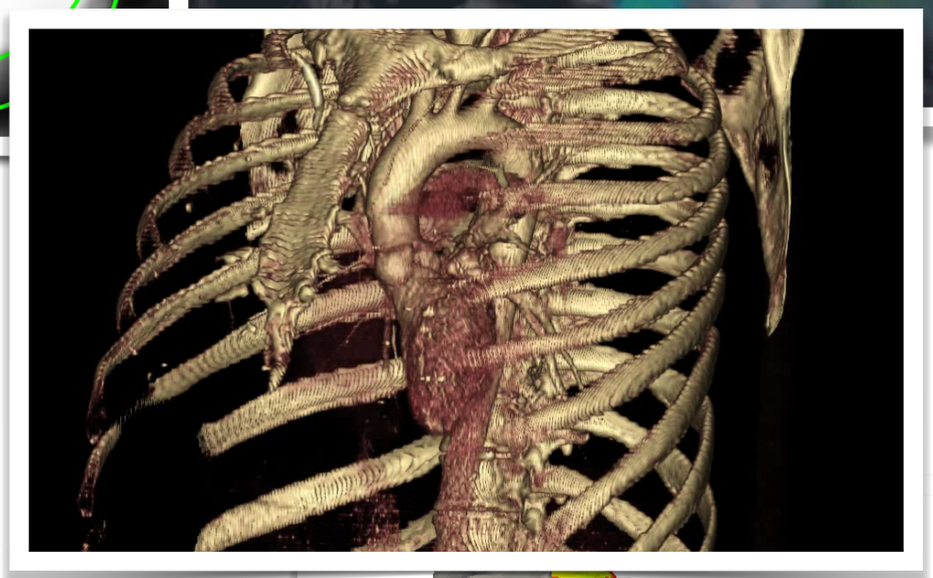


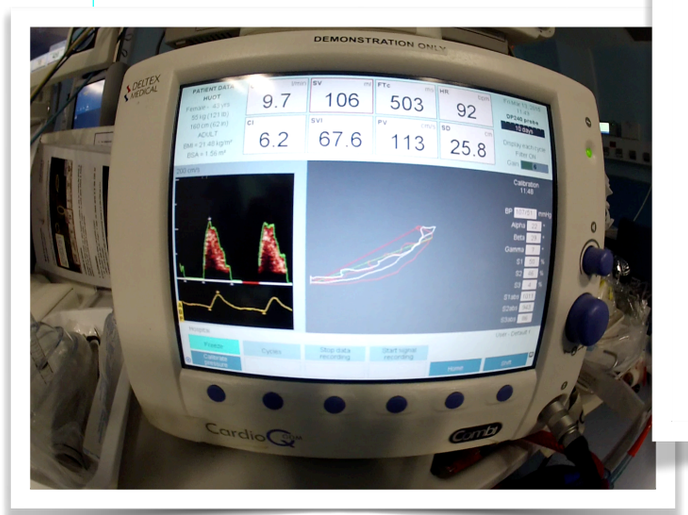
Data interaction challenges: big data or data big

