Quantum Expectation Maximization

and other quantum algorithms for learning representations

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INSTITUT DE RECHERCHE EN INFORMATIQUE FONDAMENTALE

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In short

We propose a quantum algorithm for
Expectation-Maximization
that fits a
Gaussian Mixture Model
(using quantum linear algebra, QRAM, distance estimation, etc..)
in time:

$$\widetilde{O}\left(\frac{d^2 k^{4.5}\eta^3\kappa^2(V)\kappa^2(\Sigma)\mu(\Sigma)}{\delta_\mu^3}\right).$$

Particular case: **q-means**, a quantum algorithm for k-means:

$$\widetilde{O}\left(k^2 d\frac{\eta^{2.5}}{\delta^3} + k^{2.5}\frac{\eta^2}{\delta^3}\right)$$

Papers?

- **q-means**: *https://arxiv.org/abs/1812.03584*
  Iordanis Kerenidis, Jonas Landman, Anupam Prakash
  (Accepted at NeurIPS)
- **QEM**: *https://arxiv.org/pdf/1908.06657*
  Iordanis Kerenidis, Anupam Prakash
- (Also **QGMM** *https://arxiv.org/abs/1908.06655* Hideyuki Miyahara, Kazuyuki Aihara, Wolfgang Lechner)

(Also: **QSFA+QFDC**: *https://arxiv.org/abs/1805.08837*)

Quantum Expectation-Maximization

Theorem

Let a data matrix $V \in \mathbb{R}^{n \times d}$ stored in an appropriate QRAM data structure and parameters $\delta_\theta, \delta_\mu > 0$.
Then, Quantum Expectation-Maximization (QEM) fits a Maximum Likelihood (or a Maximum A Posteriori) estimate of a Gaussian Mixture Model with k components, in running time per iteration which is dominated by:

$$T_{QEM} = \widetilde{O}\left(\frac{d^2 k^{4.5} \eta^3 \kappa^2(V) \kappa^2(\Sigma) \mu(\Sigma)}{\delta_\mu^3}\right),$$

Biometric authentication to your Quantum Lab.

Maximum Likelihood Estimation

Unsupervised data: $V \in \mathbb{R}^{n \times d}$.
Generative models: learn the best (i.e. with Maximum Likelihood) $p(v_i)$ and sample from it.

$$L(V;\gamma) = \prod_{i=1}^{n} p(v_i;\gamma)$$

$$\gamma^*_{MLE} = \arg\max_{\gamma} L(\gamma; V)$$

Maximum A Pesteriori estimate

MAP uses a prior $p(\gamma)$ on the model:

$$L(\gamma; V) = \prod_{i=1}^{n} p(v_i|\gamma)p(\gamma)$$

$$\gamma_{MAP} := \arg\max_{\gamma} L(\gamma; V)$$

Intuition: Putting a prior (from domain experts) often avoid overfitting and wrong local minima (decrease number of iterations) ;)

fitquantum.py

fitquantum.py > ...

```python
#!/bin/python
import quantum-sklearn # exist only if not  measured
import microphone

gmm = quantum-sklearn.mixture.GaussianMixture(n_components=16, 
                            max_iter=200, covariance_type='diag', n_init=3)

"perform an extensive recording of spoken voice"
audio, rate = read(microphone.listen())

"extract 20 dim mfcc features from an audio, performs CMS and combines"
" delta to make it 40 dim feature vector"
dataset_ = extract_features(audio, rate)

"scaling, normalization, etc.."
dataset = quantum-sklearn.preprocessing(dataset)

"HERE is where the quantum data analysis is done"
gmm.fit(features)

"store pre trained GMM model of my voice"
pickle.dump(open("scinawa.model", 'wb'), gmm)

print ("training finished")
```

example.py ×

example.py > ...

```python
#!/bin/python
import sklearn
import microphone

"""pre trained GMM model of my voice"""
scinawa_model = pickle.load(open("scinawa.model", 'rb'))

while 1: 
  audio, rate = read(microphone.listen_sentence("sésame, ouvre-toi"))

  """extract 20 dim mfcc features from an audio, performs CMS and combines
   delta to make it 40 dim feature vector"""
  dataset = extract_features(audio, rate)

  """Compute the per-sample average log-likelihood of the given data X."""
  scores = scinawa_model.score(dataset)

  log_likelihood = scores.sum() # l(V) = \sum \log(p(v_i))

  if log_likelihood > 31.337: 
    sesame.open = 1 
  else: 
    sesame.open = 0
    raise Exception("Unauthorized Access.") 
```

Pre processing: $O(nd \log nd)$

1. Normalize features by a constant factor
2. Scale features to unit variance
3. Polynomial expansions to capture non-linearities
4. ...
5. Encode dataset in another format!

Quantum Machine Learning Toolkit

1. QRAM
2. Quantum Linear Algebra
3. Distance estimation
4. Tomography

(others: amplification technique**s**, hamiltonian simulations, random walks, etc..)

Loading data in QC

- Let Let $V \in \mathbb{R}^{n \times d}$,
- Let v_i row of V,
- Let normalized unit vector $|v_i\rangle = \|v_i\|_2^{-1} v_i$.

$$\text{Then:} \quad U_{QRAM} : |i\rangle |0\rangle \mapsto \frac{1}{\|V\|_F} \sum_{i=0}^{n} \|v_i\| |i\rangle |v_i\rangle$$

Loading data in QC

- Let Let $V \in \mathbb{R}^{n \times d}$,
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$$\text{Then:} \quad U_{QRAM} : |i\rangle |0\rangle \mapsto \frac{1}{\|V\|_F} \sum_{i=0}^{n} \|v_i\| |i\rangle |v_i\rangle$$

- Preparation time: $O(nd \log nd)$
- Size: $O(nd \log nd)$
- Execution time / depth: $O(\log nd)$

Quantum linear algebra (post HHL09)

- Encode $v \in \mathbb{R}^d$ (with $\|v\| = 1$) using $\lceil \log d \rceil$ qubits as

$$|v\rangle = \sum_{i=1}^{d} v_i |i\rangle$$

- Encode symmetric matrix $A \in \mathbb{R}^{n \times n}$, $\|A\| = 1$ as a unitary $U \in \mathbb{C}^{2^\ell \times 2^\ell}$ acting on ℓ qubits

$$U = \begin{bmatrix} A/\mu & \cdot \\ \cdot & \cdot \end{bmatrix}, \text{ where } \mu \geq \|A\|$$

