Projet Pack Quantique

Région îledeFrance

GAUPRO

Portfolio risk assessment GAUssian PROcesses

Proposé par

NATIXIS CORPORATE AND INVESTMENT BANKING

GROUPE BPCE

MULTIVERSE COMPUTING

Avec l'accompagnement du

GENCI

Le calcul intensif au service de la connaissance

GAUPRO PARTNERS

NATIXIS CORPORATE AND INVESTMENT BANKING

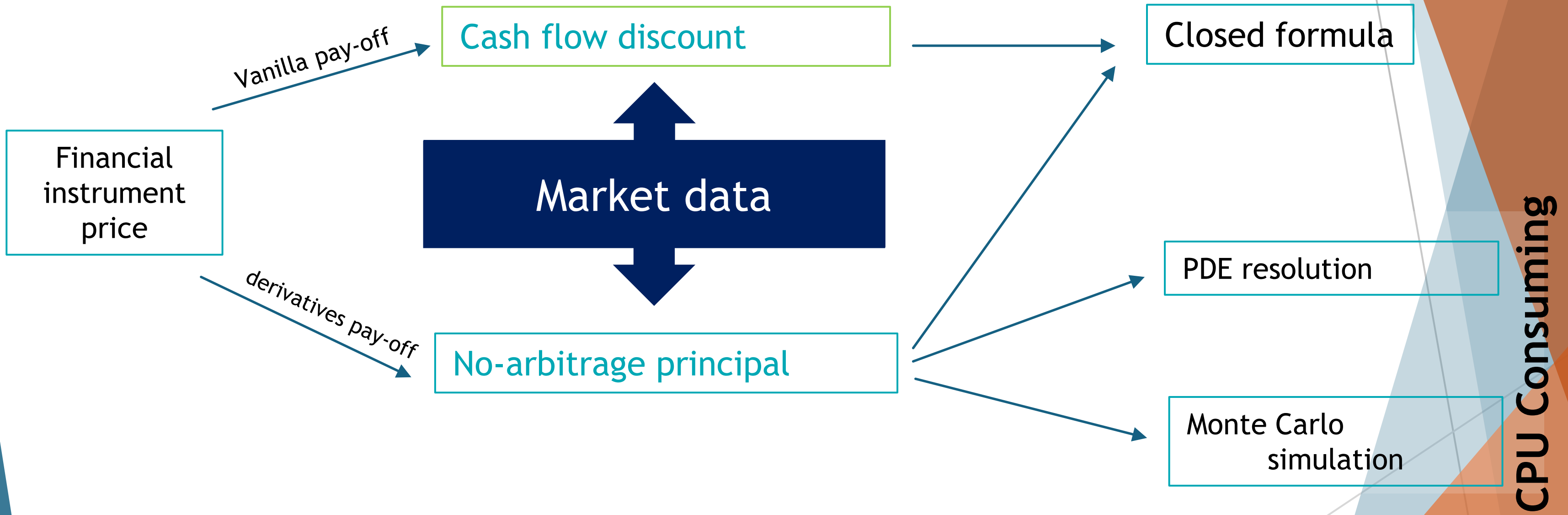
Natixis CIB is a subsidiary of Groupe BPCE, France's second-largest banking group, and acts as the group's investment, corporate, insurance and financial services bank. Natixis provides services in three main areas: retail banking, asset & wealth management and insurance.

MULTIVERSE

European leader in the development of quantum and quantum-inspired software

Pricing of Financial pay-off

- **Overview of pricing methods**



With the no-arbitrage principal the model may consume a lot of CPU (greater than 1s)

Risk measures

▶ Different type of risk measures

The pricing model are used for theses principals risk measures

Sensitivities

Are the responsiveness to changes in external factors, such as interest rates, market conditions, or other relevant variables. It measures the degree to which the product's value or performance is influenced by changes in these factors

Value at risk (VaR)

is a statistical measure used to quantify the level of financial risk within a firm or a portfolio over a specific time frame. It represents the maximum potential loss.

Counterparty Credit Risk

Is the risk that a counterparty in a financial transaction will default on its obligations. This risk arises in various financial transactions, such as derivatives, loans, and other forms of credit exposure. Financial institutions and market participants assess and manage counterparty credit risk to minimize potential losses and ensure the stability of their operations.