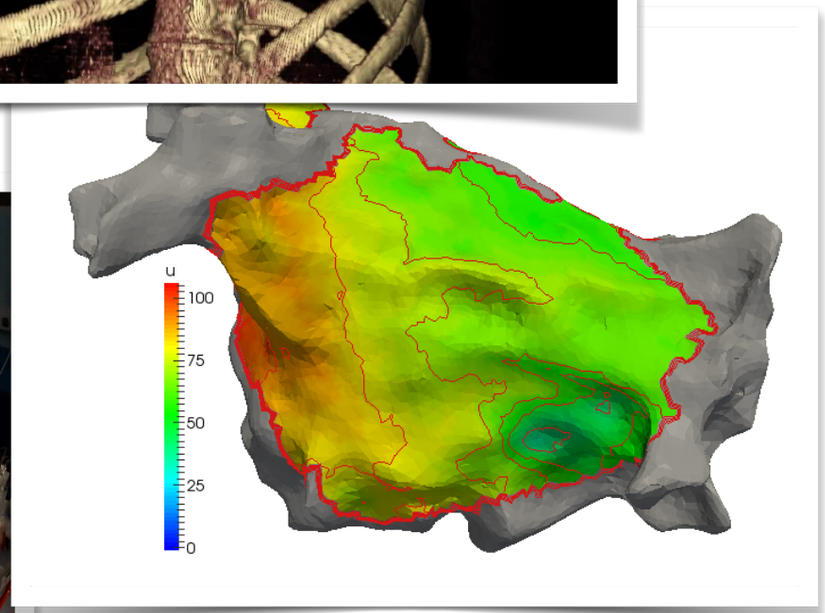
VS



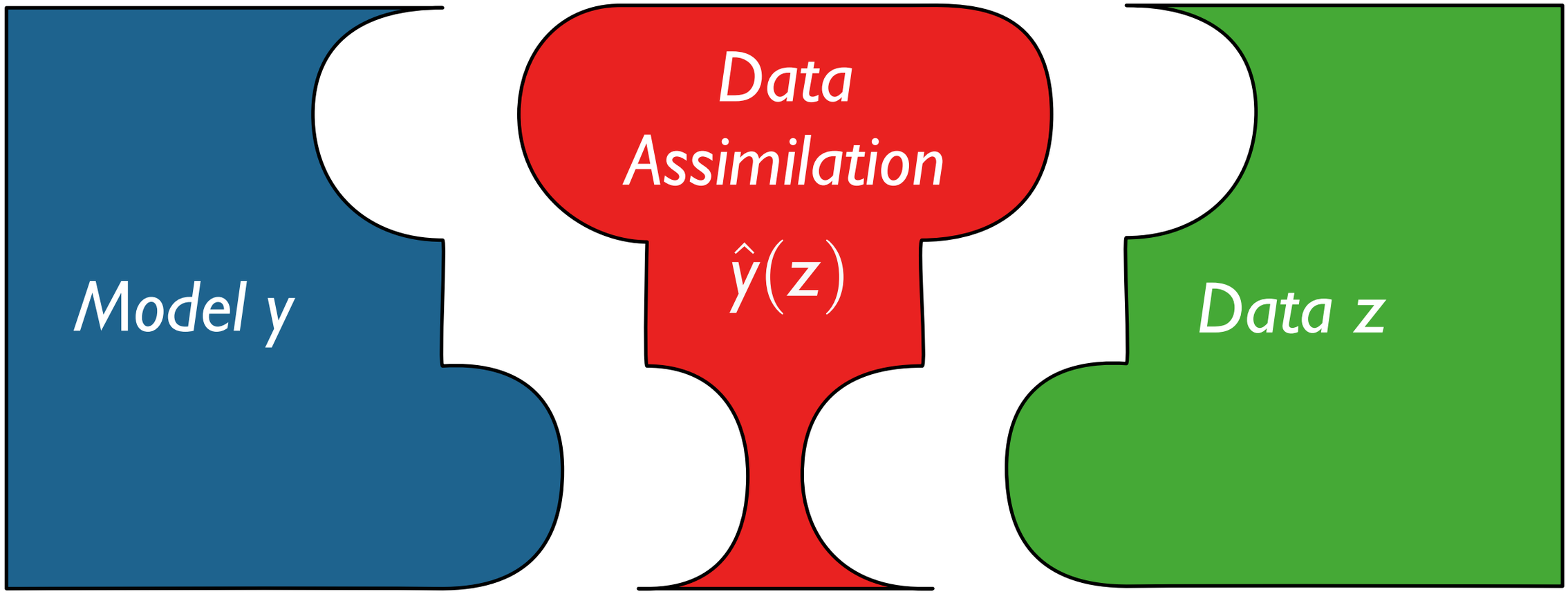








Model-data interaction by sequential data assimilation