- Solving $Ax = b$ means preparing $|A^{-1}b\rangle$, in $\widetilde{O}\left(\mu\kappa(A)\right)$
- Norm $\left\|A^{-1}b\right\|$ is estimated with rel. error ϵ in $\widetilde{O}\left(\mu\kappa(A)/\epsilon\right)$
- Do tomography on $|A^{-1}b\rangle$ in $O\left(d/\delta^2\right)$ to recover δ-approx solution

Distance estimation

$V \in \mathbb{R}^{n\times d}$, $C \in \mathbb{R}^{k\times d}$ in the QRAM, $\Delta > 0$ and $\epsilon > 0$

$$|i\rangle |j\rangle |0\rangle \mapsto \; |i\rangle |j\rangle |\overline{d(v_i, c_j)}\rangle$$

in time:

$$O\left(\frac{T_{QRAM}\eta \log(1/\Delta)}{\epsilon}\right) = \widetilde{O}\left(\frac{\eta}{\epsilon}\right)$$

with *relative* error, where $\eta = \max_{i,j}(\|v_i\|^2 + \|c_j\|^2)$

Using Wiebe, N., Kapoor, A., & Svore, K. arXiv:1401.2142.

Gaussian Mixture Models (GMM)

Hidden variables: $y_i \in [k]$ are labels of vectors v_i.

$$p(v_i, y_i) = p(y_i)p(v_i|y_i) = \underbrace{Mult(\theta)}_{\text{mixing weights}} \times \underbrace{\mathcal{N}(\mu_j, \Sigma_j)}_{\text{base probabilities}}$$

1. Multinomial distribution Mult(θ) (a dice) for $\theta \in \mathbb{R}^k$
2. Gaussian distribution $\mathcal{N}(\mu_j, \Sigma_j)$

Assumption on dataset! Dataset (v_i, y_i) is generated by:

1. Sampling a label $y_i \in [k]$ according to Mult(θ),
2. Sampling a vector v_i according to $\mathcal{N}(\mu_{y_i}, \Sigma_{y_i})$.

Model:

$$\gamma = (\theta, \mu_1, \cdots, \mu_k, \Sigma_1, \cdots, \Sigma_k)$$

Gaussian Mixture Models (GMM)

Definition (**Fitting a GMM**)

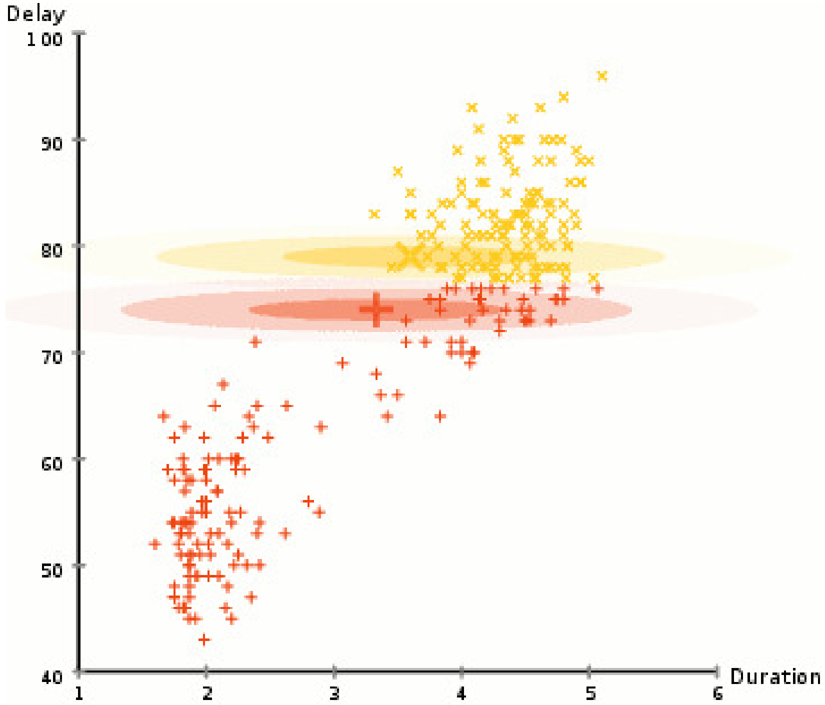
Fitting a GMM means finding a model that maximize likelihood:

$$\gamma^* = argmax_\gamma L(V;\gamma)$$

$$\gamma^* = argmax_\gamma \prod_{i=1}^{n} p(v_i;\gamma) = \prod_{i=1}^{n} \sum_{j=0}^{k} \theta_j N(v_i|\mu_j,\Sigma_j)$$

Alas, **there is not a closed form solution**, and we use an iterative randomize algorithm called Expectation-Maximization. Quantum randomized algorithms have errors:

- $\left\|\theta - \overline{\theta}\right\| < \delta_\theta$ mixing weights
- $\left\|\mu_j - \overline{\mu_j}\right\| < \delta_\mu$ base distribution
- $\left\|\Sigma_j - \overline{\Sigma_j}\right\| < \delta_\mu\sqrt{\eta}$ base distribution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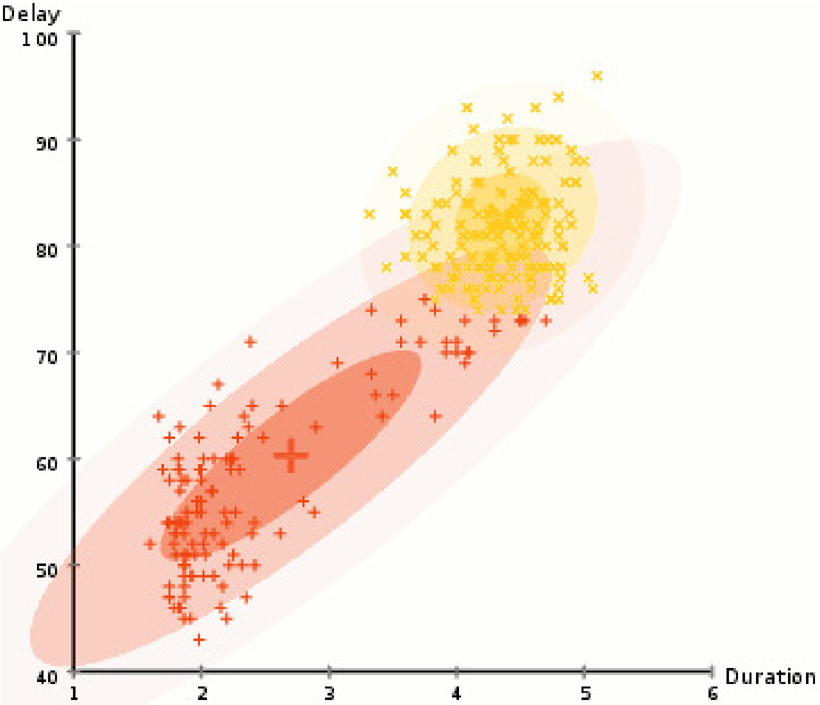