Stress test

Is a simulation or analysis conducted to evaluate the potential impact of adverse events or market shocks on a financial institution, investment portfolio,

Theses measures call for an instruments several times the pricer.

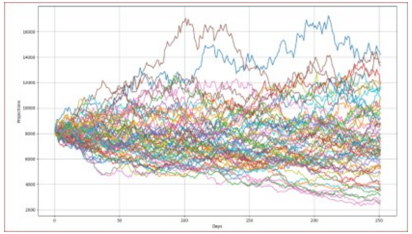
- **Example**: 1000 calls of a pricer with 30s response time need almost ~8H

Risk measures

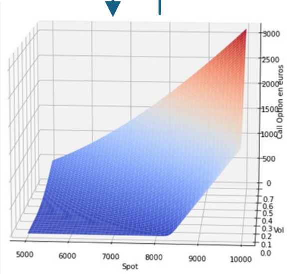
▶ GPR in a nutshell

Full pricing | **Transformation** | **Alternative pricing**

Market data context



Pricing model

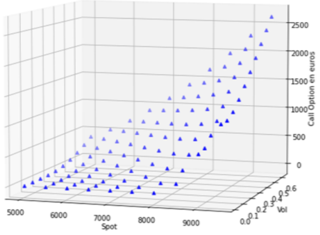


Multiple call of the pricing function of the pay-off (CPU consuming)

Idea: Replace the price function by another function calibrated minimising some error. The procedure consists in two phases:

Offline phase: Parameters calibration using some input data.
Online phase: Regression evaluation in input data possibly different from the ones used in the calibration.

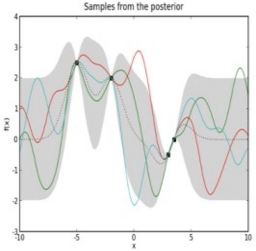
sampling



Choose an optimal sampling of the price function

Construction of a cube with a sample of price

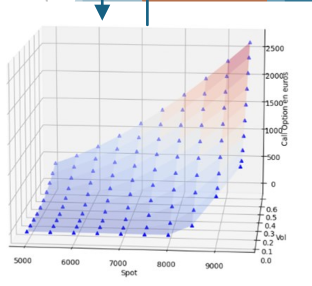
+



Calibrate a Gaussian regression process on the sample

Construction of a cube with a sample of price

smoothing



Multiple call of the interpolated price function based on a GPR (<<0,5s)

This approach has a high dimensionality on classical computer but seem a way to improve the risk measure

Related Work

Market Risk Assessment of a trading book using Statistical and Machine Learning

Noureddine Lehdili, Pascal Oswald and Harold Gueneau (*)

Natixis, Enterprise Risk Management Market and Counterparty Risk Modelling, 30, avenue Pierre Mendes-France, 75013 Paris. (*) AiYO - 3, Rue Bellini, 92800 Puteaux

Emails : noureddine.lehdili@natixis.com ; pascal.oswald@natixis.com; harold.gueneau@aiyo.fr

Abstract

Machine Learning algorithms have received a lot of interest in recent years thanks to the recent increases in computing power coupled with the availability and quantity of data in business applications ([1], [2]). In this field, many solutions have already implemented and many more being studied and explored (cf. [3], [4], [5], [6]). In finance, these algorithms are deployed for many applications ([7], [8], [9], [10]) as pricing, fraud detection, credit scoring, portfolio management, etc. In the same time, a recent study ([11]) outlines that even if several applications of machine learning dedicated to the risk management in banks and financial institutions such as credit risk, market risk, operational risk and liquidity risk has been explored, many areas in financial risk management that could significantly benefit from the power of machine learning techniques to address specific problems. Based on this observation and the fact that recently the internal models of banks are questioned by regulators (TRIM[1], FRTB[2]) that request to upgrade and to improve the underlying quantitative risk methodologies and to implement others new quantitative rules (FRTB), we are interested in how machine learning algorithms can help banks to complete the probate process of their internal models used to compute the regulatory capital. To address this question, it worth to focus on the more challenging aspect of market risk measurement task such as the practical implementation of the value-at-risk and the expected shortfall models of the trading portfolio of the bank. Given the important number of risk factors behind the trading portfolio, these risk models are implemented using several proxies and simplistic assumptions for the performance purposes. These limitations are largely due to the valuation models that cannot in general directly used to revalue all derivatives instruments in all simulation's scenarios since they are more time consuming. It is therefore quite legitimate to wonder whether it exits a robust machine learning algorithm able to repeatedly valuate a reasonable computing time a large portfolio of derivatives instruments. The recent research indicates that the answer is yes, and the Bayesian Gaussian processes could be the appropriate tool when the problem is well-formulated.