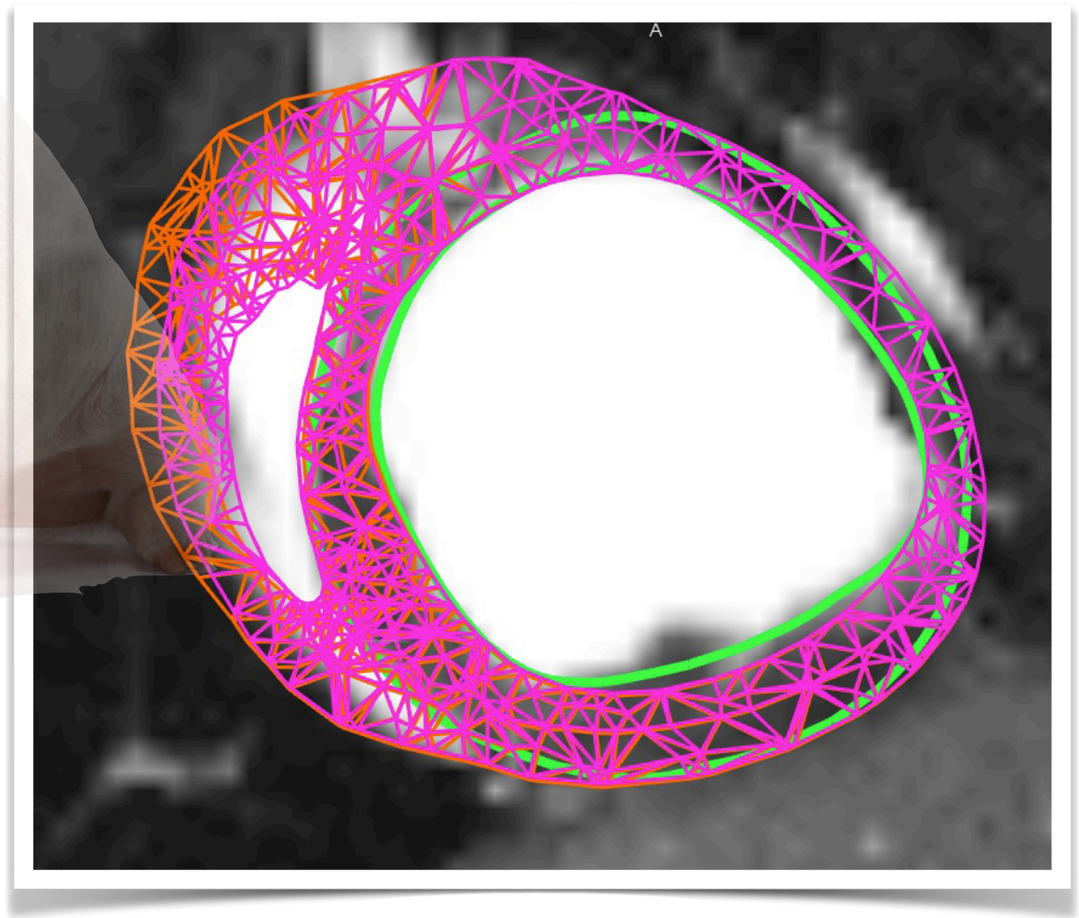


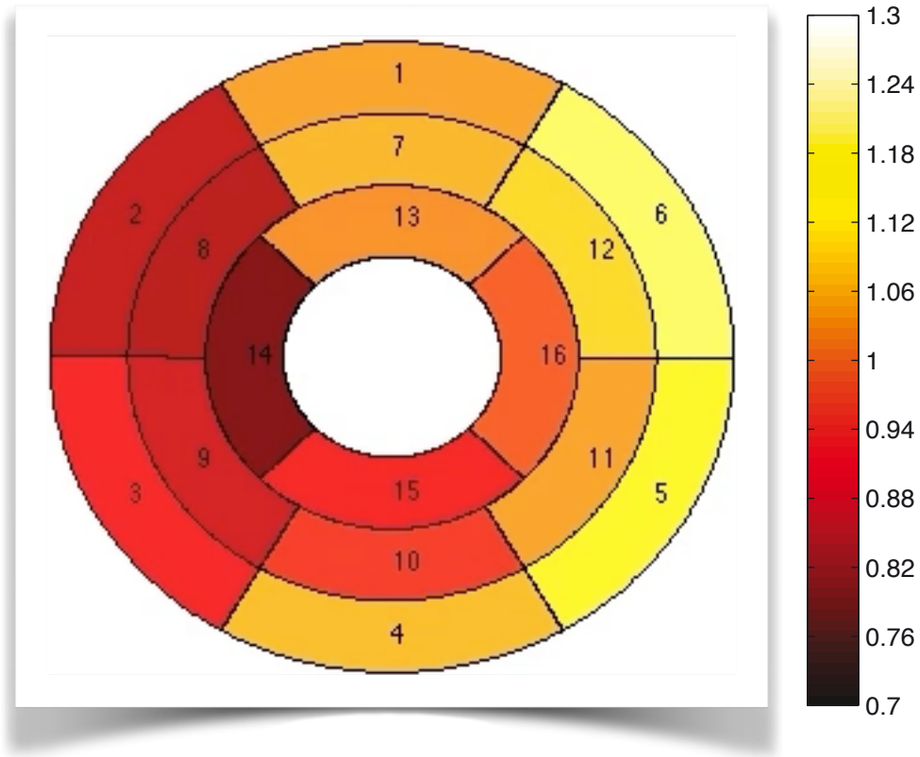
Add an additional term that incorporates the data z in the dynamics of y