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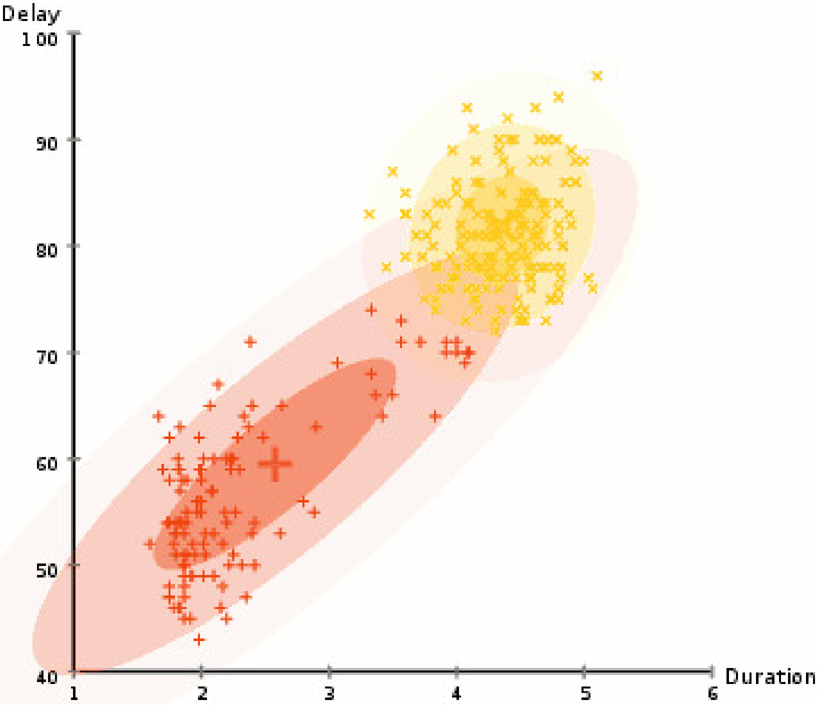

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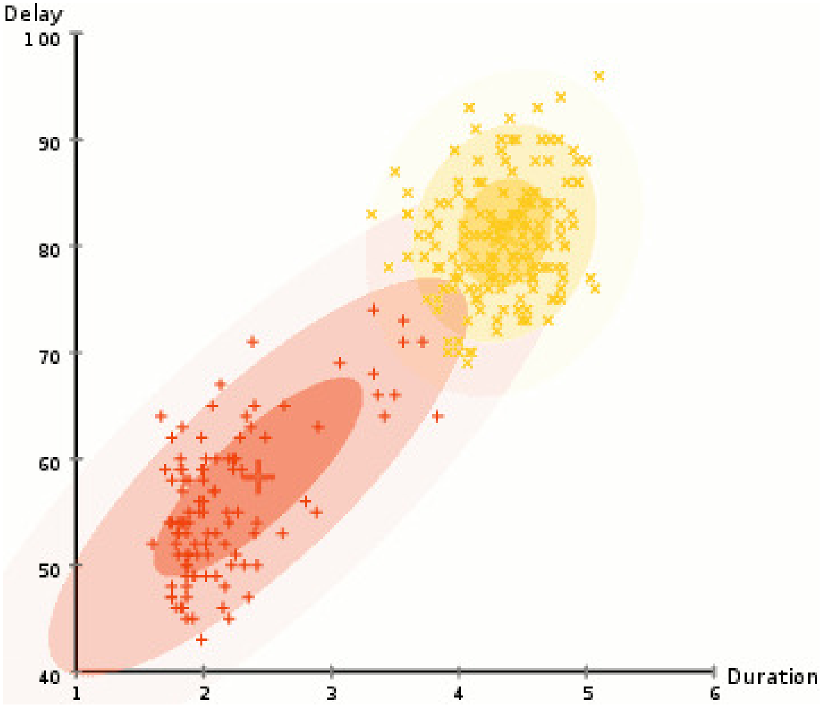

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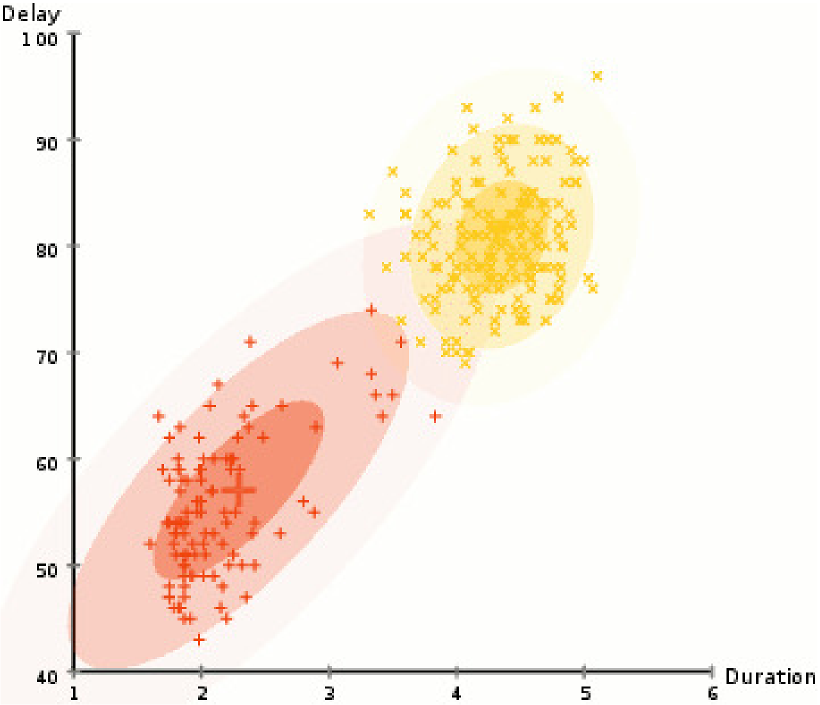
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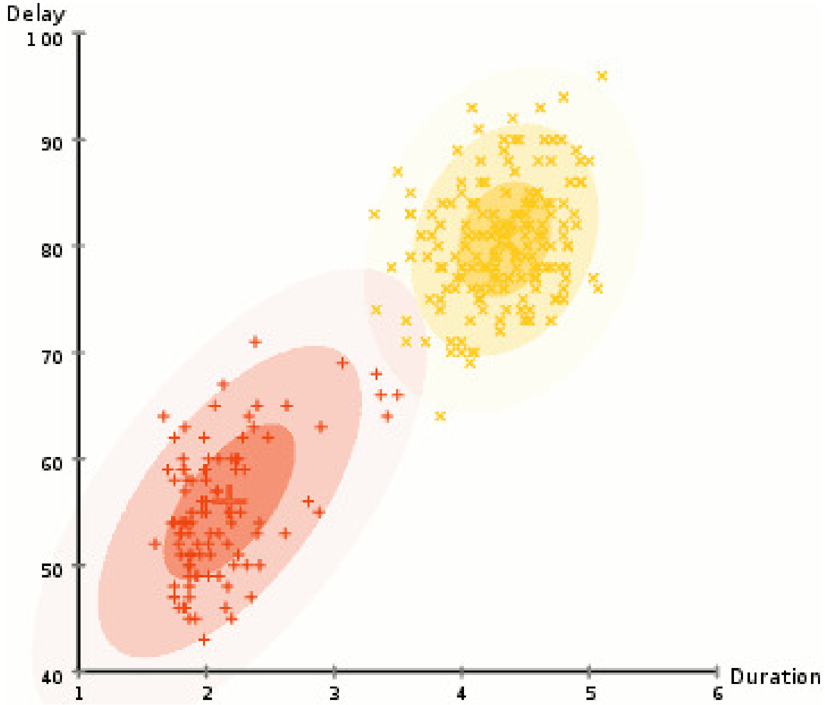