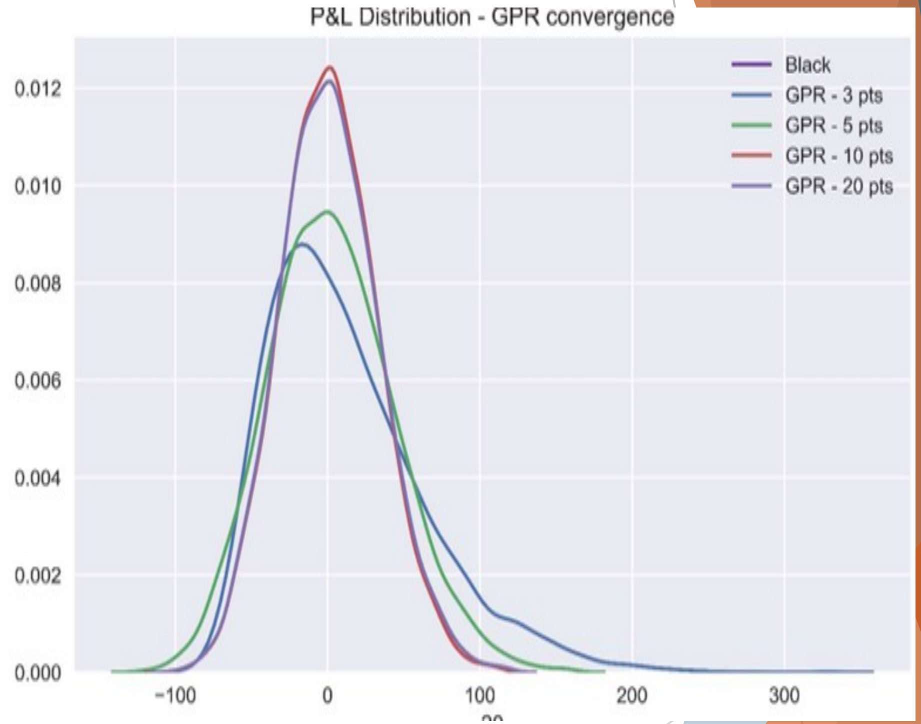
In this paper, we roll out Gaussian processes (GP), a powerful algorithm for both regression and classification, to tackle the revaluation performance problem of the whole derivatives securities trading portfolio. To do this, the GP algorithm must learn the valuation task of all pricing models by training it directly on a data set generated by sophisticated pricing models. This is possible since we postulate that the pricing models are available and used only for the valuation purposes. Then, the trained GP algorithm takes over to carry out the repeated revaluation of the derivatives security's portfolio in a fast and efficient way. Notice here that no need to learn to the GP algorithm to separately revalue each derivative security composing the portfolio. Nevertheless, the whole portfolio can be rearranged, if necessary, in terms of sub portfolios and the risk factors nature to parallelize the training stage. The numerical tests show that the Gaussian process regression (GPR) can drastically improve the computing time whilst ensuring an excellent level of accuracy. For example, the 99% Value a Risk over one day horizon computed with one million of simulations is achieved with around five seconds. In addition, the speed-up of the calculation of the VaR and the expected shortfall within the GPR framework is independent of the size and the composition of the trading portfolio, which is spectacular. For this, it seems to us that it is more profitable for the Banks to build their risk models using the powerful of the GPR techniques in terms of the calculation accuracy and the speed-up.

Keywords: Market Risk Management, Value-at-Risk, Expected Shortfall, Machine Learning, Regression, Classification, Gaussian processes, Kernel function, Monte Carlo simulations, Convex Optimization, Conditional Probabilities, Bayes theorem, Pricing models.

