Constraint Programming

Connections with HPC

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Outline

1. What is Constraint-Programming?
2. How does it work?
3. How does HPC contributes to Constraint-Programming?

What is Constraint-Programming?

A Paradigm

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Constraint-Programming

A programming paradigm between AI and OR to solve constrained decision problems in a generic way.

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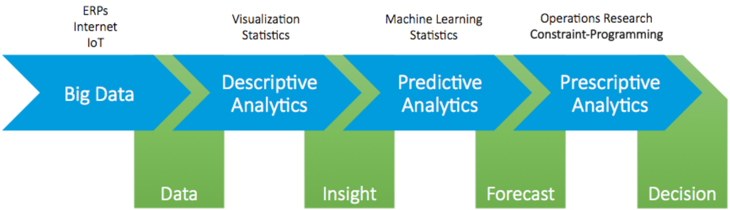

Problem oriented paradigm

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Focus

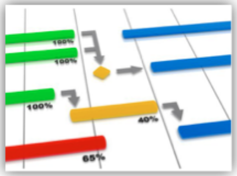
- It is not about Data (input)
- It is about solving problems
- Models are problem-specific

$\rightarrow$ CP is a generic toolbox to design algorithms

Applications

Applications

Applications



Applications

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How does it work?

Definition

Constraint Satisfaction Problem (CSP)

Defined by

- a set of variables $X = \{x_1, .., x_n\}$ (unknowns)
- a set of domains $D = \{d_1, .., d_n\}$ (possible values)
- a set of constraints $C = \{c_1, .., c_m\}$ (restrictions)

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Variable	Domain	Example
Binary	two integers	$\{0, 1\}$
Integer (bounded)	integer interval	$[\![3, 11]\!]$
Integer (enum)	integer set	$\{-2, ..., 3, 12\}$
Set	(int) set interval	$[\{2\}, \{0, 2, 3, 5\}]$
Graph	graph interval	
Real	real interval	$[3.14, 12.7]$

Definition

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Constraints	Examples
Arithmetic	$+, -, \times, \div, =, >, \leq, \neq$
Logical	$\vee, \wedge, \Rightarrow, \Leftrightarrow$
Set	$\cap, \cup, \subseteq, card, partition$
Global	$AllDifferent, Cumulative, Regular, Circuit...$
Graph	$Degrees, NCliques, NSCC, Tree$

No restrictions!

Example: TSP

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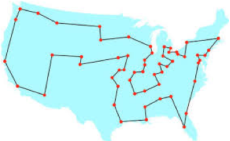
```java
// --- Circuit: successors[i] = k <=> once at client i, travel to client k
IntVar[] successors = model.intVarArray("succ",n, 0, n - 1, false);
model.circuit(successors).post();

// --- Unit travel cost
IntVar[] travelDistances = model.intVarArray(n, min, max, true);
for(int i=0; i<n ; i++){
    model.element(travelDistances[i], distances[i], successors[i]).post();
}

// --- Objective function
IntVar totalDistance = model.intVar("total distance",min*n, max*n, true);
model.sum(travelDistances, "=", totalDistance).post();
model.setObjective(Model.MINIMIZE, totalDistance);

// --- Solving
Solver s = model.getSolver();
while (s.solve()){
    System.out.println(totalDistance);
}
```



The Holy Grail

Constraint Programming represents one of the closest approaches computer science has yet made to the Holy Grail of programming: the user states the problem, the computer solves it. [Eugene Freuder]

Main algorithm

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Solving loop

- **Filtering**
  - Search space reduction (inconsistent values removal)
  - Propagating deductions until **fixpoint**
- **Search**
  - Search space exploration (hypothesis)
  - Applying branching **heuristics** to compute **decisions**

Until all variables are fixed to a value (solution)

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Optimization

- Compute solution
- Post $z < c$ (cost of previous solution)

Until no better solution exists

Filtering

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From constraints to filtering

Every constraint comes with one or many **filtering algorithms** to remove inconsistent values (domain modifications)

Filtering

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From constraints to filtering

Every constraint comes with one or many **filtering algorithms** to remove inconsistent values (domain modifications)