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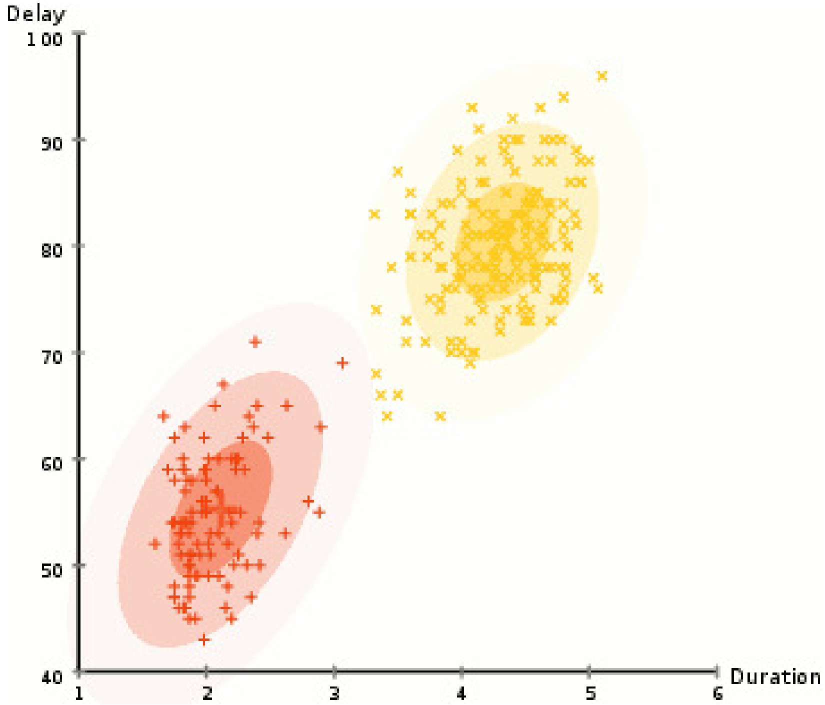
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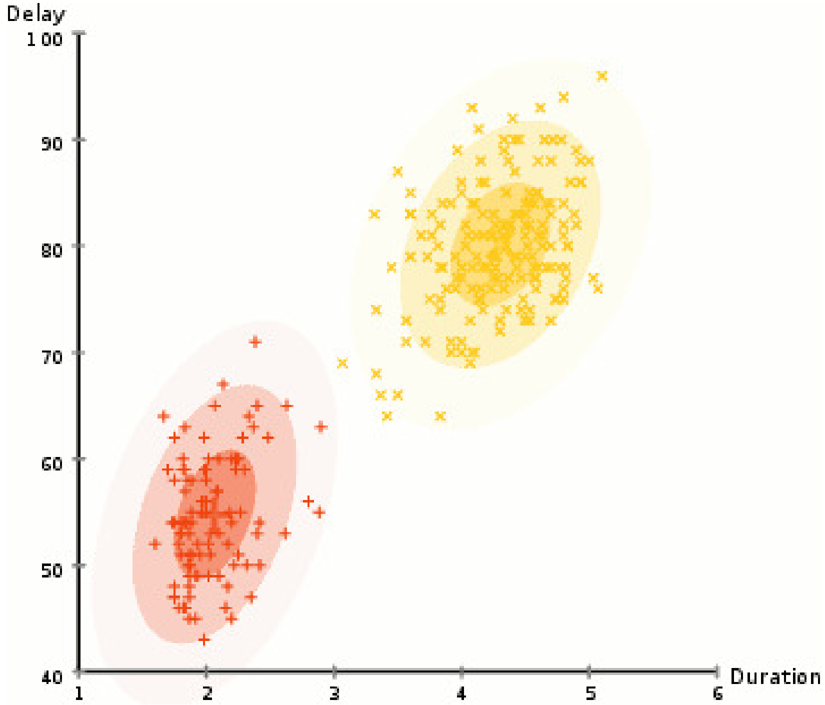