Declaration of interest: *The views expressed in this paper are solely those of the authors and do not necessarily reflect the views and policies of Natixis*

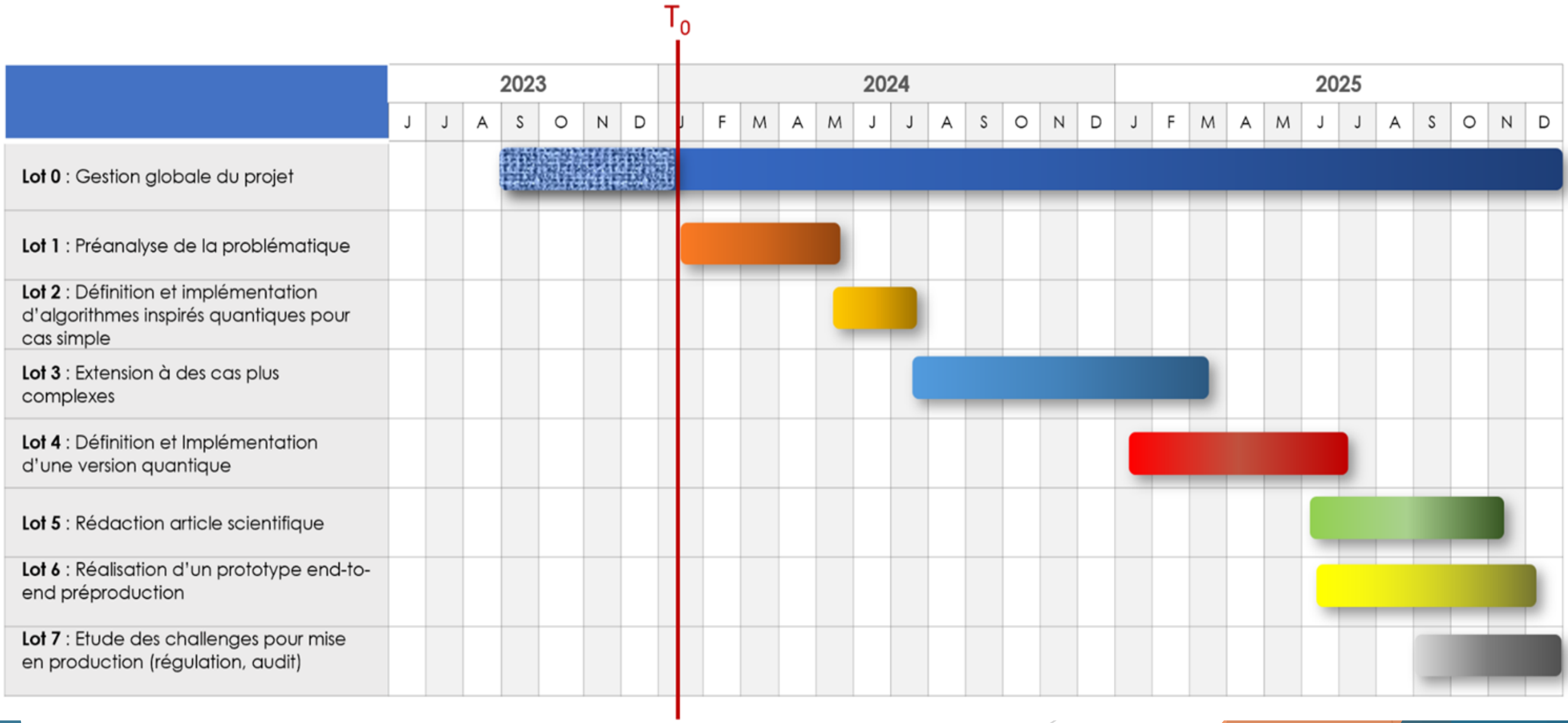
[1] The target review of internal models, or TRIM, is an European Central Bank (ECB) project to assess whether the internal models currently used by banks comply with regulatory requirements, and whether their results are reliable and comparable. One major objective of TRIM is to reduce inconsistencies and unwarranted variability when banks use internal models to calculate their risk-weighted assets (a commonly used regulatory metric that "weighs" a bank's assets based on their riskiness and constitutes a key factor in determining the bank's own funds requirements).