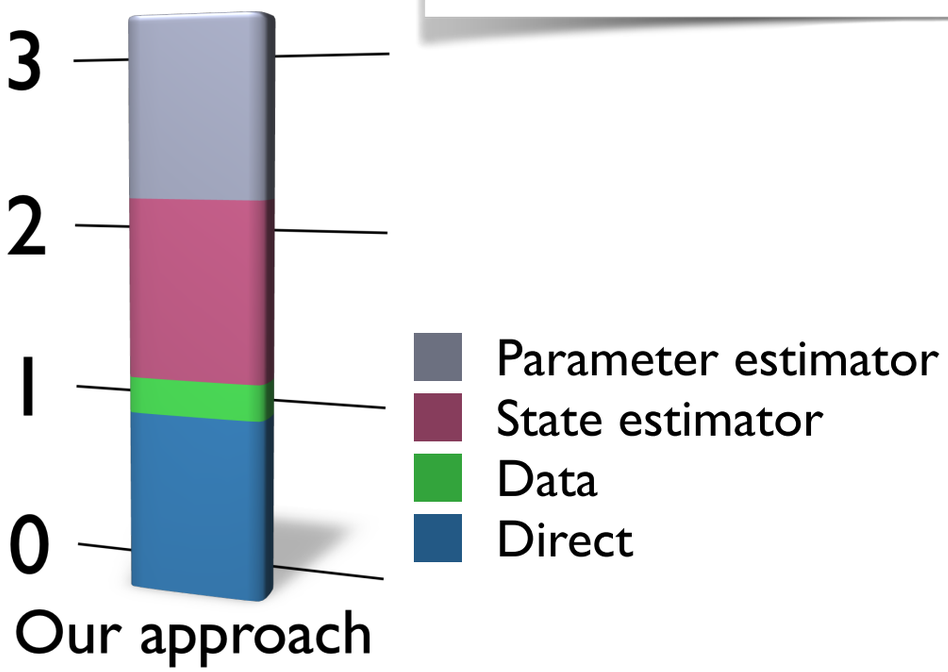
$$\begin{cases} \dot{\hat{y}} = A(\hat{y}, \hat{\theta}, t) + B_y(z, \hat{y}), \quad t > 0; \\ \dot{\hat{\theta}} = B_\theta(z, \hat{y}), \quad t > 0; \\ \hat{y}(0) = y_0 \textit{ a priori} \text{ knowledge}; \\ \hat{\theta}(0) = \theta_0 \textit{ a priori} \text{ knowledge}. \end{cases}$$