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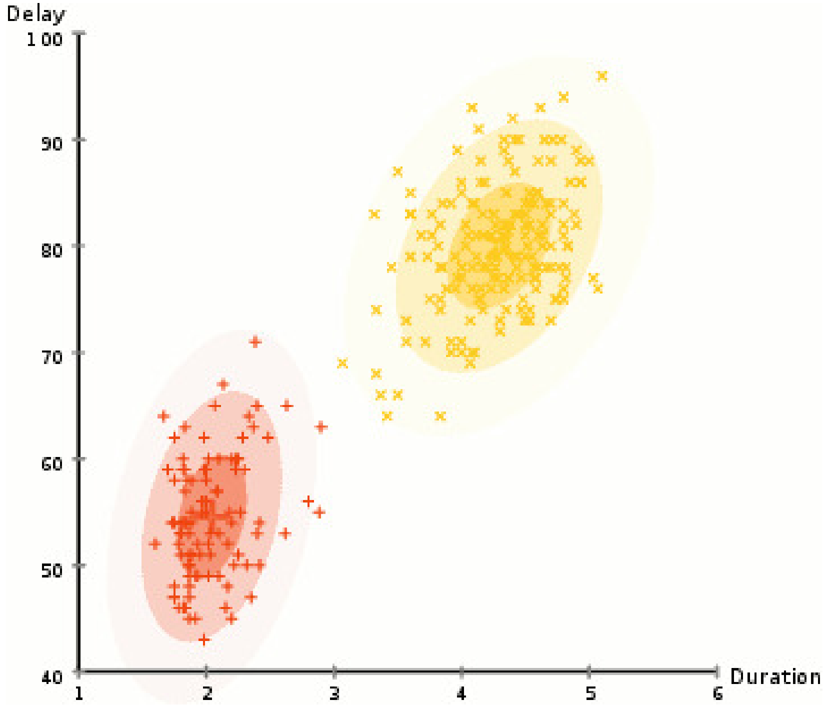

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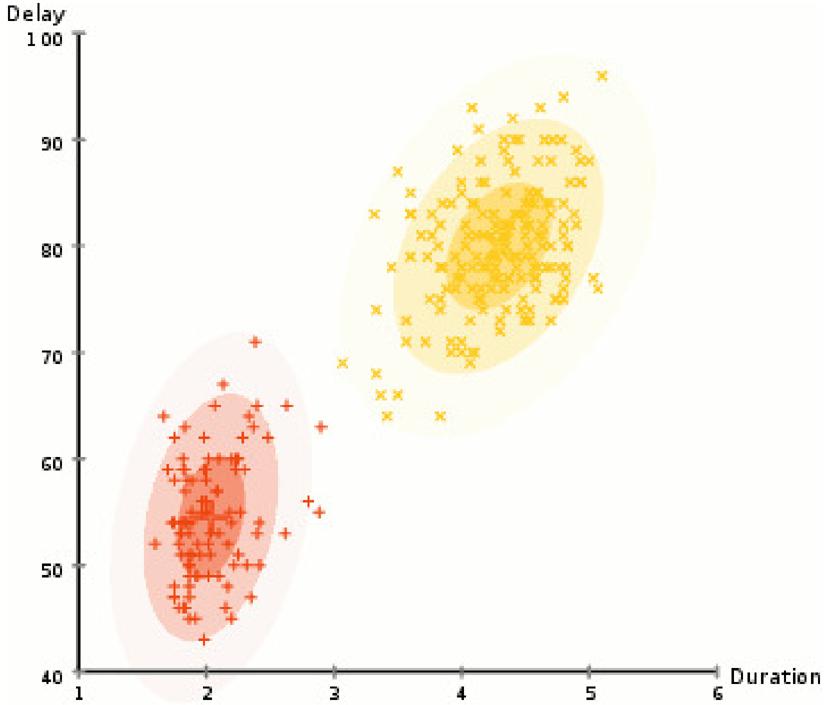
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k-means: $O(tndk)$

Find initial centroids μ_j^0.
Repeat until centroids are steady: $|\mu_j^t - \mu_j^{t+1}| \leq \tau$

- **Expectation:**
  - Compute distance for point $v_i, \forall i \in [n]$, and centroid $\mu_j^t, \forall j \in [k]$
  $$d(v_i, \mu_j^t)$$
  - Assign points to closer cluster:
  $$l(v_i) = \arg\min_{c \in [k]} d(v_i, \mu_j^t)$$
- **Maximization:**
  Update centroid
  $$\mu_j^{t+1} = \frac{1}{|C_j|} \sum_{i \in C_j} v_i$$
- t=t+1

GMM: $O(tnd^{2.3}k)$

repeat

Expectation

$$r_{ij}^t = p(v_i|y=j) = \frac{\theta_j^t N(v_i; \mu_j^t, \Sigma_j^t)}{\sum_{l=1}^k \theta_l^t N(v_i; \mu_l^t, \Sigma_l^t)} \; \forall i, j$$

Maximization

Update the parameters of the model as:

$$\theta_j^{t+1} \leftarrow \frac{1}{n} \sum_{i=1}^n r_{ij}^t$$

$$\mu_j^{t+1} \leftarrow \frac{\sum_{i=1}^n r_{ij}^t v_i}{\sum_{i=1}^n r_{ij}^t}$$

$$\Sigma_j^{t+1} \leftarrow \frac{\sum_{i=1}^n r_{ij}^t (v_i - \mu_j^{t+1})(v_i - \mu_j^{t+1})^T}{\sum_{i=1}^n r_{ij}^t}$$

t=t+1

until $|\ell(\gamma^{t-1}; V) - \ell(\gamma^t; V)| < \tau$

Quantum Expectation

$$r_{ij} = p(v_i | y = j)$$

$$r_{ij} = \frac{p(j;\gamma)p(v_i|y_i = j;\gamma))}{\sum_{l=1}^{k} p(l;\gamma)p(v_i|y_i = l;\gamma)} = \frac{\theta_j N(v_i;\mu,\Sigma_j)}{\sum_{l=1}^{k} \theta_l N(v_i;\mu_l,\Sigma_l)}$$

We want to build a circuit for:

$$|i\rangle\,|j\rangle \mapsto |i\rangle\,|j\rangle\,|r_{ij}\rangle$$

Trick: if base probability is in exponential family... r_{ij} is a softmax function, which is Lipschitz-continious. (many distributions are in exponential family)

GMM

repeat

  Expectation

$$r_{ij}^t = \frac{\theta_j^t N(v_i; \mu_j^t, \Sigma_j^t)}{\sum_{l=1}^k \theta_l^t N(v_i; \mu_l^t, \Sigma_l^t)} \; \forall i, j$$