NATIXIS
CORPORATE AND INVESTMENT BANKING

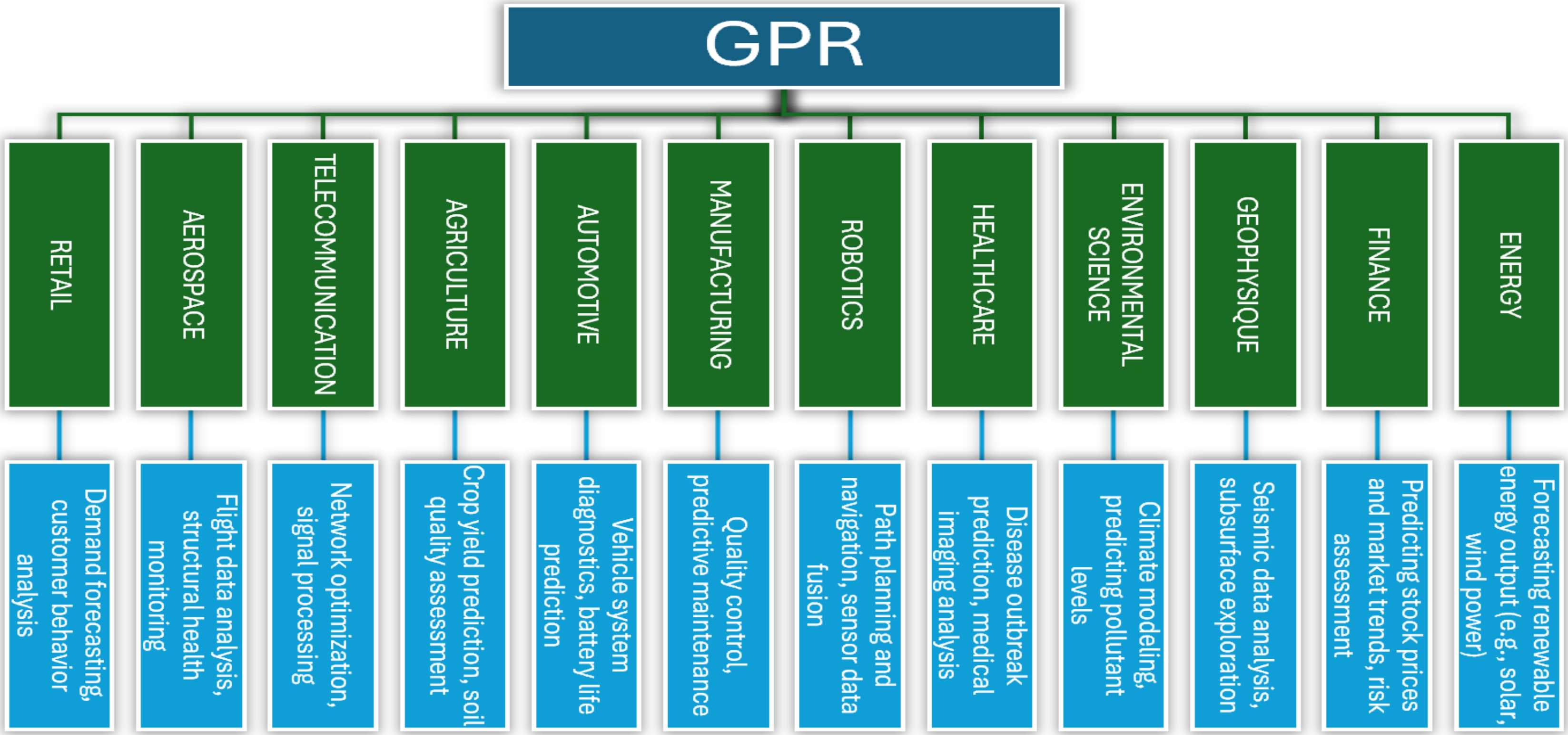


GPR algorithm has time complexity $O(N^3)$ where N is size of the training sample