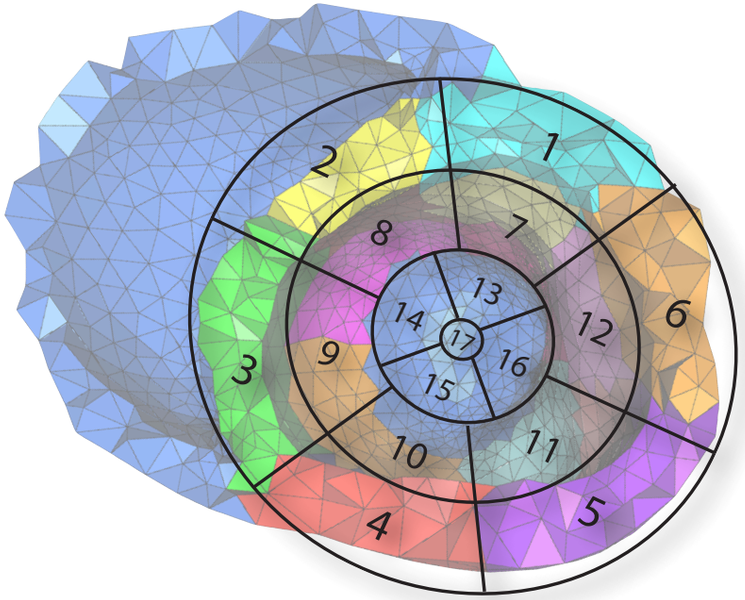
Can also correct the physical parameters θ