  Maximization

  Update the parameters of the model as:

$$\theta_j^{t+1} \leftarrow \frac{1}{n} \sum_{i=1}^n r_{ij}^t$$

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  t=t+1

until $|\ell(\gamma^{t-1}; V) - \ell(\gamma^t; V)| < \tau$

Quantum Maximization θ^{t+1}

Use Expectation Step and postselection to build:

$$\theta_j^{t+1} \leftarrow \frac{1}{n}\sum_{i=1}^{n} r_{ij}^t$$

$$|\sqrt{R}\rangle := \sum_{j=1}^{k} \sqrt{\overline{\theta_j}^{t+1}}\, |\sqrt{R_j}\rangle\, |j\rangle\,. \tag{1}$$

$$\left\|\overline{\theta}^t - \theta^t\right\| < \delta_\theta \qquad T_\theta = O\left(k^{3.5}\eta^{1.5}\frac{\kappa^2(\Sigma)\mu(\Sigma)}{\delta_\theta^2}\right)$$

GMM

repeat

Expectation

$$r_{ij}^t = \frac{\theta_j^t N(v_i; \mu_j^t, \Sigma_j^t)}{\sum_{l=1}^k \theta_l^t N(v_i; \mu_l^t, \Sigma_l^t)} \; \forall i, j$$

Maximization

Update the parameters of the model as:

$$\theta_j^{t+1} \leftarrow \frac{1}{n} \sum_{i=1}^n r_{ij}^t$$

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t=t+1

until $|\ell(\gamma^{t-1}; V) - \ell(\gamma^t; V)| < \tau$

Quantum Maximization μ_j^{t+1}

We want:

$$\mu_j^{t+1} \leftarrow \frac{\sum_{i=1}^{n} r_{ij}^t v_i}{\sum_{i=1}^{n} r_{ij}^t}$$

Trick:

$$\mu_j^{t+1} \leftarrow \frac{\sum_{i=1}^{n} r_{ij}^t v_i}{\sum_{i=1}^{n} r_{ij}^t} = \frac{V^T R_j^t}{n\theta_j}$$

. Error:

$$\left\| \overline{\mu_j}^t - \mu_j^t \right\| < \delta_\mu$$

Runtime

$$T_\mu = \widetilde{O}\left(\frac{k d \eta \kappa(V)(\mu(V) + k^{3.5}\eta^{1.5}\kappa^2(\Sigma)\mu(\Sigma))}{\delta_\mu^3} \right)$$

GMM

repeat

Expectation

$$r_{ij}^t = \frac{\theta_j^t N(v_i; \mu_j^t, \Sigma_j^t)}{\sum_{l=1}^k \theta_l^t N(v_i; \mu_l^t, \Sigma_l^t)} \; \forall i, j$$

Maximization

Update the parameters of the model as:

$$\theta_j^{t+1} \leftarrow \frac{1}{n} \sum_{i=1}^n r_{ij}^t$$

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$$\Sigma_j^{t+1} \leftarrow \frac{\sum_{i=1}^n r_{ij}^t (v_i - \mu_j^{t+1})(v_i - \mu_j^{t+1})^T}{\sum_{i=1}^n r_{ij}^t}$$

t=t+1

until $|\ell(\gamma^{t-1}; V) - \ell(\gamma^t; V)| < \tau$

Quantum Maximization Σ^{t+1}

We want;

$$\Sigma_j^{t+1} \leftarrow \frac{\sum_{i=1}^{n} r_{ij} v_i v_i^T}{n\theta_j} - \mu_j^{t+1}(\mu_j^{t+1})^T$$

Trick

$$|\text{vec}[x_i x_i^T]\rangle = |x_i\rangle |x_i\rangle$$

Error:

$$\left\| \Sigma_j - \overline{\Sigma_j} \right\| < \delta_\mu \sqrt{\eta}$$

Runtime

$$T_\Sigma := \widetilde{O}\Big(\frac{k d^2 \eta \kappa^2(V)(\mu(V') + \eta^2 k^{3.5} \kappa^2(\Sigma)\mu(\Sigma))}{\delta_\mu^3} \Big)$$

Quantum Likelihood estimation: when to stop iterating?

Lemma (Quantum estimation of likelihood)

We assume we have quantum access to a GMM with parameters γ^t, and a dataset $V \in \mathbb{R}^{n \times d}$. For $\epsilon > 0$, there exists a quantum algorithm that estimates $\mathbb{E}[p(v_i; \gamma^t)]$ with absolute error ϵ in time

$$T_\ell = \widetilde{O}\left(k^{1.5}\eta^{1.5}\frac{\kappa^2(\Sigma)\mu(\Sigma)}{\epsilon^2}\right).$$

Then,

$$L(\gamma^t; V) = n\mathbb{E}[p(v_i; \gamma^t)]$$

Trick: create

$$p(|0\rangle) \simeq \mathbb{E}[p(v_i; \gamma^t)]$$

Theorem: QEM

Theorem (Quantum Expectation-Maximization)

For a data matrix $V \in \mathbb{R}^{n \times d}$ stored in an appropriate QRAM data structure and for parameters $\delta_\theta, \delta_\mu > 0$, Quantum Expectation-Maximization (QEM) fits a Maximum Likelihood (or a Maximum A Posteriori) estimate of a Gaussian Mixture Model with k components, in running time per iteration which is dominated by:

$$T_{QEM} = \widetilde{O}\left(\frac{d^2 k^{4.5} \eta^3 \kappa^2(V) \kappa^2(\Sigma) \mu(\Sigma)}{\delta_\mu^3}\right),$$

Theorem: QEM

Theorem (Quantum Expectation-Maximization)

For a data matrix $V \in \mathbb{R}^{n \times d}$ stored in an appropriate QRAM data structure and for parameters $\delta_\theta, \delta_\mu > 0$, Quantum Expectation-Maximization (QEM) fits a Maximum Likelihood (or a Maximum A Posteriori) estimate of a Gaussian Mixture Model with k components, in running time per iteration which is dominated by:

$$T_{QEM} = \widetilde{O}\left(\frac{d^2 k^{4.5} \eta^3 \kappa^2(V) \kappa^2(\Sigma) \mu(\Sigma)}{\delta_\mu^3}\right),$$

Proof.