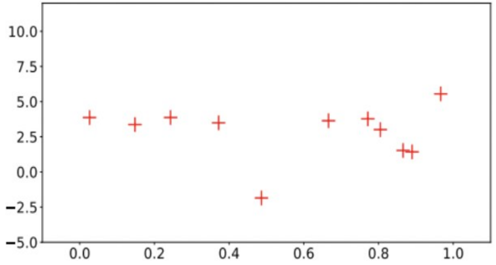
Modified project timeline

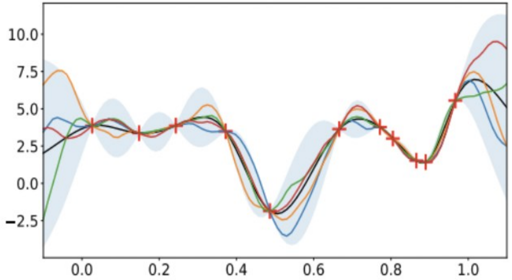


Multifaceted Applications of GPR in Industries



Bottlenecks for implementing Gaussian Processes



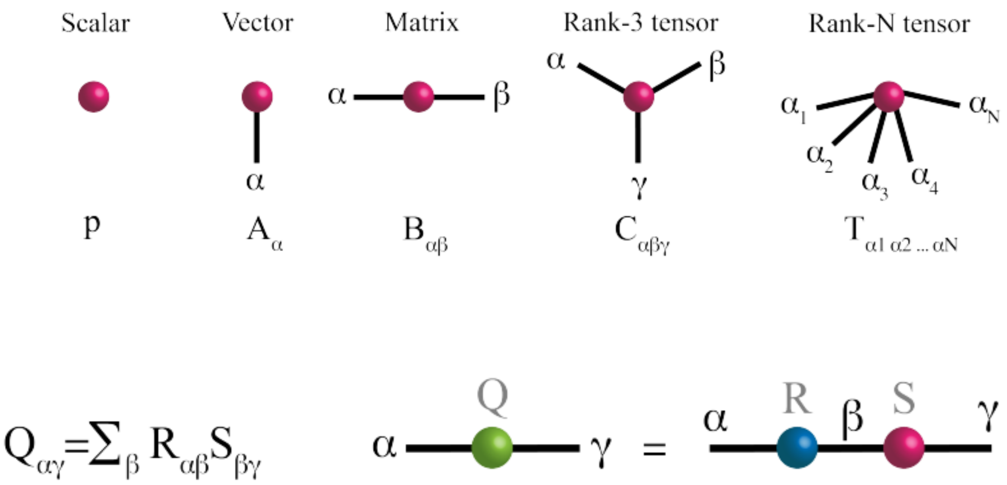


Training a GPR and/or making inferences with the trained model requires inverting the covariance matrix. The overall computation complexity is $O(N^3)$, where N is the number of datapoints/observations, and the memory consumption is quadratic.

➔Computational complexity and memory usage are clear bottlenecks for implementing Gaussian Processes (GP) with big datasets. Sparse GP can be used to tackle regression tasks with big datasets, but it often requires doing some approximation to the kernel function.

Defining Quantum-Inspired Algorithms network with Tensor Networks

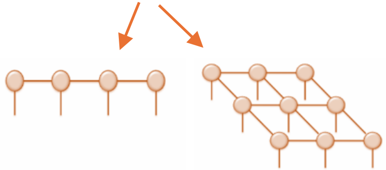
Tensor Networks (TN) are mathematical decompositions of highly correlated structures



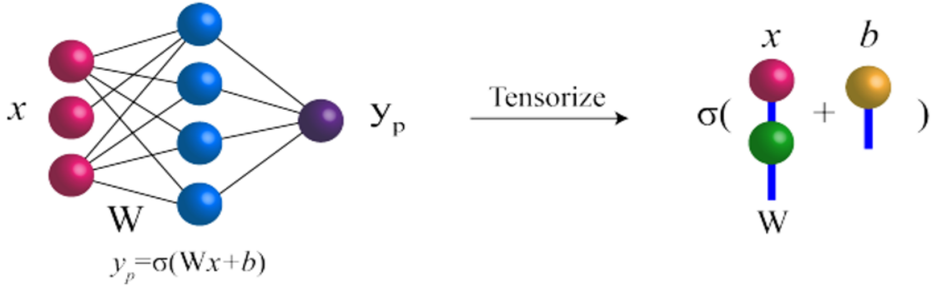
Defining Quantum-Inspired Algorithms network with Tensor Networks

- TN are widely used by physicists for computing properties of quantum and classical systems
- They reduce dimensionality and can be used to tackle quantum mechanics simulation problems

$$|\Psi_{\text{many body}}\rangle = \sum_{\alpha_1,\alpha_2,..,\alpha_n}^{n} \Psi_{\alpha_1,\alpha_2,...,\alpha_n} |\alpha_1, \alpha_2, \ldots, \alpha_n\rangle$$