Data interaction challenges: adaptation with real data







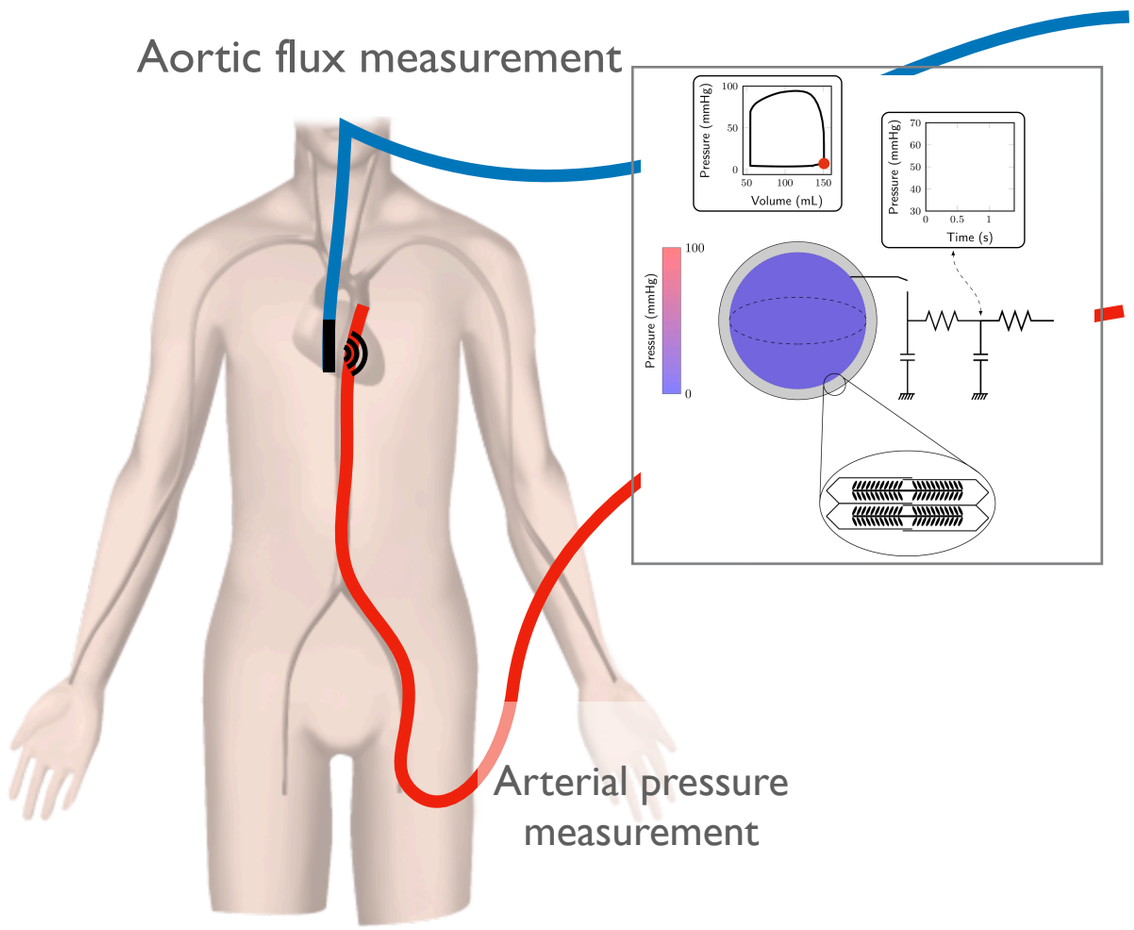


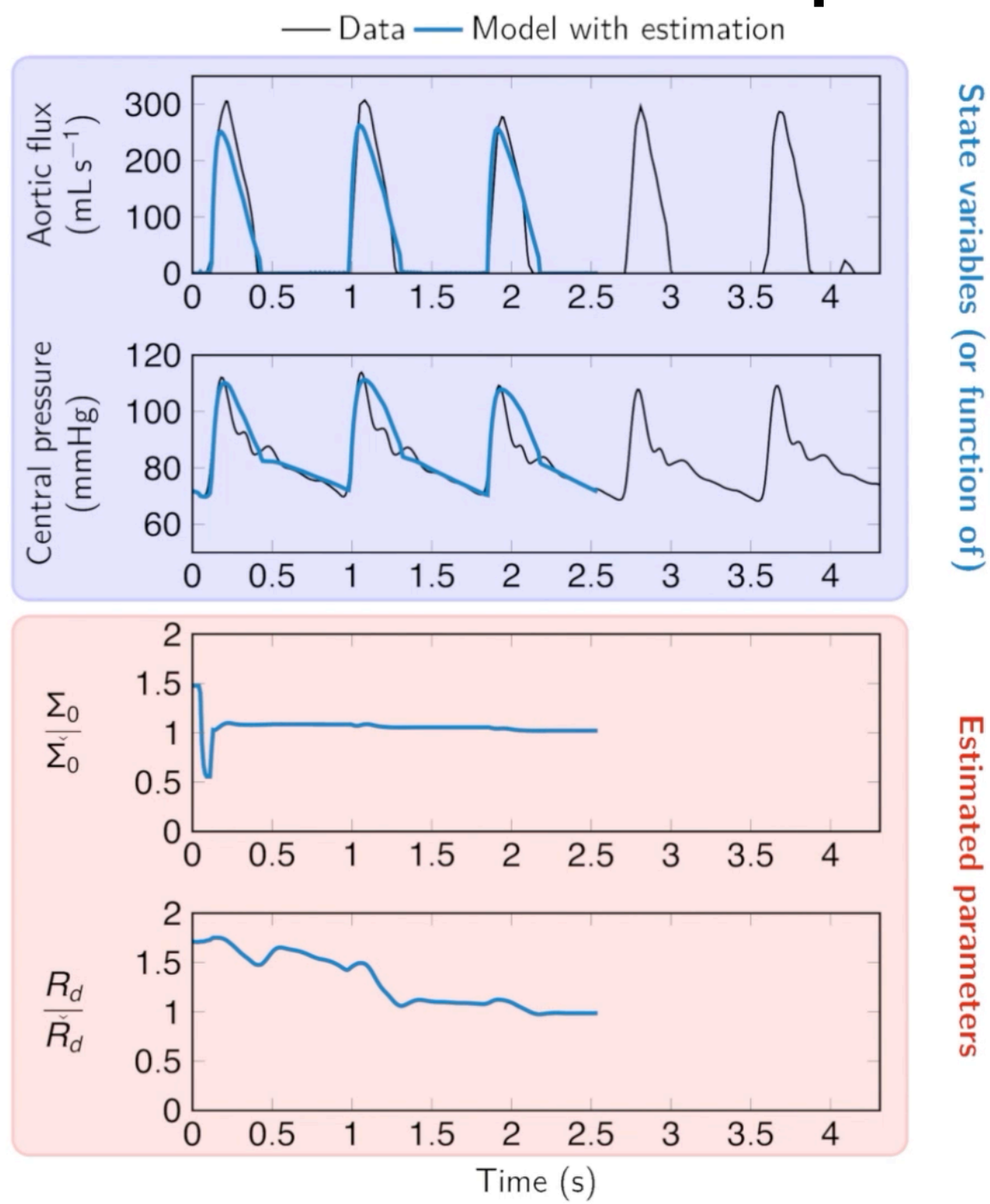
Challenges: sometimes unanticipated

Operating room