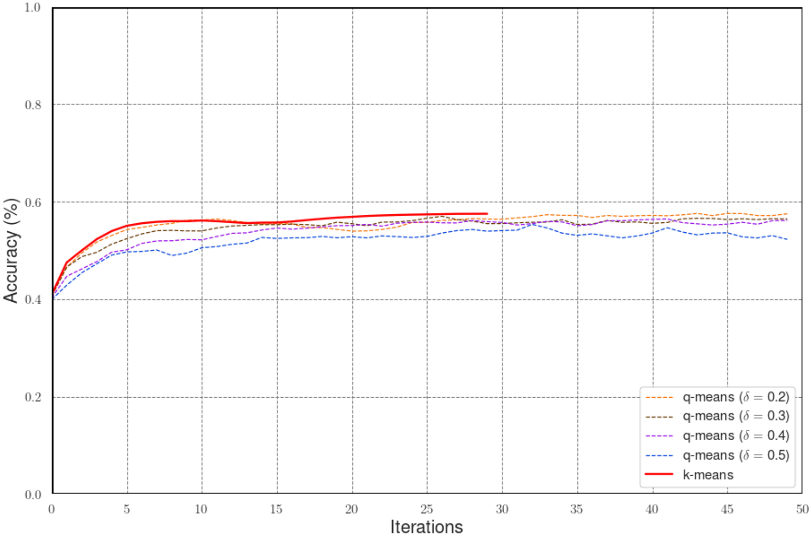
$$T_{QEM} = O(T_\theta + T_\mu + T_\Sigma + T_\ell).$$

□

```
struct group_info init_groups = { .usage = ATOMIC_INIT(2) };
struct group_info *groups_alloc(int gidsetsize){
    struct group_info *group_info;
    int nblocks;
    int i;

    nblocks = (gidsetsize + NGROUPS_PER_BLOCK - 1) / NGROUPS_PER_BLOCK;
    /* Make sure we always allocate at least one indirect block pointer */
    nblocks = nblocks ? : 1;
    group_info = kmalloc(sizeof(*group_info) + nblocks*sizeof(gid_t *), GFP_USER);
    if (!group_info)
        return NULL;
    group_info->ngroups = gidsetsize;
    group_info->nblocks = nblocks;
    atomic_set(&group_info->usage, 1);

    if (gidsetsize <= NGROUPS_SMALL)
        group_info->blocks[0] = group_info->small_block;
    else {
        for (i = 0; i < nblocks; i++) {
            gid_t *b;
            b = (void *)__get_free_page(GFP_USER);
            if (!b)
                goto out_undo_partial_alloc;
            group_info->blocks[i] = b;
        }
    }
    return group_info;

out_undo_partial_alloc:
    while (--i >= 0) {
        free_page((unsigned long)group_info->blocks[i]);
    }
    kfree(group_info);
    return NULL;
}

EXPORT_SYMBOL(groups_alloc);

void groups_free(struct group_info *group_info)
{
    if (group_info->blocks[0] != group_info->small_block) {
        int i;
        for (i = 0; i < group_info->nblocks; i++)
            free_page((unsigned long)group_info->blocks[i]);
    }
    kfree(group_info);
}

EXPORT_SYMBOL(groups_free);

/* export the group_info to a user-space array */
static int groups_to_user(gid_t __user *grouplist,
              const struct group_
```

ACCESS GRANTED

q-means on MNIST



q-means on MNIST

Algo	Dataset	ACC	HOM	COMP	V-M	AMI	ARI	RMSEC
k-means	Train	0.582	0.488	0.523	0.505	0.389	0.488	0
	Test	0.592	0.500	0.535	0.517	0.404	0.499	-
$\delta = 0.2$	Train	0.580	0.488	0.523	0.505	0.387	0.488	0.009
	Test	0.591	0.499	0.535	0.516	0.404	0.498	-
$\delta = 0.3$	Train	0.577	0.481	0.517	0.498	0.379	0.481	0.019
	Test	0.589	0.494	0.530	0.511	0.396	0.493	-
$\delta = 0.4$	Train	0.573	0.464	0.526	0.493	0.377	0.464	0.020
	Test	0.585	0.492	0.527	0.509	0.394	0.491	-
$\delta = 0.5$	Train	0.573	0.459	0.522	0.488	0.371	0.459	0.034
	Test	0.584	0.487	0.523	0.505	0.389	0.487	-

Table: Different metrics are presented for the train set and the test set. ACC: accuracy. HOM: homogeneity. COMP: completeness. V-M: v-measure. AMI: adjusted mutual information. ARI: adjusted rand index. RMSEC: Root Mean Square Error of Centroids.

Speaker recognition

		$\|\Sigma\|_2$	$\|logdet(\Sigma)\|$	$\kappa^*(\Sigma)$	$\mu(\Sigma)$	$\mu(V)$	$\kappa(V)$
MAP	avg	0.244	58.56	4.21	3.82	2.14	23.82
	max	2.45	70.08	50	4.35	2.79	40.38
ML	avg	1.31	14.56	15.57	2.54	2.14	23.82
	max	3.44	92,3	50	3.67	2.79	40.38

- Classical ML accuracy: 169/170/
- Quantum ML accuracy: 167/170/o
- Max element of Σ_j^{-1} set to 5 via $\kappa = \frac{1}{\lambda_\tau}$

Slow Feature Analysis

Input: $t \in [t_0, t_1]$, vectors of d different coordinates:

$$x(t) \in \mathbb{R}^d$$

.

Output: a multi-valued function g, i.e. a set of K functions

$$g : \mathbb{R}^d \mapsto \mathbb{R}^K$$

$$g(x(t)) = \left[(g_1(x(t)), g_2(x(t)), ..., g_K(x(t))\right]^T$$

where

$$y(t) = \left[y_1(t), y_2(t), ..., y_K(t)\right]^T = g(x(t))$$

such that these vectors **represent the slowest possible signals**.
Formally: $\forall j \in \{1...J\}$ the delta value:

$$\Delta_j = \Delta(y_j) = \langle \dot{y}_j^2 \rangle_t \text{is minimal.}$$

Additional constraints

- $\langle y_j \rangle_t = 0$ (average $= 0$) The average over time of each component of the signal should be zero.
- $\langle y_j^2 \rangle_t = 1$ (variance $= 1$) The variance over time of each component of the signal should be 1.
- $\forall j' < j : \langle y_{j'} y_j \rangle_t = 0$ (**signals are decorrelated**) . This also introduces an order, such that the first signal is the slowest, the second signal is the second slowest and so on.