- Or optimization , AI , … among others



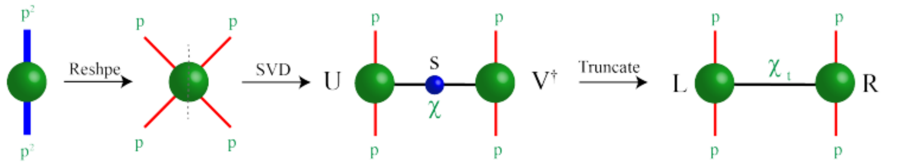
Possible approaches to reduce the dimensionality Quantum inspired enhanced GPR

Training a GPR and/or making inferences with the trained model requires inverting the covariance matrix, which is a costly process.

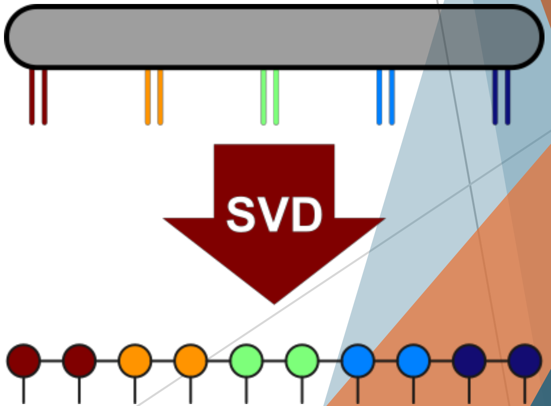
We can make this computation without explicitly inverting the matrix A with TN, by solving the linear equation A x = f.

1. Represented the covariance matrix with a Tensor Network.
2. Use TN algorithms (DMRG) to solve the associated variational problem

$$x^* \longrightarrow \text{argmin}(\, x;\ ||A\, x - f||^2\,)$$

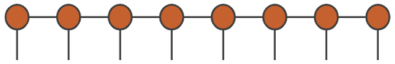


MPS/TT representation

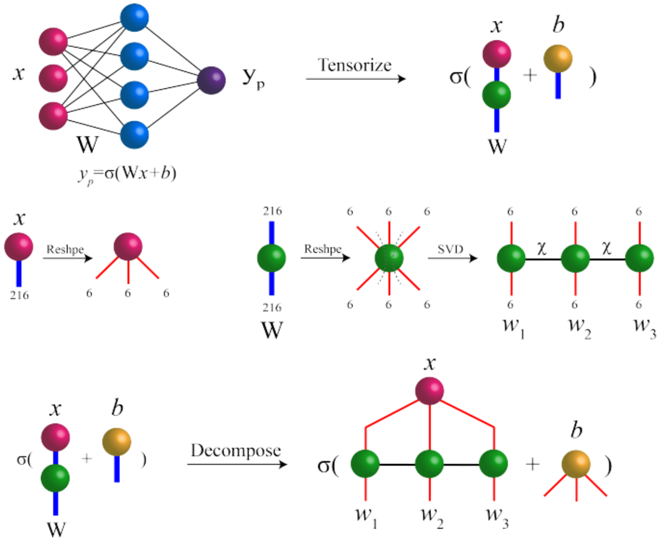


Possible approaches to reduce the dimensionality
Neural Kernel MPS

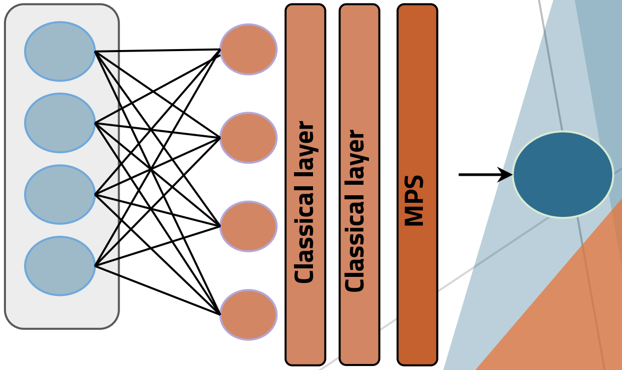
The Gaussian Process can be approximated with an MPS



Since the representation power of pure tensor networks is limited to linear functions, extra layers are added in order to introduce non linearities. The layer neural network is used as the kernel function, which is then fed as input to the MPS.



Neural Kernel MPS



Thank you !