Example

- $x = \{0, 1, 2, 3, 5, 6\}$ and $y = \{2, 3, 5, 6, 10\}$
- constraint: $x > y$
- Filtering $x \rightarrow y$: $y = \{2, 3, 5, \cancel{6}, \cancel{10}\}$
- Filtering $x \leftarrow y$: $x = \{\cancel{0}, \cancel{1}, \cancel{2}, 3, 5, 6\}$

Filtering

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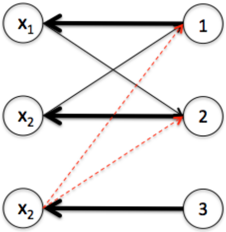
From constraints to filtering

Every constraint comes with one or many **filtering algorithms** to remove inconsistent values (domain modifications)



Example: *AllDifferent*

Let $X = \{x_1, x_2, x_3\}$
with $x_1 \in \{1, 2\}$, $x_2 \in \{1, 2\}$ and $x_3 \in \{1, 2, 3\}$
such that $x_1 \neq x_2$, $x_1 \neq x_3$, $x_2 \neq x_3$

$x_1 \in \{1, 2\}$, $x_2 \in \{1, 2\}$ and $x_3 \in \{\cancel{1}, \cancel{2}, 3\}$

Filtering

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From constraints to filtering

Every constraint comes with one or many **filtering algorithms** to remove inconsistent values (domain modifications)

Scope

- Each filtering algorithm is **local** to a constraint
- Filtering algorithms are called **incrementally** until reaching a fix point

Search

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Search

Heuristic domain modification to drive the solving process

- Triggers propagation and filtering
- Guides how to reach a solution

Binary branching

At each node a **decision** (var, op, val) is taken

- Left child: apply decision (guess)
- Right child: negate decision (deduction)

Search

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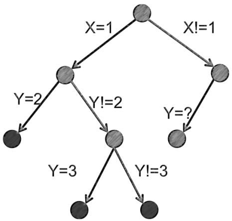
Search heuristics

Which decision (var, op, val) to generate?

- Huge impact on results



Many strategies

- Black-box v.s. User-defined strategy
- Best-First v.s. Fail-First principle

Summary

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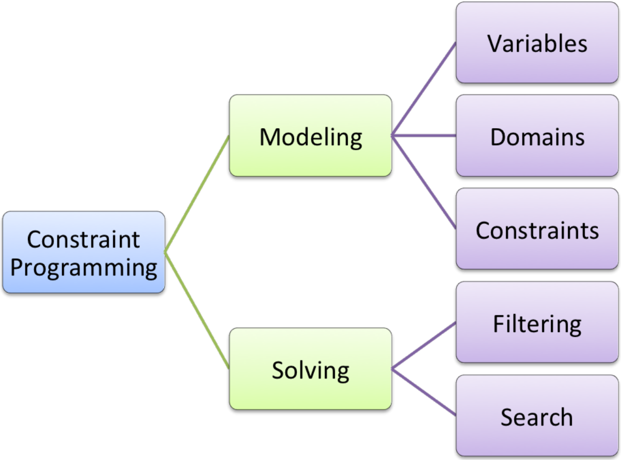

How does HPC contributes to Constraint-Programming?

Parallelism

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Parallelism

Can we achieve significant gains using parallel computing?

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Parallelism

Can we achieve significant gains using parallel computing?

Filtering parallelization

Filtering algorithms could be called in parallel, but:

- Domain cloning overhead
- Unit propagation is (usually) fast
- Propagation is an iterative process

$\Rightarrow$ Not used in practice

Parallelism

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Parallelism

Can we achieve significant gains using parallel computing?