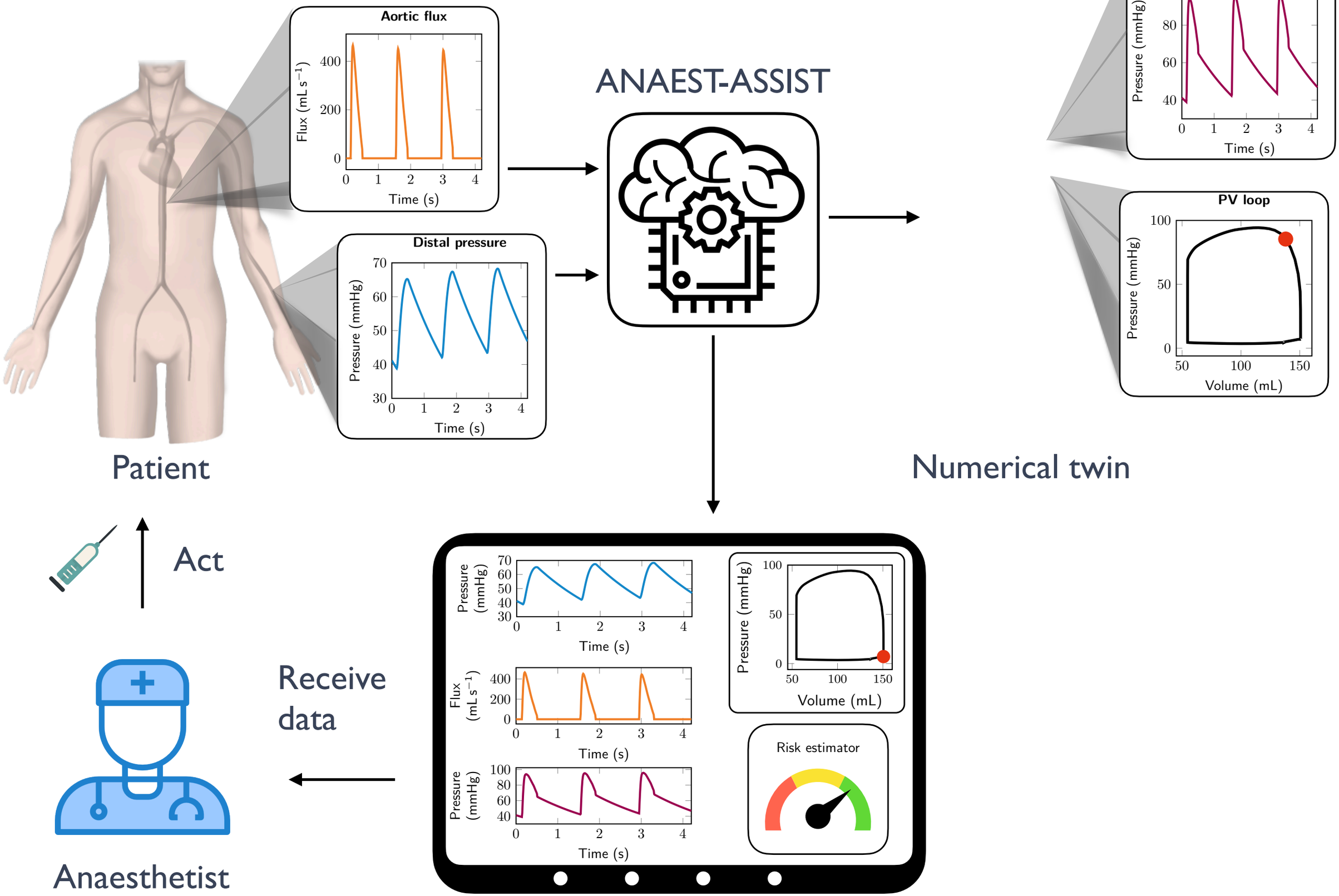
Data assimilation (Kalman filter) to bridge model and data

Real patient data





Important remark

- Model adapted from data of this patient, data not from a database

Opportunity