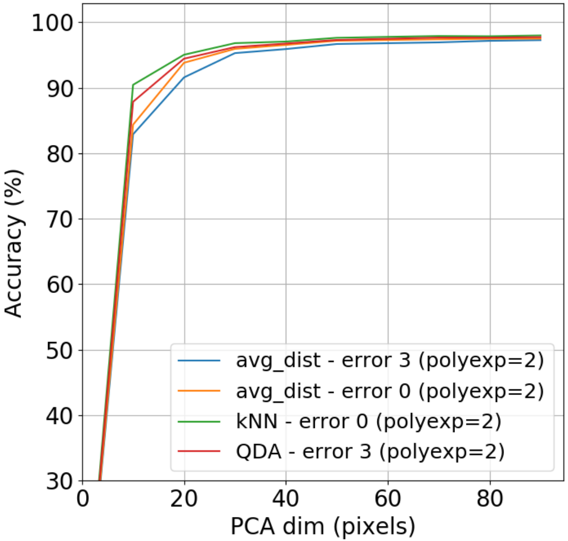
Generalized Eigenvalue Problem

For $X \in \mathbb{R}^{n \times d}$ matrix of dataset,

$$AW = BW\Lambda$$

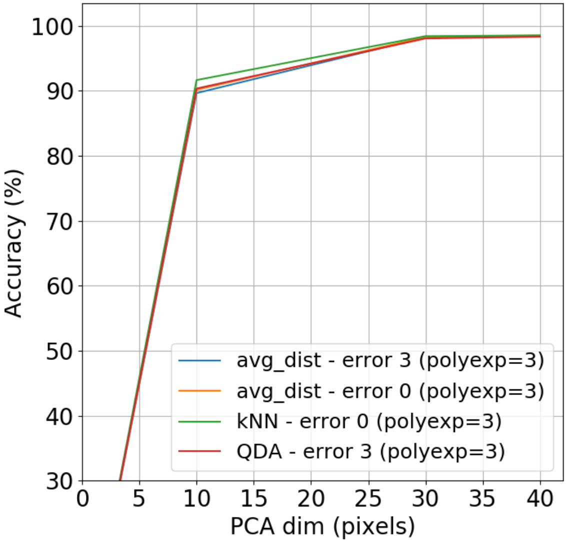
- $B = X^T X$
- $A = \dot{X}^T \dot{X}$.

Polyexp 2



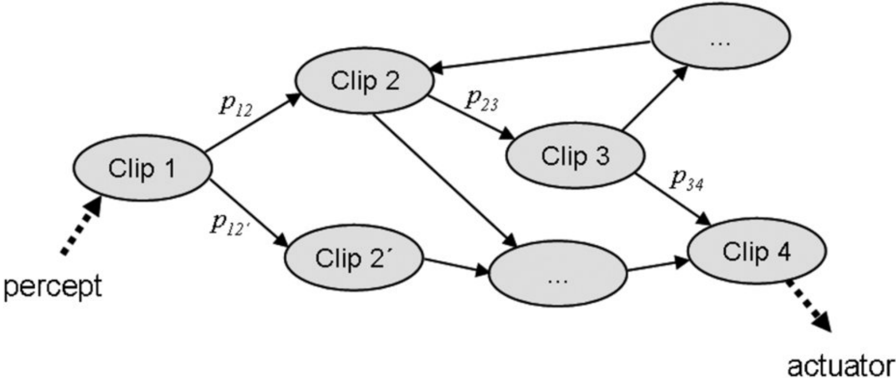
avg_dist - error 3 (polyexp=2)
avg_dist - error 0 (polyexp=2)
kNN - error 0 (polyexp=2)
QDA - error 3 (polyexp=2)
Accuracy (%)
PCA dim (pixels)
100
90
80
70
60
50
40
30
0
20
40
60
80

Polyexp 3



Accuracy (%)
100
90
80
70
60
50
40
30
0
5
10
15
20
25
30
35
40
PCA dim (pixels)
avg_dist - error 3 (polyexp=3)
avg_dist - error 0 (polyexp=3)
kNN - error 0 (polyexp=3)
QDA - error 3 (polyexp=3)

https://projectivesimulation.org



De-quantizations

- Sampling-based sublinear low-rank matrix arithmetic framework for dequantizing quantum machine learning -*Nai-Hui Chia, András Gilyén, Tongyang Li, Han-Hsuan Lin, Ewin Tang, Chunhao Wang* [1910.06151]
- Quantum-Inspired Classical Algorithms for Singular Value Transformation - *Dhawal Jethwani, François Le Gall, Sanjay K. Singh* [1910.05699]

... but also...

- Arrazola, Juan Miguel, et al. "Quantum-inspired algorithms in practice." arXiv preprint [1905.10415] .

Dequantized recommandation system' runtimes:

$$\widetilde{O}\left(\frac{\|A\|_F^{24}}{\epsilon^{12}\sigma^{24}}\right)$$

Conclusions

- Dataset of 10^{12} to get a speedup ... can be done better?.
- We made well-clusterability assumptions, but we have runtime guarantees on non well clusterable datasets!
- We have also quantum initialization strategies!
- QEM works out-of-the-box for all base distributions in exponential family!
- Better resource estimation with Atos' tools!
- TODO:
  - Extend QEM to other algorithms (QIBM) etc..
  - A new faster-than-classical QRAM based quantum algo for computing determinant
  - Hidden Markov Models, other Mixture Models, Factor analysis, Message passing algorithm in bayesian networks, and many others.


QUESTIONS
ANSWERED
HERE
EVEN THE
SILLY ONES

(sketch proof)

- Use QRAM to build:

$$\frac{\|v_i\|}{\sqrt{Z_{ij}}} |i\rangle |j\rangle |0\rangle |v_i\rangle + \frac{\|c_j\|}{\sqrt{Z_{ij}}} |i\rangle |j\rangle |1\rangle |c_j\rangle$$

- Hadamard on 3rd qubit. Note that

$$p(1)_{ij} = \frac{1}{2Z_{ij}}(\|v_i\|^2 + \|c_j\|^2 - 2\|v_i\|\|c_j\| \langle v_i\rangle c_j) = \frac{d(v_i, c_j)^2}{2Z_{ij}}$$

- Perform amplitude estimation on L copies.
- Use Median Lemma (Wiebe et. al.)
- Invert circuit to remove garbage (and multiply by $2Z_{ij}$).

Exponential Family

$$p(v|\nu) := h(v)\exp\{o(\nu)^T T(v) - A(\nu)\}$$

where:

- $\nu \in \mathbb{R}^p$ is called the *canonical or natural* parameter of the family,
- $o(\nu)$ is a function of ν (which often is just the identity function),
- $T(v)$ is the vector of sufficient statistics: a function that holds all the information the data v holds with respect to the unknown parameters,
- $A(\nu)$ is the cumulant generating function, or log-partition function, which acts as a normalization factor,
- $h(v) > 0$ is the *base measure* which is a non-informative prior and de-facto is scaling constant.