Search parallelization

- Solver portfolios
- Distributed search
  - Work stealing
  - Embarrassingly Parallel Search

Portfolio approaches

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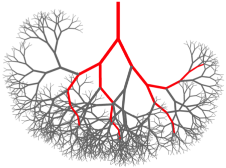
Portfolio

- Several instances of **same model** but **Different search strategies** run in parallel
- Satisfaction: no communication at all (luck)
- Optimization: Bound sharing (collaboration)



Many strategies

- Exponential speedup
- State-of-the-art for few cores

Portfolio approaches

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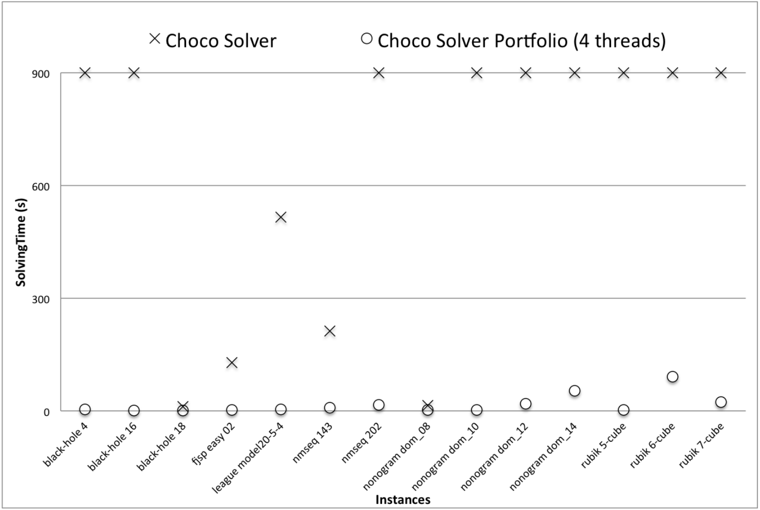

Distributed DFS

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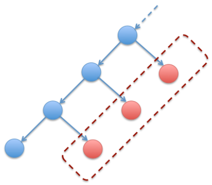
Work stealing [Jaffar et. al.]

- Master records solutions & bounds
- Workers (dynamical)
  - Pop or steal node
  - Explore node
  - Push right child in stack



Issue

- Communication cost
- Hard to find a balanced distribution
- Hard to scale beyond 100 cores

Distributed DFS

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Embarrassingly Parallel Search [Regin et. al.]

- **Static** decomposition into **consistent** subproblems
- Based on depth-bounded DFS
- Communicate with master only when subproblem is solved
- ≈ 40 subproblems per workers $\rightarrow$ balanced workload

Issue

- Almost no communication cost but decomposition cost
- State of the art over large number of cores

EPS speedup

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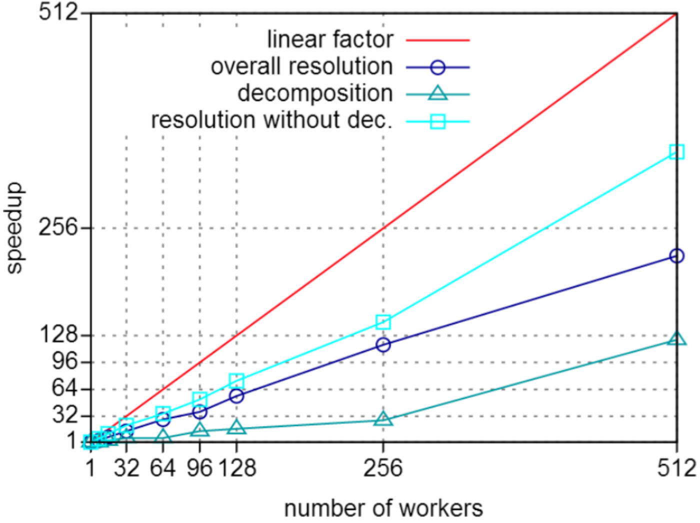

Distributed DFS

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Insight

- Independent subproblems are efficiently solved in parallel
- Automated decomposition may be costly/intricate

Distributed DFS

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Insight

- Independent subproblems are efficiently solved in parallel
- Automated decomposition may be costly/intricate

Good news!

It is often possible to decompose a problem

- Stochastic optimization based on different scenarios
- Optimization of different orders (logistics)
- Routing optimization of different areas (transportation)
- Frequent user request presolving (web-app)

Conclusion

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HPC

- Simple and affordable
- Pushes forward the limits of Constraint Programming
- Opens real-time application opportunities

Thank you for your attention

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References

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Main references

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- Global Constraint Catalogue: http://sofdem.github.io/gccat/
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Journals & conferences

- Artificial Intelligence, Constraints
- CP, CPAIOR, AAAI, ECAI, JFPC